OBESITY EXPERTISE

A first-principles curriculum in 196 books

BOOK 0 1st of 196 in the series

Ground floor

Numbers, units, change, uncertainty, the body's chemistry, and reading evidence

THE FOUNDATION EVERY LATER BOOK STANDS ON

Dr. Harsh Maheshwari

MD Community Medicine Resident, AIIMS Raipur

drharshmaheshwari.com · version 1.2

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Numbers, units, change, uncertainty, the body's chemistry, and reading evidence

Dr. Harsh Maheshwari

MD Community Medicine Resident, AIIMS Raipur

contact@drharshmaheshwari.com · drharshmaheshwari.com · ORCID 0000-0001-8848-1918

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CONTACT	contact@drharshmaheshwari.com
ORCID	https://orcid.org/0000-0001-8848-1918
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NOT ADVICE

This book is for learning. It is not medical, legal or financial advice, and it does not replace a qualified professional who knows your situation.

About the author



Dr. Harsh Maheshwari

MD Community Medicine Resident, AIIMS Raipur

Harsh Maheshwari completed his MBBS at King George's Medical University (KGMU), Lucknow, and is a resident in MD Community Medicine at the All India Institute of Medical Sciences (AIIMS), Raipur. His MD thesis, on chrononutrition and obesity, was selected by the Indian Council of Medical Research (ICMR) for funding in 2026.

He learns best from first principles: rebuilding an idea from what came before it until the formula feels like the only reasonable answer. This series grew out of that habit.

His long-term aims are to reduce obesity in India and to help make India healthy.

Email	contact@drharshmaheshwari.com
Website	drharshmaheshwari.com
ORCID	0000-0001-8848-1918

Introduction

Why this book exists

Every other book in this series stands on this one. It teaches, once, the ground floor the other 195 books assume: numbers you can trust, units and dimensions, relationships and change, uncertainty, the chemistry of a body, and how to read evidence. It assumes you can read English and use a calculator. It assumes nothing else: no medicine, no biology, no remembered mathematics.

How to read it

Read it in order. Each section assumes everything before it and nothing after it. If a section looks easy, read it anyway and read it quickly: it is here because something later needs it.

Every section has the same shape. **Definition** states the idea exactly. **In plain terms** says it again in everyday words. **Illustration** works through one real case, usually with a source you can open yourself. **Where this picture breaks** says where the illustration stops being true. **Must-know points** are what should change what you do. Then come **exercises** and, in most sections, a graded **practice set**.

Do the exercises before you look anything up. The worked answers are in the appendix at the back, under the same numbers. The references for each Part follow that Part. The glossary lists every term of art in the plain words it was first given. The last pages list all 196 books in the series and show where this one sits.

If you are new to this

On a first reading, read each section's definition, plain-terms explanation, illustration and must-know points, and do the exercises. Worked derivations can be skimmed and revisited once the idea feels familiar. Keep a calculator beside you: every worked answer can be followed with pen, paper and a calculator. Anything beyond the stated prerequisites is explained where it first appears, or marked as beyond this book. The **Symbols used in this book** page lists every sign and Greek letter with how to read it and where it is first used.

How to send feedback

If something is wrong, unclear, or could be better, please write to **contact@drharshmaheshwari.com**. Put the book number, the version and the section in the subject line (for example: *Book 0, v1.2, section C5*). Say what the text says, what you think it should say, and, if you can, the source.

Corrections are made in new versions. The version number and the date of the last update are on the copyright page of every book, and the newest version of every book is at **drharshmaheshwari.com**.

WHERE THIS BOOK SITS

Book 0 of the series' 196 books (Book 0 and 195 rung books). It is the first. After it: Book 1, Energy balance and dynamic weight-change modelling · Rung 1. The full list is at the back.

Symbols used in this book

Every sign and Greek letter this book prints, how to read it aloud, what it means in this series, and the section where it first appears. Each is also explained where it is first used.

Symbol	Read as	Meaning	First in
SIGNS			
$\int$	the integral of	The area under a curve, or the total accumulated over a range	C8 · Accumulation, a running total, without the machinery
$\geq$	greater than or equal to	An inequality that includes its boundary	D3 · Conditional probability through the two-way table
°	degree(s)	Degrees of temperature (°C) or of angle	D7 · Random error and systematic error as different things
Δ	delta, change in	A change or difference: Δt is a change in time	E4 · Heat and temperature; how heat moves
MARKS ON LETTERS			
x^2, 10^{-3}	superscript	Small raised characters are powers: $x^2 = x \times x$, and $10^{-3} = 0.001$	A6 · Powers, roots and scientific notation

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Part A · Numbers you can trust

A1 · Counting, place value, and what a calculator is actually doing

DEFINITION. A number written in digits carries two things at once: which digits were used, and the place each one occupies. The place fixes what the digit is worth.

Each place is worth ten times the place immediately to its right. That is the whole of what a written number means, and it is why the system is called base ten.

A negative number is a number less than zero. On the number line it lies as far below zero as the matching positive number lies above it, and it is written with a minus sign in front.

The operations in a written calculation are carried out in a fixed, agreed order: anything in brackets first, then powers, then multiplication and division, then addition and subtraction. Within multiplication and division, and within addition and subtraction, the work runs from left to right. [1, 2, 3]

IN PLAIN TERMS. Take 4,505. There are two fives in it, and one of them is worth five hundred while the other is worth five. Move one step to the left and a digit is worth ten times as much.

Now the part that catches people in India. Where you put the commas is a different question from what the digits are worth. And India does not do it the way most places do. Here the last three digits go together and everything above that is grouped in pairs: 10,00,000. In most other places they run in threes all the way along: 1,000,000. Same number, same value, different commas.

The names follow the commas. India counts thousand, lakh, crore. Most other places count thousand, million, billion. One lakh is a hundred thousand and one crore is ten million, so the two ladders never line up.

A calculator does base ten for you, fast and without complaint, but it cannot read your mind. Type one zero too many and it will work the sum out correctly and hand you an answer ten times too big.

Numbers do not stop at zero. Take 240 away from 52 and the 52 runs out after 52 steps. The other 188 steps carry on below zero, and you land on minus 188. It is written -188, with the sign in front of the number.

So the minus sign has two jobs. Between two numbers it means take away, as in 52 minus 240. In front of one number it says the number is below zero, as in -188. People say “minus 3” out loud for both, so ask which job the sign is doing.

A negative answer makes sense only if the thing you are measuring can go below zero. A temperature can fall below zero. A bank balance can be a debt. A room cannot hold minus two people. When a thing like that comes out negative, something went wrong before the answer. Either a mistake happened earlier, or a rule was run past the point where it holds.

A written line of sums is also worked in a fixed order. You multiply and divide before you add and take away. So 3 plus 2 times 4 is 11, not 20. Nobody discovered that order. Everybody agreed to it, so that one line of sums has one answer. Anything in brackets comes first, so (3 plus 2) times 4 is 20.

A calculator that shows the whole line as you type it, as most phone and scientific ones do, follows this order. A basic desk calculator does not. It works each step out as you press the next key. Test yours with 3 plus 2 times 4. If it shows 11, it follows the order.

The catch is the equals key. Press it part way through a line, and you have chosen the order yourself.

ILLUSTRATION. Open the Food Safety and Standards Act, 2006, which you can open at https://indiacode.gov.in/handle/123456789/586348. Search inside it for “misleading advertisement”.

Every Indian Act opens with a list of its section names, so your first hit is the line 53. Penalty for misleading advertisement in that list. Search again and take the second hit, which is section 53 itself.

It says ten lakh rupees. Write that in digits, the Indian way, and then the international way.

```
10,00,000    1,000,000
```

Count the zeros in each. Five, and five. The number has not changed. Only the commas moved.

Somebody reading the Act writes a note in English for a reader abroad. Ten lakh, they think, and lakh sounds like million. So ten lakh becomes ten million, and they type this.

```
1,00,00,000
```

Ten million is one crore. They have multiplied the penalty by ten, in a note that other people will quote back at them.

Ten lakh is one million. One crore is ten million.

Section 53’s ceiling is ten lakh and section 51’s is five lakh. Type one million, then divide by five hundred thousand, and it returns 2. That is right: the first ceiling is twice the second. Now type it again and leave one zero off the second number. It returns 20.

The calculator did not object either time, and it never will. Checking the question is your job. Counting the digits before you press equals is how you do it.

Two more things the calculator does without telling you. It handles numbers below zero, and it follows an order.

Start with a number line. Draw a line, mark 0 in the middle, and mark 1, 2, 3 going right. Now keep marking to the left of 0, at the same spacing: -1, -2, -3. Every number left of zero is negative. The further left, the smaller it is, so -200 is smaller than -3.

Work out 52 minus 240 on it. Start at 52 and step left 240 times. After 52 steps you are at zero, and 188 steps are still to go. They carry on past zero.

```
240 minus 52 = 188    (steps still to go at zero)
52 minus 240 = -188
```

Take away another 48 and you go further left, not back up.

```
-188 minus 48 = -236
```

Now look at your calculator. The key that takes one number from another is the minus key. Many calculators have a second key for the sign of one number. A key marked (-) types a minus in front of the next number. A key marked +/- flips the sign of the number already showing.

Type 52, the minus key, 240, and equals. The display shows -188. The minus key did the taking away. The dash on the display is the other job: the sign of one number.

Three rules cover everything this book does with negatives. Here is each one once.

Adding a negative number is the same as taking away the positive one. Taking away a negative number is the same as adding the positive one.

```
10 plus -4 = 6
10 minus -4 = 14
```

Multiplying and dividing share one rule. If the two numbers have the same sign, the answer is positive. If the signs differ, the answer is negative.

```
3 times -4 = -12
-3 times -4 = 12
-12 divided by 4 = -3
-12 divided by -4 = 3
```

So a negative divided by a positive is negative. And two negatives multiplied give a positive.

Now the check that matters most. A negative number means something only if the quantity can go below zero. Some quantities have a direction built in. A temperature can fall below zero. A budget can run a deficit, which is spending more than there was. A bank balance can be a debt. For these, the minus sign says which way, and it is a real answer.

Other quantities are amounts of something: the mass of a thing, a count of people, a volume of water. None of these can be less than nothing.

So when one of them comes out negative, stop. Do not report it as the amount, and do not round it to zero. Report that the figures do not add up. Something went wrong before the answer. A figure was copied wrongly, a step was done in the wrong order, or a sign was dropped. Or a rule was run past the point where it holds, like a stock worked forward past the day it runs out. Go back and find which.

The last thing a calculator does is follow an order. Type 3 plus 2 times 4. Should it add first, or multiply first? The two choices give different answers.

```
adding first
3 plus 2 = 5
5 times 4 = 20
multiplying first
2 times 4 = 8
3 plus 8 = 11
```

Both lines of working are correct arithmetic, so the arithmetic cannot settle it. People settled it by agreement. The order of operations is a rule that was chosen, not a fact that was found. It exists so that one written line has one answer, and here that answer is 11.

The order runs like this.

1. Anything inside brackets.
2. Powers, meaning a number multiplied by itself a set number of times. A later section teaches them.
3. Multiplying and dividing.
4. Adding and taking away.

Within one level, work from left to right. So 12 divided by 2 times 3 is 18, not 2. And 10 minus 4 plus 3 is 9, not 3.

```
3 plus 2 times 4 = 11
(3 plus 2) times 4 = 20
12 divided by 2 times 3 = 18
10 minus 4 plus 3 = 9
```

Brackets are how you get the other answer when you want it. Put them round the part that has to happen first.

Here is where a calculator catches people. A calculator that shows the whole line as you type it follows this order. Press equals part way through, and you have chosen the order yourself.

Go back to the two ceilings in the Act. Say a note needs the section 53 ceiling plus twice the section 51 ceiling. As one line, that is ten lakh plus 2 times five lakh. Type it whole and press equals once.

```
10,00,000 plus 2 times 5,00,000 = 20,00,000
```

Now press the same keys in a different rhythm. Press equals straight after the 2, then carry on.

```
10,00,000 plus 2 = 10,00,002
10,00,002 times 5,00,000 = 5,00,00,10,00,000
```

Same keys, different answer. The right one has seven digits and the wrong one has twelve. Counting the digits catches it here, as it caught the extra zero.

Some small desk calculators work out each step as soon as you press the next key. That is the same as pressing equals every time. Test yours. Type 3 plus 2 times 4 and press equals once. If it shows 11, it follows the order. If it shows 20, it does not, and you must press the keys in the order you want the work done.

Figures quoted in this illustration are from [4].

WHERE THIS PICTURE BREAKS. Place value is a rule about writing numbers down. It tells you nothing about whether ten lakh rupees is a large penalty or a small one.

The commas break in a second way, and quietly. A spreadsheet or a website prints the grouping it was built for, and does not say which one that is. So count the digits, not the commas. Where the two disagree, the digits are right.

A calculator is only as good as what you typed. It will divide a sum of money by a count of children and hand you an answer. That answer is arithmetic rather than sense.

The below-zero check works on amounts, not on changes. A mass cannot be negative, but a change in mass can. A count of people cannot, but the change in that count from one year to the next can. So before you run the check, say whether the number is an amount or a change.

The order of operations is an agreement about how to read a written line. It says nothing about which order the sum needed. A line typed without the brackets it needed is read by the agreed order, and gives the wrong answer without complaint.

MUST KNOW POINTS FOR YOU.

- A lakh is not a million. One lakh is a hundred thousand. One crore is ten million.
- Before you press equals, count the digits in each number you typed. Then say the number out loud, in the naming system you are writing in.
- Indian statutes, budgets and ministry papers state money in lakh and crore. Journals and international agencies state it in million and billion. Every time a figure crosses between the two, write it in digits first and count them.
- Asked on the spot how big a crore is, do not answer “a lot”. Say ten million, say a hundred lakh, and say how many digits that is.
- A trainee who cannot move between the two groupings will hand you a broken table. When a table looks odd, check the conversion before you check the arithmetic.
- When a quantity that cannot go below zero comes out negative, do not report it as the amount and do not round it to zero. Report that the figures do not add up. A mass, a

count of people or a volume below zero means a figure, a step or a sign went wrong earlier. Or a rule was run past where it holds. Go back and find which.

- A calculator that shows the whole line follows the agreed order, and only on a line you type whole. A basic desk calculator does not follow it at all, so test yours with 3 plus 2 times 4. Press equals part way and you have chosen the order yourself. Type mixed sums as one line, with brackets round whatever must happen first.

EXERCISE 1 (calculation). Section 50 of the Food Safety and Standards Act, 2006 sets a ceiling of five lakh rupees. A proviso lowers it to twenty-five thousand rupees for small food businesses — petty retailers, hawkers, temporary stalls and the like. Write both figures in digits, in both groupings. Then say how many times larger the ceiling is than the proviso figure.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is a sentence from a report.

> *The maximum penalty for a misleading food advertisement in India is one million rupees. That is about ten crore.*

Find the mistake, and say which step it happened at.

PRACTICE. Seventeen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. In the number 90,807, say what each digit is worth. Then say how many digits the number has.

2. Work these out. 15 minus 40. 8 minus -5. -6 plus -9. -6 times -7. -36 divided by 4.

3. Write each of these in digits, twice: once in the Indian grouping and once in the international one. Two lakh. Twelve lakh. Three crore.

4. Work these out in the agreed order. 5 plus 3 times 6. (5 plus 3) times 6. 20 divided by 4 times 5. 18 minus 6 divided by 3. Then say which of the four would give a different answer if you worked strictly from left to right.

5. Which is larger, 90,90,000 or 9,900,000? Say how you decided, in one line.

6. A school kitchen has a budget of 3,50,000 rupees for a month and spends 4,20,000 rupees. Its store starts the month with 40 sacks of rice, receives 25 sacks, and records 70 sacks issued. All of these figures are made up for this problem. Work out the money left and the sacks left. For each answer, say whether its sign makes sense.

7. Section 34(1) of the Consumer Protection Act, 2019 gives the District Commission complaints worth up to one crore rupees. Section 47 gives the State Commission up to ten crore. Write both in both groupings and count the digits in each.

8. The second schedule of the National Food Security Act, 2013 sets a lower primary meal at 450 calories. A school serves 1,200 of those meals in a week, and adds 50,000 calories of fruit. The 1,200 meals and the 50,000 calories of fruit are made up for this problem. Write the total as one

line and work it out. Then say what a calculator shows if you press equals straight after the first addition.

9. A district serves 1,00,000 lower primary meals. The second schedule of the National Food Security Act, 2013 sets that meal at 450 calories. Work out the total calories. Write the answer in digits in both groupings, and name it in both naming systems.

10. A colleague is checking two ceilings in the Food Safety and Standards Act, 2006. Section 53 sets ten lakh rupees for a misleading advertisement. Section 51 sets five lakh rupees for sub-standard food. They want to know how many times larger the first one is. Here is what they typed, and what came back.

```
10000000 divided by 500000 = 20
```

> *The advertising ceiling is twenty times the sub-standard food ceiling.*

Find the step that broke, and say why the answer looks reasonable.

11. A colleague wants the average number of meals served over two days: the two days added together, then divided by 2. Monday had 1,150 meals and Tuesday 1,250. These figures are made up for this problem. They type one line into a phone calculator, and here is what came back.

```
1150 plus 1250 divided by 2 = 1775
```

> *The average is 1,775 meals a day.*

Find the step that broke, and say why the answer looks reasonable.

12. A colleague adds three penalty ceilings — ten lakh, three lakh and five lakh — and reports the total as “18,00,000, or about 1.8 crore”. Find the step that broke and say what the total should be called.

13. A ration shop’s register shows 120 sacks of wheat at the start of the month, 30 sacks received and 165 sacks issued. These figures are made up for this problem. A clerk works out the stock at the end of the month.

```
120 plus 30 = 150
150 minus 165 = 15
```

> *Stock at the end of the month: 15 sacks.*

Find the step that broke, and say why the wrong answer looks reasonable.

14. Here is a press line.

> *India's food regulator can fine a company up to ten million rupees for a misleading food advertisement.*

You have the Food Safety and Standards Act, 2006 open. Decide what to check, check it, then say what your answer does not establish.

15. In a meeting somebody says this.

> *Enrolment at the school changed by plus 300 last year and by minus 400 this year. So enrolment is now minus 100.*

The figures are made up for this problem. Decide what to compute, compute it, and say what your answer does not establish.

16. In a meeting somebody says this.

> *The scheme costs 400 crore, which is about 4 billion rupees. That is roughly 4,000 rupees for each of the ten lakh people it reaches.*

Decide what to compute, compute it, and say what your answer does not establish.

17. A note to a district office says this.

> *Cost per child: 4,00,000 plus 2,00,000 divided by 400 children, which comes to 1,500 rupees a child.*

The figures are made up for this problem. Decide what to compute, compute it, and say what your answer does not establish.

A2 · Fractions

DEFINITION. A fraction is a division that has not yet been carried out. The number below the line, the denominator, states how many equal parts the whole was cut into. The number above the line, the numerator, states how many of those parts are being counted.

Multiplying the numerator and the denominator by the same amount, or dividing both by the same amount, leaves the value of the fraction unchanged. [5]

IN PLAIN TERMS. A fraction is a sum you have not done yet.

Cut a roti into fourteen equal pieces and take nine of them. Now cut the same roti into seventy pieces and take forty-five. You are holding exactly the same amount of roti. Both numbers in the second fraction are five times the numbers in the first, so the two fractions are one number written twice.

Multiply the top and the bottom by the same thing, or divide both by the same thing, and the value stays put. Change one of them on its own and you have a different number.

To add two fractions, first make the bottoms match.

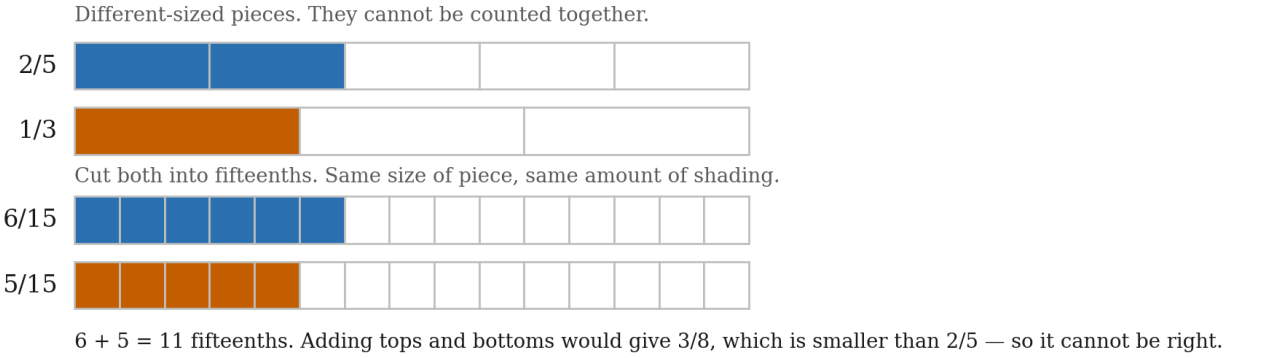
Take two fifths and one third. Fifths and thirds are different sizes of piece, so there is nothing yet to add. Find a size that both can be cut into. Fifteen works, because five goes into fifteen and so does three.

Now rewrite each one at that size. Two fifths: multiply top and bottom by three, giving six fifteenths. One third: multiply top and bottom by five, giving five fifteenths. Neither number changed. You cut the same amount into smaller pieces.

Six fifteenths and five fifteenths are the same size of piece, so count them. Eleven fifteenths.

Add the tops and the bottoms instead and you get three eighths. That is less than the two fifths you started with, and adding something to a number cannot make it smaller. That is your check.

A fraction is always a fraction of something. So before you compare two fractions, or add them, check that they were cut from the same whole. If they were not, the arithmetic still works and the answer still means nothing.



Two fifths and one third, then the same two amounts cut into fifteenths. The shaded length does not change when the pieces are re-cut, which is the whole of why a common denominator is allowed.

ILLUSTRATION. Open the National Food Security Act, 2013, which you can open at https://indiacode.gov.in/handle/123456789/496105. Search inside it for the words NUTRITIONAL STANDARDS. It is the second schedule, and it sets what a meal under these schemes has to deliver.

The standard for a lower primary hot cooked meal is 450 calories and 12 grams of protein. The standard for an upper primary one is 700 calories and 20 grams. The Act sets what a meal has to deliver. It does not say what any child receives.

Calories count food energy and grams count weight. A number with its label stripped off is not a number any more.

What fraction of the upper primary energy standard is the lower primary one?

Write it as a fraction, energy over energy, and work it down.

```
450 over 700
= 45 over 70    (both ended in a zero, so divide both by ten)
= 9 over 14    (both are in the five times table, so divide both by five)
```

Somebody hears that and says: a lower primary child gets nine fourteenths of what a child needs.

They have swapped the whole. Nine fourteenths is the lower primary standard as a fraction of the upper primary standard. Neither is what a child needs in a day, and those two rows of the Act say nothing whatever about a day's requirement. The fraction is right. The sentence built on it is not.

Do the protein column yourself now, the same way. Twelve over twenty. Divide both by four and you have three fifths.

Nine fourteenths is a little under two thirds. Three fifths is a little over half.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. Fractions only behave if the parts are equal.

This section tells you how to handle a fraction once you have one. It cannot tell you what the whole was. It comes from the document, the table heading or the person who handed you the number. If none of them says, you do not have a fraction yet.

MUST KNOW POINTS FOR YOU.

- A fraction on its own is not a finding. Nine fourteenths is only a fact once you can say what the fourteen was.
- To simplify a fraction, divide the top and the bottom by the same number. To compare or add two, put them over the same denominator first.
- Name which standard, and name the group it was written for.
- Asked in public what fraction of a child's need a meal covers, do not answer with the fraction you happen to have.
- Ask for the denominator every time, of your own numbers first.

EXERCISE 1 (calculation). The second schedule of the National Food Security Act, 2013 gives a take-home ration for children aged six months to three years of 500 calories. For a malnourished child aged six months to six years it gives 800 calories. Write the first as a fraction of the second and reduce it as far as it will go. Then say, in one sentence, what that fraction is and is not a fraction of.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is a line from a draft note.

> *Lower primary children get 12 grams of protein and upper primary children get 20 grams. So lower primary children are getting three fifths of their protein.*

The arithmetic is right. Say what is wrong with the sentence, and rewrite it so it is defensible.

PRACTICE. Ten problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Reduce each of these as far as it will go. Show the number you divided by at each step.

```
6 over 8    20 over 50    36 over 48
```

2. Which is larger, three fifths or five eighths? Put them over the same denominator first.

3. Add two fifths and one third. Then say why you could not simply add the tops and the bottoms.

4. The same schedule sets protein at 12 grams for a lower primary meal and 20 for an upper primary meal. It sets calories at 450 for the lower primary meal and 500 for a take-home ration for a child of six months to three years. Reduce each of those two pairs to a fraction.

5. A block has 400 households. Two hundred and forty of them lift their full grain entitlement in a month. Write the share that did as a fraction reduced as far as it will go, and say what the whole was.

6. A block has 320 households. Three eighths of them lifted grain in the first fortnight of the month and a quarter of them lifted it in the second. No household is in both counts. What fraction of the block's households lifted grain in the month, and how many households is that?

7. Here is a worked answer. Find the step that broke.

```
1 over 4 + 1 over 4 = 2 over 8 = 1 over 4
```

8. Here is a line from a note.

> *The lower primary meal is 450 calories and the upper primary meal is 700. So the lower primary meal is about two thirds of a child's daily energy.*

The arithmetic behind "two thirds" is right. Find what broke.

9. Here is a line from a district note.

> *Two fifths of our anganwadi centres and half of our schools reported a shortage, so nine tenths of our facilities had one.*

Decide what to compute, compute it, and say what your answer does not establish.

10. Somebody hands you a one-line finding.

> *Protein provision improved: the meal now meets three fifths of the standard, up from a half.*

Decide what to compute, compute it, and say what your answer does not establish.

A3 · Decimals, rounding, and significant figures

DEFINITION. A decimal continues place value to the right of a point. Each place to the right is worth one tenth of the place on its left, so the places run tenths, hundredths, thousandths.

Rounding is choosing which place to stop at and adjusting the last kept digit for what follows it.

The significant figures of a number are the digits that carry information about the quantity, counted from the first digit that is not zero. A quantity calculated from other quantities cannot carry more significant figures than the least precise of them. [7]

IN PLAIN TERMS. A decimal is the same place-value rule, carried on past the point. To the right, each step is ten times smaller: tenths, then hundredths, then thousandths.

Rounding is deciding where to stop. Pick the place you want to stop at and look at the very next digit. If it is four or less, leave your last digit alone. If it is five or more, push it up by one.

Significant figures are the digits that carry the information. Start counting at the first digit that is not a zero. Stop at the last digit written down. Everything in between counts, zeros included.

So 0.00420 has three of them: the four, the two and the final zero. The zeros in front are only holding the place. The zero at the end is holding nothing, so somebody put it there on purpose, and it says the measurement reached that far.

Whole numbers ending in zeros are the awkward case. Write 4,200 and nobody can tell whether you counted to the nearest hundred or to the nearest one. The digits cannot tell you. The person, the table heading or the document has to. If none of them does, you do not know how precise the number is. Do not treat it as exact until somebody tells you it is.

Your answer cannot be more exact than the numbers you put in. Divide something you know to two digits by something you know to two digits, and you know the answer to about two digits.

ILLUSTRATION. Open the National Food Security Act, 2013 again and search inside it for “five kilograms”. The first is section 3(1), which gives a person in a priority household five kilograms of foodgrains per person per month.

How much is that a day?

Type five, divide by thirty, and read what the calculator says.

```
5 divided by 30 = 0.1666666667
```

Here is the mistake, and it goes into real documents. The figure is copied straight across, and a report goes out saying each person receives 0.1666666667 kilograms of grain a day.

Count the digits after the point. Ten of them. Nothing you did could possibly support that.

The five is exact: it is not a measurement, it is a figure written into an Act, and it is five and nothing else. The thirty is not exact at all. You chose it. A month is twenty-eight days, or thirty, or thirty-one.

So work the two ends instead of one middle.

```
5 divided by 31 = 0.16129…
5 divided by 28 = 0.17857…
```

The honest answer sits somewhere between 0.16 and 0.18 kilograms a day. Two digits, not ten.

Now round the middle case properly, so you can see the rule work. You have 0.1666666667 and you want two places after the point. The next digit along is a 6, which is five or more, so the 6 you are keeping goes up to 7.

```
0.1666666667 rounds to 0.17
```

Round to 0.17, then multiply by 30 to get back to a month.

```
0.17 times 30 = 5.1
```

You started with exactly 5 and came back with 5.1. The rounding leaked.

If somebody needs a daily figure, give them 0.16 to 0.18 kilograms and say where the spread came from.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. Significant figures are a rough guide to precision, not a measure of it. They do not tell you how wrong the number might be.

The rounding rule itself has an edge. When the digit you are dropping is exactly five and nothing follows it, the rule above always pushes up. That nudges a long column of numbers slightly high. It rarely matters. It matters when you are rounding thousands of values and then adding them.

MUST KNOW POINTS FOR YOU.

- The digits on a calculator display are not precision. They are what the division produced.
- Keep full precision while you work and round once, at the end, to what your weakest input earned.
- Five kilograms in an Act is exactly five. Five kilograms on a scale is not. Ask which kind you are holding before you decide how to round it.
- Indian entitlements are commonly written per month while reporting wants a daily figure. That one division is where false precision enters.
- When a number you are given carries more digits than its source could support, say so at the time.

- Do not quote a spuriously precise figure in public even when it is the one you were handed.

EXERCISE 1 (calculation). A household of six people is in a priority household under section 3(1) of the National Food Security Act, 2013. Work out the household's monthly entitlement, then its daily entitlement. Give the daily figure to a precision you can defend, and say in one line why you stopped there.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A district report reads: "Average grain lifted per beneficiary was 4.8732 kg per month against an entitlement of 5 kg." Nothing here is arithmetically wrong. Name what you would ask about the figure 4.8732, and say what you expect the answer to be.

PRACTICE. Ten problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. In the number 0.408, say what each digit after the point is worth. Then put these four numbers in order, smallest first.

```
0.7    0.65    0.089    0.7000
```

2. Round each of these to two places after the point.

```
0.3849    12.0761    0.0955    7.2950
```

3. How many significant figures does each of these carry?

```
450    0.0072    6.030    1,200
```

4. Work out 17 divided by 7. Write the calculator's full display, then give the answer to three significant figures.

5. Section 3(1) of the National Food Security Act, 2013 gives five kilograms of foodgrains per person per month. Work out the figure per week, using 4 weeks in a month, and give it to a precision you can defend.

6. A hot cooked meal is set at 450 calories by the second schedule of the National Food Security Act, 2013. A child attends on 196 of the year's school days. Work out the year's calories from the meal, and say how many significant figures the answer deserves.

7. A store issues 1,850 kilograms of grain to 387 households in a month. Work out the average per household and report it to a precision you can defend. Say in one line what fixed the precision.

8. Here is a worked answer. Find the step that broke.

```
2.4 times 3.6 = 8.64
8.64 rounded to 2 significant figures = 8.6
8.6 divided by 4 = 2.15
final answer, to 2 significant figures = 2.2
```

9. A district report reads: “Coverage stands at 64.8135 per cent.”

It was computed from a count of covered households and a count of total households, both taken from a register. Find what is wrong with the figure as reported.

10. Somebody sends you a one-line finding and asks whether to publish it.

> *Mean intake fell from 2,014.6 to 1,987.3 calories a day, a fall of 1.36 per cent.*

Decide what to compute, compute it, and say what your answer does not establish.

A4 · Percentages, and the percentage of a percentage

DEFINITION. A percentage is a fraction whose denominator is one hundred. Per cent means per hundred, so 24 per cent is 24/100, which is 0.24.

A percentage change is the change divided by the value it started from, multiplied by one hundred.

A percentage point is the arithmetic difference between two percentages. A per cent change is that difference divided by the first percentage.

A percentage of a percentage is the product of the two fractions: 40 per cent of 30 per cent is 0.40 times 0.30, which is 0.12, or 12 per cent of the original whole. [8]

IN PLAIN TERMS. Per cent means per hundred. Twenty-four per cent is twenty-four out of a hundred, which is 0.24.

The hard half is what the hundred was.

A percentage change is worked out against the number you started from. Because the starting number sits underneath, going up and coming back down are not the same percentage.

If something goes from 2 per cent to 3 per cent, it has gone up by one percentage point. It has also gone up by 50 per cent, because one is half of two.

ILLUSTRATION. Go back to the two rows you read in the National Food Security Act, 2013. A lower primary meal is set at 450 calories. An upper primary meal is set at 700.

How much more is the upper primary standard?

Work out both.

The plain difference first.

```
700 minus 450 = 250 calories more
```

Now as a percentage. The rule is: the change, divided by what you started from, times a hundred.

```
250 divided by 450 = 0.5555…
0.5555… times 100 = 55.6 per cent    (rounded to one place, as the last section had it)
```

Now turn it round, because this is where people slip. Start at 700 and come down to 450.

```
250 divided by 700 = 0.3571…
0.3571… times 100 = 35.7 per cent
```

The lower primary meal is about 36 per cent less. Not 56. The number sitting underneath changed, so the percentage changed with it.

Ask which end it was measured from, every single time.

Here is a sentence of the kind that reaches a front page.

> *The share of children affected rose by fifty per cent in a decade.*

Suppose the real figures behind it are two in a hundred rising to three in a hundred.

```
3 minus 2 = 1 percentage point
1 divided by 2 = 0.5, so 50 per cent
```

Both are true of the same change. The writer picked the fifty.

Percentage points subtract. Per cent divides.

Ask for the count as well as the percentage, always.

Suppose 30 per cent of schoolchildren in a district eat a packaged snack on a school day. Of those who do, suppose 40 per cent buy it within a hundred metres of the school gate. Multiply the fractions.

```
0.40 times 0.30 = 0.12, so 12 per cent of all schoolchildren
```

Those two percentages are made up, to show the move. A percentage of a percentage is smaller than either.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. Percentages hide the size of the thing they are a percentage of, and they hide it by design.

A percentage change and a percentage point look alike on the page and are different quantities. When a source says “per cent” you often cannot tell which it meant, and you cannot repair it by reading harder. Go back to the two original numbers, or say in your own writing that you could not.

MUST KNOW POINTS FOR YOU.

- A percentage point and a per cent are different quantities. Whichever is quoted at you, ask for the other.
- For any percentage in front of you, ask two questions before you use it: per cent of what, and how many people is that.
- A rise and the fall that undoes it are different percentages, because the starting value sits underneath. Say which end you measured from whenever you quote a change.
- A percentage of a percentage multiplies, so it is always smaller than either.

- In a committee a relative change will be offered when it is the larger number and an absolute one when it is the smaller. Bring both to the meeting yourself.
- When a journalist gives you a percentage on the phone, do not react to it. Ask what it is a percentage of and what the two underlying counts were.

EXERCISE 1 (calculation). The second schedule of the National Food Security Act, 2013 sets 12 grams of protein for a lower primary meal. It sets 20 grams for an upper primary meal. Express the step from 12 to 20 as a percentage, and the step from 20 back to 12 as a percentage. Then say which of the two you would put in a note, and why.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (teaching). Explain to a journalist, in three minutes and without jargon. The claim is that obesity in this group rose by fifty per cent. Say why that may be a smaller story than it sounds, and why it may also be a larger one. Give them one sentence they can print.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Write each of these as a decimal, and then as a fraction over one hundred.

```
7 per cent    40 per cent    125 per cent    0.5 per cent
```

2. Work out 35 per cent of 840. Then work out 840 as a percentage of 1,200.

3. A figure goes from 80 to 92. Express the change as a percentage. Then express the step from 92 back to 80 as a percentage.

4. Work out 40 per cent of 30 per cent. Then work out 5 per cent of 60 per cent. Give each answer as a percentage of the whole you started with.

5. Section 3(2) of the National Food Security Act, 2013 says entitlements shall extend up to 75 per cent of the rural population. For the urban population the figure is up to 50 per cent. A district has 18 lakh rural and 6 lakh urban residents. Work out the ceiling on the number covered in each.

6. In a survey, 45 per cent of respondents buy a packaged snack in a week. Of those who do, 20 per cent buy it at a shop next to a school. What share of all respondents is that? Then say how many people that is, out of 2,400 respondents.

7. Here is a worked answer. Find the step that broke.

```
a figure rises from 4 per cent to 6 per cent
6 minus 4 = 2
so the rise is 2 per cent
```

8. Here is a worked answer. Find the step that broke.

```
in a survey, 30 per cent of respondents buy a packaged snack on a school day
of those who do, 40 per cent buy it at a shop by the school gate
30 plus 40 = 70
so 70 per cent of all respondents buy a snack at a shop by the school gate
```

9. Here is a worked answer. Find the step that broke.

```
a budget of 100 is cut by 20 per cent
the following year it is raised by 20 per cent
20 minus 20 = 0, so the budget is back to 100
```

10. Here is a line you are asked to comment on.

> *Overweight among women in this district rose by sixty per cent in a decade.*

Decide what to compute, compute it, and say what your answer does not establish.

11. A colleague brings you two sentences about the same programme and asks which to put in the brief.

> *Coverage rose by 50 per cent.*

> *Coverage rose by 4 percentage points, from 8 per cent to 12 per cent.*

Decide what to compute, compute it, and say what your answer does not establish.

A5 · Ratios, rates and proportions

DEFINITION. A ratio compares two quantities of the same kind by division, and is written a to b. It has no unit, because the units cancel.

A proportion is a part divided by the whole it came from. It runs from zero to one, and a percentage is a proportion multiplied by one hundred.

A rate is a count divided by the quantity of something else it was counted over: time, or people, or both at once.

Two rates may be compared only when their denominators are the same. [9]

IN PLAIN TERMS. Twelve grams against twenty grams is a ratio of twelve to twenty, or three to five.

If three of every five children in a room had the meal, the proportion is three fifths, or 0.6. A proportion can never be more than one.

Five kilograms a month is a rate. Fifteen deaths per thousand people per year is a rate.

The bottom of a rate is not a footnote. It is part of the number. Change it and you have a different quantity, even when the digits look the same.

ILLUSTRATION. Go back to section 3(1) of the National Food Security Act, 2013, which you opened two sections ago. It gives a person in a priority household five kilograms of foodgrains per person per month. This time read the sentence after it.

A proviso just below sets a different figure for households covered by the Antyodaya Anna Yojana. It is thirty-five kilograms of foodgrains per household per month. That figure applies to the extent the Central Government specifies for each State under that scheme.

Here is the sentence somebody writes.

> *Antyodaya households get seven times as much as priority households.*

Look at where that seven came from. Thirty-five divided by five. The arithmetic is perfect and the claim is meaningless, because the five is per person and the thirty-five is per household. Two different denominators went into one division.

Make them the same and the picture changes. Take a household of six people.

```
5 kilograms per person per month, times 6 people
= 30 kilograms per household per month
```

```
35 against 30
35 minus 30 = 5 kilograms more
5 divided by 30 = 0.1666…, so about 17 per cent more
```

Seventeen per cent more, not seven times as much.

Now change the household and watch the answer move.

```
household of 7:    5 times 7 = 35, so the two are identical
household of 3:    5 times 3 = 15, so 35 is more than twice as much
household of 10: 5 times 10 = 50, so 35 is less
```

The comparison does not have one answer. It depends entirely on the denominator that the original sentence dropped.

One more thing to take from this, because it comes up in every table you will read. Rates for uncommon events are written per thousand or per hundred thousand people rather than per person. The only reason is to avoid a string of zeros after the point. Fifteen per thousand and 0.015 per person are the same rate.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. Making the denominators match is arithmetic. Deciding which denominator is the right one is not, and this section cannot help you with it.

So the conversion above is a tool for catching a false comparison. It is not a way of deciding who is better off.

And a rate per thousand people says nothing about which thousand. One figure for a state can hold districts that differ several times over. The rate will not show you that. It was never built to.

MUST KNOW POINTS FOR YOU.

- Two rates with different denominators cannot be compared, and dividing one by the other still produces a number.
- Say the denominator out loud before you use any rate: per what, and over how long. If either half is missing you do not have a quantity yet.
- Indian entitlements and scheme figures move between per person, per household and per beneficiary without warning. Read the whole provision, including the proviso, before you take a number out of it.
- When two schemes are stated on different denominators, find the point at which they are equal and quote that.
- Converting a per-household figure to a per-person one using an average household size assumes every household is average.
- Asked to say which of two programmes delivers more, do not answer with the larger number. Ask what each is divided by.

- Rates per thousand or per hundred thousand exist only to keep the zeros off the page. Convert freely, and check the denominator first whenever two tables disagree.

EXERCISE 1 (calculation). A block has 1,200 priority households with an average of five people each, and 300 Antyodaya households. Use section 3(1) of the National Food Security Act, 2013 and the proviso beneath it. Work out the monthly foodgrain entitlement of each group, and of the block as a whole. Then state the grain per household per month for each group and say which is higher.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is a line from a district presentation.

> *Grain offtake rose from 1,850 tonnes to 2,050 tonnes, so coverage improved by 11 per cent.*

The arithmetic in the percentage is right. Name the step that is missing, and say what you would ask for.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Reduce each ratio as far as it will go.

```
18 to 24    45 to 15    200 to 500
```

2. Of 250 households in a block, 90 lift their full entitlement. Write that as a ratio of lifting to not lifting, and as a proportion of all households.

3. A rate is 15 per 1,000 people per year. Write the same rate per 100,000 people per year, and per person per year.

4. Section 3(1) of the National Food Security Act, 2013 gives 5 kilograms per person per month. A household of four is entitled under it. Work out the household's entitlement per month and per year, and state the denominator of each answer.

5. Under the proviso to section 3(1), an Antyodaya household gets 35 kilograms per household per month. Work out the household size at which a priority household on 5 kilograms per person per month receives the same amount.

6. A block has 3,200 people and records 48 events in a year. A second block has 12,500 people and records 160 events in a year. Work out each rate per 1,000 people per year and say which is higher.

7. Here is a worked answer. Find the step that broke.

```
in a block, 72 households lift their full entitlement and 108 do not
the ratio of lifting to not lifting is 72 to 108, or 2 to 3
72 divided by 108 = 0.6666…
so about 67 per cent of households lift their full entitlement
```

8. Here is a worked answer. Find the step that broke.

```
block A: 15 cases per 1,000 people
block B: 40 cases per 100,000 people
40 is more than 15, so block B is worse
```

9. Here is a worked answer. Find the step that broke.

```
a scheme delivers 2 kg per person per week
a month is 4 weeks
so it delivers 8 kg per person per month
a household has 5 people
so it delivers 8 kg per household per month
```

10. A note crosses your desk with one line in it.

> *Households here average five people, so an Antyodaya household gets 40 per cent more grain.*

Section 3(1) of the National Food Security Act, 2013 gives 5 kilograms per person per month. The proviso beneath it gives an Antyodaya household 35 kilograms per household per month. Decide what to compute, compute it, and say what your answer does not establish.

11. A journalist calls with one line and wants a comment by the evening.

> *District A has twice the case rate of district B, so it needs twice the resources.*

Decide what to compute, compute it, and say what your answer does not establish.

A6 · Powers, roots and scientific notation

DEFINITION. A power is repeated multiplication of one number by itself. The number being multiplied is the base and the small raised number is the exponent, which counts how many copies of the base are multiplied together.

A negative exponent means repeated division by the base rather than repeated multiplication, so 10^{-3} is one thousandth. Any base raised to the power zero is one.

Multiplying two powers of the same base adds their exponents, and dividing subtracts them.

A root is the inverse of a power. The square root of a quantity is the number which, multiplied by itself, yields that quantity. Equivalently it is that quantity raised to the exponent one half, which is the case that shows an exponent need not be a whole number.

Scientific notation writes a quantity as a number between one and ten multiplied by a power of ten. [10]

IN PLAIN TERMS. Ten to the power three is written 10^3. The ten is the base. The small raised three is the exponent, and all it does is count how many tens are being multiplied. Ten to the power three is one thousand.

A quick way to read any power of ten: the exponent is the number of zeros.

A minus sign in the exponent flips it from multiplying to dividing. So 10^{-3} is one divided by a thousand, which is 0.001.

Multiply two powers of ten and you add the exponents. Divide, and you subtract them.

Squaring a number means multiplying it by itself. Three squared is nine.

A square root runs that backwards. The square root of nine is three, because three times three is nine. Ask it as a question. What number, multiplied by itself, gives me this one?

Some roots come out whole. The square root of 144 is 12. Most do not. The square root of two starts 1.414 and never stops. Take what the calculator gives you and round it the way the section on rounding told you to.

There is a second way to write a root, and you need it further on. An exponent of one half means the square root. So $9^{(1/2)}$ is 3, the same answer, written in the language of powers.

That is worth a pause, because it breaks the rule you started with. An exponent does not have to count copies. Once one half is allowed, so is 1.6, and $10^{1.6}$ is an ordinary number the calculator gives as about 40. Logarithms, two sections on, are built on exactly that.

Write a quantity as a number between one and ten, times a power of ten. Thirty-five thousand becomes 3.5 times 10^4.

ILLUSTRATION. Build the ladder of powers of ten yourself, in two columns, with the Indian names on one side and the international names on the other.

power	written out	Indian name	international name
10^1	10		
10^2	100		
10^3	1,000	thousand	thousand
10^4	10,000		
10^5	1,00,000	one lakh	
10^6	10,00,000		one million
10^7	1,00,00,000	one crore	
10^8	10,00,00,000	ten crore	
10^9	1,00,00,00,000		one billion

Read the gaps. A lakh is 10^5 and a crore is 10^7, so the Indian ladder steps by two powers of ten. A million is 10^6 and a billion is 10^9, so the international ladder steps by three.

Now the failure, and it is one of the commonest in real work. A spreadsheet has a column of grain tonnages in it. The column is narrow, so the spreadsheet prints one cell like this.

```
1.5E+06
```

Somebody copies the visible part into a report and writes 1.5 tonnes.

Read what the cell actually said. That E means “times ten to the power”, so the cell held 1.5 times 10^6 tonnes.

```
1.5 times 10^6
= 1.5 times 1,000,000
= 1,500,000
```

Take a lakh and multiply it by a hundred.

```
1,00,000 times 100
= 10^5 times 10^2
= 10^(5 plus 2)
= 10^7
= 1,00,00,000, which is one crore
```

Try the other direction. Divide a crore by a lakh.

```
10^7 divided by 10^5
= 10^(7 minus 5)
= 10^2
= 100
```

A crore is a hundred lakh.

WHERE THIS PICTURE BREAKS. "The exponent is the number of zeros" is a rule about powers of ten and nothing else. Two to the power three is eight, and there is no zero anywhere in it.

MUST KNOW POINTS FOR YOU.

- A negative exponent does not make a number negative. It makes it small. Ten to the power minus three is one thousandth, an ordinary positive quantity.
- Whenever a number contains an E, or a small raised number, write it out in full before you use it.
- The Indian ladder steps by two powers of ten: lakh is 10^5 and crore is 10^7. The international ladder steps by three: million is 10^6 and billion is 10^9.
- Write any large figure as a digit or two times a power of ten before you compare it with another.
- Asked to check a colleague's calculation quickly, check the exponents first and the digits second.
- The square root of a sum is not the sum of the square roots. Work out what sits under the root first, then take the root of that one number.
- An exponent does not have to be a whole number. One half means the square root, and a value in between is an ordinary number the calculator will give you.

EXERCISE 1 (calculation). Write each of these as a number between one and ten times a power of ten. The list: one lakh, thirty-five thousand, ten lakh, one crore, one thousandth. Then work out, using the exponent rules alone, how many lakh there are in a crore and how many thousand there are in a million.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is a colleague's note.

> *Daily production is 2.4E+05 units, which is about 2.4 lakh. So annual production is roughly 8.8E+07, or about 8.8 crore.*

Check every step. Say which are right and which are not.

EXERCISE 3 (calculation). A square plot of land covers 2,025 square metres. Work out the length of one side.

Then do the same for a second square plot covering 2,000 square metres. Say how many digits of that answer you would be willing to write down.

Record your confidence as a percentage before turning to the answer.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Write each of these out as an ordinary number.

```
10^2    10^5    10^0    10^-2    9^(1/2)    144^(1/2)
```

2. Work these out using the exponent rules alone, without writing the numbers in full.

```
10^4 times 10^3    10^8 divided by 10^5    10^3 times 10^-1
```

3. Write each of these in scientific notation, as a number between one and ten times a power of ten.

```
6,200    93,00,000    0.0047    1,00,00,000
```

4. Section 34(1) of the Consumer Protection Act, 2019 caps District Commission complaints at one crore rupees. Section 47 caps State Commission ones at ten crore. Write both in scientific notation and say, from the exponents alone, how many times larger the second is.

5. A district serves 1.8 times 10^5 meals in a year. The second schedule of the National Food Security Act, 2013 sets each meal at 450 calories. Work out the total calories in scientific notation, showing the exponent step.

6. A square plot covers 8.0 times 10^5 square metres. Write the area out in full, then work out the length of one side. Give the side to the number of digits the area gives you, and check your rounded answer by multiplying it back.

7. A value is given as 2.5E+04 in a spreadsheet cell and another as 3.0E-02. Write both out in full, then multiply them and give the answer in scientific notation.

8. Here is a worked answer. Find the step that broke.

```
10^6 times 10^3
= 10^(6 times 3)
= 10^18
```

9. Here is a worked answer. Find the step that broke.

```
two square plots are to be replaced by one square plot of the same total area
the first covers 900 square metres and the second 1,600 square metres
900^(1/2) = 30 and 1,600^(1/2) = 40
so the new plot has a side of 30 plus 40 = 70 metres
```

10. Here is a worked answer. Find the step that broke.

```
a concentration is 5 times 10^-3
a second is 2 times 10^-6
10^-3 is smaller than 10^-6 because -3 is greater than -6
so the first is the smaller concentration
```

11. Here is a line from a note.

> *Annual production is 1.2E+06 tonnes, so at 3 lakh tonnes a quarter the plant is running close to capacity.*

Decide what to compute, compute it, and say what your answer does not establish.

12. In a meeting somebody says this.

> *We reach 2 crore people, so a rupee a head is only 2 lakh rupees.*

Decide what to compute, compute it, and say what your answer does not establish.

A7 · Logarithms — what they are for before how they work

DEFINITION. The logarithm of a number to base ten is the power to which ten must be raised to produce that number.

It is defined only for numbers above zero, because no power of ten produces zero or a negative number. The logarithm of one is zero.

A logarithm converts multiplication into addition.

On an axis marked in logarithms, equal distances represent equal multiplications rather than equal additions. [11]

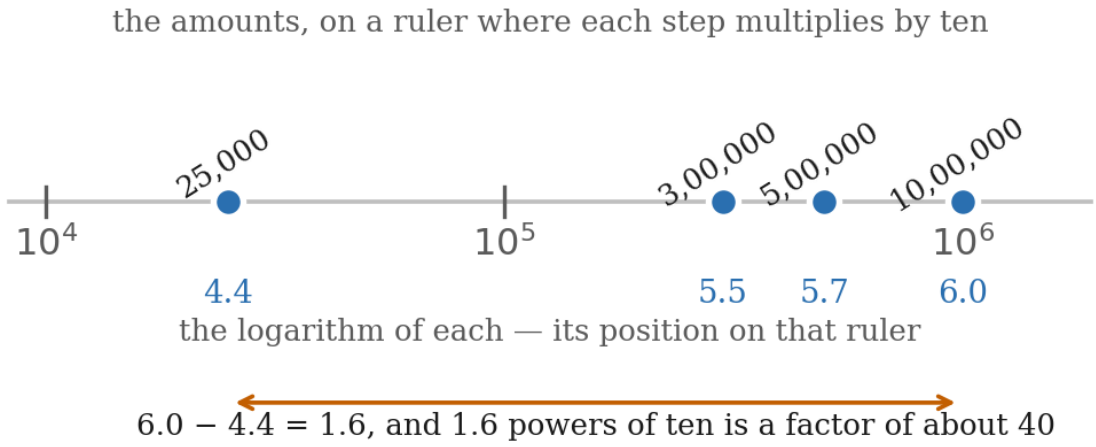
IN PLAIN TERMS. You have a pile of numbers and they are wildly different sizes. Put them on an ordinary scale and the small ones vanish into the left edge.

So ask a different question of each number. Not how big is it, but how many powers of ten is it?

Ten is 10^1, so its logarithm is 1. That is the whole definition: the logarithm is the exponent.

Numbers in between get an answer in between. Five hundred sits between a hundred and a thousand, so its logarithm sits between 2 and 3. A calculator will tell you it is about 2.7. You do not have to work that out by hand, and you will not be asked to.

Put logarithms on the side of a graph and equal distances up the page stop meaning equal amounts. They mean equal multiples. One step is ten times, whether you are stepping from 1 to 10 or from 100 to 1,000.



The four penalties on a ruler where each step multiplies by ten. The logarithm of an amount is simply where it sits on that ruler, so the distance between the smallest and the largest is a subtraction that has done a division.

ILLUSTRATION. Picture the side of the graph marked like this, bottom to top.

```
1    10    100    1,000    10,000
```

The marks are evenly spaced up the page. Somebody looks at it and says this.

> *The rise from 100 to 1,000 is the same size as the rise from 1 to 10.*

Do the subtraction yourself and watch the claim fall apart.

```
10 minus 1 = 9
1,000 minus 100 = 900
```

Nine against nine hundred. The gaps on the page are equal and the amounts are a hundred times apart.

So what is equal? Do the division instead of the subtraction.

```
10 divided by 1 = 10
1,000 divided by 100 = 10
```

Both steps multiply by ten.

Open the Food Safety and Standards Act, 2006 again, the one you opened in section A1, and search inside it for “twenty-five thousand”. One hit this time, in the proviso to section 50: a penalty not exceeding twenty-five thousand rupees for small food businesses.

For the other three, search the section headings rather than the money. So search for “Penalty for misbranded food”, then “Penalty for sub-standard food”, then “Penalty for misleading advertisement”.

Write the four down and turn each into a power of ten. Use a calculator for the logarithm column; that is what it is for.

25,000	2.5 times 10^4	logarithm 4.4
3,00,000	3 times 10^5	logarithm 5.5
5,00,000	5 times 10^5	logarithm 5.7
10,00,000	1 times 10^6	logarithm 6.0

The four you wrote down run from 4.4 to 6.0. That is a span of 1.6, and 1.6 powers of ten is a factor of about forty.

The heaviest of your four is about forty times the lightest.

```
100 times 1,000 = 1,00,000
logarithm 2 plus logarithm 3 = 5
and 10^5 is indeed 1,00,000
```

Adding the logarithms did the multiplying.

Figures quoted in this illustration are from [4].

WHERE THIS PICTURE BREAKS. A logarithmic axis makes very different numbers fit on one page, and it does that by squashing the differences between the large ones. A tenfold rise and a hundredfold rise look similar on the page.

It also cannot show zero, and it cannot show anything below it.

MUST KNOW POINTS FOR YOU.

- On a logarithmic axis equal distances are equal multiples, not equal amounts.
- Before reading any figure, look at the axis and decide which kind it is. If the marks go 1, 10, 100, 1,000 the steps are multiplications. If they go 1, 2, 3 they are additions.
- A logarithmic axis cannot show zero or anything below it, because no power of ten produces them. Ask what happened to those observations.
- A log axis makes a large difference look modest. If you are arguing about the size of an effect rather than its presence, that axis is working against you.
- Steps of one lakh, two lakh, three lakh are an ordinary scale. Steps of one lakh, ten lakh, one crore are a log scale. Read the steps rather than the words.
- Asked in a viva what the logarithm of one is, the answer is zero, because ten to the power zero is one.
- What you must be able to do is say what one means and read an axis built from them. And notice when a figure has been drawn on one without saying so.

EXERCISE 1 (interpretation). A figure has a vertical axis marked 1, 10, 100, 1,000, 10,000, with the marks evenly spaced. A line climbs steadily across it, at a constant angle, from the bottom left to the top right. Say what a straight line on this figure means about the quantity, and say what the same data would look like on an ordinary scale.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (calculation). Take the four penalty ceilings from the Food Safety and Standards Act, 2006 in the worked example. Say how many times larger the largest is than the smallest, first by dividing and then by subtracting the logarithms. Then say what the logarithm of ten crore would be.

PRACTICE. Thirteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Give the logarithm to base ten of each of these, without a calculator.

> 100 10,000 1 10^9

2. Each of these logarithms belongs to a number. Say which number.

> 3 6 0 -2

3. Without a calculator, say between which two whole numbers the logarithm of 4,800 lies. Then say which of the two it is closer to, and why.

4. Work out 100 times 10,000 using logarithms instead of multiplication. Take the logarithm of each number, add the two logarithms, and turn the total back into a number. Then check the result by multiplying directly.

5. Section 21(2) of the Consumer Protection Act, 2019 caps a penalty at ten lakh rupees. Sections 34 and 47 cap District and State Commission complaints at one crore and ten crore. Give the logarithm of each figure.

6. Four vertical axes are marked like this, bottom to top, in every case with the marks evenly spaced up the page.

A	1 lakh	2 lakh	3 lakh	4 lakh
B	1 lakh	10 lakh	1 crore	10 crore
C	0	250	500	750
D	100	1,000	10,000	1,00,000

Say which are logarithmic and which are ordinary. For each logarithmic one, say what a single step up the page means.

7. Take the four penalty figures from the Food Safety and Standards Act, 2006: 25,000, 3,00,000, 5,00,000 and 10,00,000. Using a calculator for the logarithms, say how many times larger the three lakh figure is than the twenty-five thousand one, by subtracting logarithms.

8. Section 3(1) of the National Food Security Act, 2013 gives 5 kilograms of grain per person per month. A block office is planning for 2,00,000 people. Work out the kilograms needed in one month by adding logarithms rather than by multiplying, then check the answer directly.

9. A figure has a vertical axis marked 100, 1,000, 10,000 and 1,00,000, evenly spaced. Two points sit on it: one at 300 and one at 30,000. Say how far apart they are in logarithms, and how many times larger one value is than the other.

10. Here is a worked answer. Find the step that broke.

```
logarithm of 200 = 2.3
logarithm of 300 = 2.5
2.3 plus 2.5 = 4.8
so 200 plus 300 = 10^4.8 = about 63,000
```

11. Here is a claim about a figure whose vertical axis runs 1, 10, 100, 1,000 in evenly spaced marks.

> *The line rose one mark between 2015 and 2020, and one mark again between 2020 and 2025, so the increase was steady.*

Find what is wrong with the word “steady”.

12. Here is a caption under a figure in a report.

> *Values below the axis minimum are not shown. The vertical axis is logarithmic.*

Decide what to work out about the data, and say what the figure cannot establish.

13. A colleague shows you two charts of the same data and asks which to use in a briefing. One has an ordinary vertical axis, the other a logarithmic one. On the log chart the rise looks modest; on the ordinary one it looks dramatic. Decide what to work out, and say what your answer does not settle.

A8 · Orders of magnitude and the back-of-envelope sanity check

DEFINITION. The order of magnitude of a quantity is the power of ten nearest to it. Two quantities are of the same order of magnitude when their ratio is smaller than about ten.

An order-of-magnitude estimate rounds every input to one significant figure, carries out the arithmetic on the powers of ten, and claims the answer only to the nearest power of ten.

Its purpose is to test a figure, not to produce one. [12]

IN PLAIN TERMS. Round every number in sight to one digit and a power of ten. Do the arithmetic on the powers of ten, which is adding and subtracting small numbers. Then compare the answer against something whose size you already know.

What you get back is not an answer. It is a verdict on somebody else's answer.

ILLUSTRATION. Somebody asks you, in a meeting, roughly how much food energy a district's school meal programme puts out in a year. You do not have the figures. Do it anyway, on paper, in front of them.

The second schedule of the National Food Security Act, 2013 sets a lower primary hot cooked meal at 450 calories. Round it to one digit.

```
450 calories, to one digit: 5 times 10^2
```

Now the two numbers you do not have. Write each one down and mark it.

```
children in the district    1,00,000 = 10^5    ← a guess
school days in a year    200    = 2 times 10^2    ← a guess
```

Multiply, working on the powers of ten.

```
5 times 10^2, times 10^5, times 2 times 10^2
= (5 times 2) times 10^(2 plus 5 plus 2)
= 10 times 10^9
= 10^10 calories in a year
```

An estimate is only worth anything once you compare it against something.

So compare. About how much does one person eat in a year? Take two thousand calories a day and mark it a guess as well.

```
2,000 a day = 2 times 10^3    ← a guess
365 days    = 4 times 10^2    (rounded to one digit)
2 times 10^3, times 4 times 10^2 = 8 times 10^5, call it 10^6 calories a year
```

Now divide the programme by the person.

> 10^{10} divided by 10^{6} = $10^{(10 \text{ minus } 6)}$ = 10^{4}

The programme delivers about as much food energy in a year as ten thousand people eat in a year. Does that sound right for a lakh of children getting one meal on some of the days?

Check it roughly. One meal of 450 against a day of 2,000 is about a fifth. Two hundred school days out of 365 is about half. A fifth of a half is a tenth, and a tenth of a lakh of children is ten thousand. It agrees, so nothing has fallen over.

Now the failure, which is why anyone bothers with the second half. Suppose that while typing you had put the district's children at ten lakh instead of one lakh.

> 10^{6} children instead of 10^{5}
> answer becomes 10^{11} calories
> 10^{11} divided by 10^{6} = 10^{5}

A hundred thousand people's worth of annual eating, from school meals, in one district. Hold that against a district population of a few lakh and it cannot be right. The school meals would be feeding a large part of everybody, all year, on one meal a day for part of the week.

Notice what caught it. Not the arithmetic, which was faultless both times. The comparison against something whose size you already knew. That is the whole method, and the first half of it is useless without the second.

Last, what you say out loud when you have done this. Not "it is ten to the ten". Say something like this.

> *On a rough calculation this comes out around the amount ten thousand people eat in a year. I assumed a lakh of children and two hundred school days. If it is wrong by a factor of ten, one of those assumptions is wrong and I want the real figures.*

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. This method is built to catch a factor of ten and it is blind to a factor of two. It answers one question: is this number possible.

The method only works if the thing you compare against is one you actually know. Compare against another estimate and you have two guesses agreeing with each other, which feels like confirmation and is not.

And rounding to one digit cuts both ways. Round three numbers down and your answer is low by a factor of two or three before you start.

MUST KNOW POINTS FOR YOU.

- An order-of-magnitude estimate is not a rough version of the right answer. It is a different tool with a different job: deciding whether a number somebody else has given you is possible.

- Then compare the answer against a quantity whose size you already know. The comparison is not the last step. It is the step that does the work.
- Write down every number you supplied yourself and label it a guess, in the working, where somebody can argue with it.
- This catches an error of a factor of ten and never one of a factor of two. So it cannot be used to argue that an effect is big enough to matter.
- When a journalist reads you a figure on the phone, you can usually say within thirty seconds whether it is possible. Say exactly that and no more: whether it is possible, and what you assumed.
- Indian figures cross between lakh, crore and million inside one discussion, and this is where a power of ten goes missing.
- In a committee, an estimate offered with its assumptions written on the page is very hard to dismiss and very easy to improve. Someone will correct one of your guesses, which is the point.
- Asked to check a calculation under time pressure, do not redo it. Estimate it independently and see whether the two land on the same power of ten.

EXERCISE 1 (calculation). A note claims that a state's school meal programme served 2 crore meals last year. Take the 450 calorie lower primary standard from the second schedule of the National Food Security Act, 2013. Estimate the total food energy. Then estimate how many people's annual eating that represents. Write down every assumption you make and mark it. Then say whether the claim of 2 crore meals is possible for a state, and what you assumed to decide that.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A press note says a proposed scheme will "reach 12 crore children at a cost of 40 crore rupees a year". Do an order-of-magnitude check on the cost per child and say what you conclude. Then say what your check can and cannot establish.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Round each of these to one digit and a power of ten.

```
4,820    61    938,000    0.0037
```

2. Estimate each of these by rounding both numbers to one digit first. Then say, for the first one, whether your estimate is above or below the true answer.

```
412 times 7,900    61,000 divided by 290
```

3. Two quantities are 4 times 10^6 and 9 times 10^7. Are they of the same order of magnitude? Show the working that decides it.

4. Estimate the number of school meals served in one Indian district in a school year. Mark every input you supply yourself, and give your answer to the nearest power of ten.

5. Section 3(1) of the National Food Security Act, 2013 gives 5 kilograms per person per month. Estimate the tonnes a district needs in a year. Mark every input you supply, and say which one your answer depends on most.

6. Here is a worked estimate. Find the step that broke.

> 3 times 10^4 households
> each getting 20 kilograms a month
> 3 times 20 = 60
> so 60 times 10^4 = 6 times 10^4 kilograms a month

7. Here is a worked estimate. Find the step that broke.

> a scheme covers 30 lakh people
> each is to get 2 kilograms of grain a month
> 30 lakh = 3 million = 3 times 10^5
> 3 times 10^5, times 2 = 6 times 10^5 kilograms a month
> = 600 tonnes a month

8. Here is a worked estimate. Find what is wrong with it, and it is not the arithmetic.

> a district has about 10 lakh people ← a guess
> about 2 per cent need the service ← a guess
> each needs about 3 visits a year ← a guess
> 10^6 times 0.02 times 3 = 6 times 10^4 visits a year

9. Here is a worked estimate and the conclusion drawn from it. The arithmetic is right. Say what is wrong with the conclusion.

> district A 4 lakh households ← a guess
> 3 visits each a year ← a guess
> 4 times 10^5, times 3 = 1.2 times 10^6 visits
> district B 6 lakh households ← a guess
> 3 visits each a year ← a guess
> 6 times 10^5, times 3 = 1.8 times 10^6 visits
> so district B needs half as many staff again as district A

10. Here is a claim from a press note.

> *The programme will provide 50 lakh children with a daily glass of milk for 200 days at a cost of 100 crore rupees.*

Decide what to compute, compute it, and say what your answer does not establish.

11. A journalist reads you a figure over the phone and wants a comment in the next ten minutes.

> *A new tax on sugary drinks will raise 40,000 crore rupees a year.*

Decide what to compute, compute it, say what you would tell the journalist, and say what your answer does not establish.

References · Part A

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4. Government of India. The Food Safety and Standards Act, 2006. Act 34 of 2006. Consulted at ss. 91-93 (power to make rules and regulations; laying before Parliament). figures quoted in the text. India Code. https://indiacode.gov.in/handle/123456789/586348 (accessed 2026-09-19).

5. Marecek, Lynn and Anthony-Smith, MaryAnne and Mathis, Andrea Honeycutt. Prealgebra 2e. 2020. ch. 4, Fractions. https://openstax.org/books/prealgebra-2e/pages/1-introduction (accessed 2026-09-22). *Outstanding — see the note below.*

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11. OpenStax. College Algebra 2e. Openly licensed. Carries logarithms, which Prealgebra does not - the reason Part A needs two anchors rather than one. ch. 6, Exponential and Logarithmic Functions. https://openstax.org/books/college-algebra-2e (accessed 2026-09-22). *Outstanding — see the note below.*

12. Marecek, Lynn and Anthony-Smith, MaryAnne and Mathis, Andrea Honeycutt. Prealgebra 2e. 2020. ch. 10, Polynomials, scientific notation. https://openstax.org/books/prealgebra-2e/pages/1-introduction (accessed 2026-09-22). *Outstanding — see the note below.*

An entry marked outstanding has not yet been checked against the source itself. Nothing in these sections rests on one: the reasoning is a derivation you can check with a calculator, and the entry records an anchor still to be supplied.

Part B · Units and dimensions

B1 · What a unit is, and the SI base and derived units

Checked against the sources as they stood on 2026-09-22.

DEFINITION. A measurement is a number together with a unit. A number given without its unit is not a measurement.

The International System of Units (SI) is the system of units in which the numerical values of seven defining constants are fixed exactly. Since 20 May 2019 all SI units are defined in terms of constants that describe the natural world.

Seven base units keep their role inside that system: the second, the metre, the kilogram, the ampere, the kelvin, the mole and the candela.

A derived unit is a product of powers of the base units. The joule, written J, is the SI unit of energy, and J = kg m^2 s^{-2}. [1, 2, 3, 4, 5, 6, 7]

IN PLAIN TERMS. A measurement has two halves. A number, and a unit. Take one half away and the other half says nothing.

A unit is an agreed amount that everybody measures against. One metre is an agreed length.

The agreement is only worth something if it is the same agreement everywhere. That is the job the SI does.

Seven units are called base units. They are the second, the metre, the kilogram, the ampere, the kelvin, the mole and the candela.

Every other unit is built out of those seven by multiplying and dividing. A unit built that way is called a derived unit.

ILLUSTRATION. A strip of tablets is labelled 500. Five hundred what? Five hundred milligrams and five hundred micrograms are both ordinary strengths, and one is a thousand times the other. So you go and find the unit, every time, and you never carry a bare number into a calculation.

Now build some units yourself. You only need two to start: the metre for length and the second for time.

Speed is how far, divided by how long. So its unit is a length divided by a time.

```
metres divided by seconds
= m/s
```

Area is a length times a length.

```
metres times metres
= m2
```

Volume is a length three times over, which gives m^3. Density is a mass divided by a volume, which gives kg/m^3.

Now go and read the SI's own words. The page is called The International System of Units (SI), and it is published by the International Bureau of Weights and Measures (BIPM) at https://www.bipm.org/en/measurement-units. Open it and search inside the page for "J =".

```
J = kg m2 s-2
```

Say it out loud as a sentence. A joule is a kilogram, times a square metre, divided by a second squared.

A prefix is a name for a power of ten stuck on the front of a unit. It changes the size of the unit and nothing else.

prefix	multiply the unit by	so
kilo	10^3	one kilometre is 1,000 metres
centi	10^{-2}	one centimetre is 0.01 metres
milli	10^{-3}	one milligram is 0.001 grams
micro	10^{-6}	one microgram is 0.000001 grams

The symbol for kilo is k, for centi is c and for milli is m. The symbol for micro is the Greek letter mu, written μ.

One minute is 60 seconds. One hour is 3,600 seconds.

Figures quoted in this illustration are from [8].

WHERE THIS PICTURE BREAKS. Building a unit out of base units tells you what kind of quantity you are holding. It tells you nothing about whether the number beside it is right.

And all of this is decided, not discovered.

MUST KNOW POINTS FOR YOU.

- It is not a measurement at all. Send it back for the unit. Never work out which unit was meant from how big the number looks.
- Read the unit off the label or the report before you read the number.
- On its own m is the metre; in front of another unit it is milli, meaning a thousandth. So mg is a milligram.
- A derived unit is a recipe rather than a label. When an unfamiliar quantity turns up in a paper, read its unit before you read its value.
- The SI fixes every unit through seven constants with exact values. One of them is the speed of light in vacuum, at 299 792 458 metres per second.
- Getting the units right proves nothing about accuracy. A machine reading twice the true value still reports in correct units.

EXERCISE 1 (retrieval). Without looking back, write down the seven SI base units. Then say in one sentence what makes a unit a derived unit, and build two derived units yourself from base units alone.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is a column out of a block stores register. The header says Rice. The entries under it are 250, 180, 4.2 and 300.

Say what you would ask for before using any of it, and say what the 4.2 tells you.

EXERCISE 3 (teaching). A first-year resident tells you they can never remember whether the joule is kg m^2 s^{-2} or kg m s^{-2}. You have ten minutes and a whiteboard. Teach them so that the question stops arising.

B2 · Converting units as multiplication by one

DEFINITION. A conversion factor is a fraction whose top and bottom are the same quantity written in two different units. Because the top and the bottom are equal, the fraction is equal to one.

The orientation to use is the one that puts the unwanted unit where it cancels. [9]

IN PLAIN TERMS. One kilometre is 1,000 metres. That is an agreement, and you can write it as a fraction in two ways.

```
1,000 m over 1 km
1 km over 1,000 m
```

Both of those fractions are equal to one.

Multiplying by one does not change an amount. Only the name on it changes.

Which of the two do you use? The one that cancels the unit you want to get rid of.

Turn 7 kilometres into metres. You want the km gone, so put km on the bottom.

```
7 km times 1,000 m over 1 km
= 7 times 1,000 m
= 7,000 m
```

Metres are left, which is what you asked for.

ILLUSTRATION. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month. Somebody on a call asks you what that is in grams a day.

Here is the answer that comes back first. It is wrong.

```
5 divided by 1,000 = 0.005
```

Nothing is wrong with that arithmetic. What is wrong is the choice to divide.

A gram is smaller than a kilogram, so the number of grams has to be bigger than 5, not smaller. And written out as a factor, the unit that comes out is not grams at all.

```
5 kg times 1 kg over 1,000 g
= 0.005 kg squared per gram
```

Now do it properly. Put the kg where it cancels.

```
5 kg per person per month, times 1,000 g over 1 kg
= 5,000 g per person per month
```

Five thousand grams. Bigger number, smaller unit, as it had to be.

There is no agreed number of days in a month, so you have to choose one and say so. Take 30 days.

```
5,000 g per person per month, times 1 month over 30 days
= 166.67 g per person per day
```

About 167 grams a day.

Write the quantity, then each factor, then cancel.

```
5 kg per person per month
times 1,000 g over 1 kg
times 1 month over 30 days
= 166.67 g per person per day
```

A store has 45,000 grams of a commodity and the register wants kilograms.

```
45,000 g times 1 kg over 1,000 g
= 45 kg
```

The same factor, the other way up, chosen for the same reason: it puts the unwanted unit where it cancels.

Figures quoted in this illustration are from [10].

WHERE THIS PICTURE BREAKS. The 30 days is a choice, not a fact.

The Act gives a monthly entitlement. The daily figure is yours, and you say so when you quote it.

And conversion only ever works between two units of the same kind. There is no factor from kilograms to litres, or from grams to rupees.

MUST KNOW POINTS FOR YOU.

- A conversion factor is not a rule about when to multiply and when to divide. It is a fraction equal to one, and which way up you write it is settled by which unit you want to cancel.
- Check the unit that fell out of the working. If it is not the unit you were asked for, the factor was upside down.
- Write the unit beside every line while you work, not once at the end.
- Run the size check before you act on any converted dose or dispensed amount. A bigger unit gives a smaller number. A smaller unit gives a bigger number.
- There is a conversion factor only between two units of the same kind.
- A monthly entitlement becomes a daily figure only by choosing a number of days. You make that choice, and no statute makes it for you.

EXERCISE 1 (calculation). A ward register records a child's height as 96 centimetres. It records a sack of grain as 0.4 kilograms per packet, with 25 packets to the sack.

Write the height in metres. Write the sack in grams. Show the factor you used each time, and say which unit cancelled.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A colleague sends you this line and asks you to check it before it goes into a note.

> *Each packet holds 250 millilitres, so the 40-packet carton holds 10 millilitres.*

Say what has gone wrong and give the right answer. Then say what you would have them change in how they work, rather than in this one line.

PRACTICE. Eight problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Convert each of these.

```
3 km into metres
4,500 g into kilograms
2.5 m into centimetres
```

2. Write the conversion factor between kilograms and grams both ways up. Say which one turns grams into kilograms, and say why the other one does not.

3. Do each of these with two factors in one line.

```
2 hours into seconds
0.75 kg into milligrams
```

4. The proviso to section 3(1) of the National Food Security Act, 2013 covers households under the Antyodaya Anna Yojana. It entitles such a household to 35 kilograms of foodgrains per household per month. Two limits ride with that figure. The entitlement runs to the extent the Central Government specifies for each State under the scheme. And the grain comes at the prices set out in Schedule I of the Act.

Write that 35 kilograms per household per month in grams per household per month, and in kilograms per household per year. Keep the whole denominator on every answer. Then say in one line what those two limits stop you from saying about any one household.

5. A chart is written in micrograms and the label on the bottle is in milligrams.

Write 0.25 milligrams in micrograms. Then write 4,000 micrograms in milligrams. State which orientation of the factor you used each time.

6. Here is a worked answer. Find the step that broke.

```
a prescription is written as 0.5 g and the chart wants milligrams
1 gram is 1,000 milligrams
0.5 g times 1 g over 1,000 mg
= 0.0005
so the chart entry is 0.0005 mg
```

7. Here is a worked answer. Find the step that broke.

```
a plot covers 12,000 square metres and the report wants square kilometres
1 kilometre is 1,000 metres
12,000 divided by 1,000 = 12
so the plot covers 12 square kilometres
```

8. In a meeting somebody says this about two strengths of the same medicine.

> *The new tablet is 500 micrograms and the old one was 0.5 milligrams, so the new one is a thousand times stronger.*

Decide what to compute, compute it, and say what your answer does not establish.

B3 · Dimensional analysis as an error check

DEFINITION. Every measured quantity has a dimension, which is the kind of thing it is: a length, a mass, a time, or a combination of these built by multiplication and division. The unit is how that dimension is measured. Two quantities of the same dimension may be measured in different units.

An equation between physical quantities must carry the same dimension on both sides, and two quantities may be added or subtracted only when they have the same dimension.

Multiplying quantities multiplies their units, and dividing them divides their units.

Matching dimensions on both sides cannot confirm a formula, because a wrong dimensionless factor changes no unit. [11]

IN PLAIN TERMS. You can often show that a formula is wrong without knowing the right formula. You do it by looking at the units and ignoring the numbers.

Both sides of an equals sign have to carry the same kind of quantity. The word for the kind of thing a quantity is, is its dimension.

You can only add or subtract quantities of the same dimension, and then only once both are written in the same unit. Three metres plus four seconds is not seven of anything, because a length and a time are different kinds of thing.

Multiplying and dividing behave the way they do with plain numbers. Metres times metres give the unit m^2. You can cancel them the way you cancel a shared number in a ratio.

So here is the check, in four steps.

1. Throw the numbers away and keep the units.
2. Write the unit of every quantity on both sides.
3. Cancel anything that appears on the top and on the bottom.
4. Compare the two sides. Different units mean the formula is wrong.

Matching units do not make a formula right. Halving a quantity, doubling it, or multiplying it by 3.14 leaves every unit exactly where it was.

ILLUSTRATION. > The stretch is 12 km and the bus averages 40 kilometres an hour, so allow 480 minutes.

Take the numbers out and watch what the units do.

```
km times km per hour
= km² per hour
```

A square kilometre per hour is an area divided by a time. It is not a time at all.

You want hours to be left standing at the end.

```
km divided by km per hour
= km times hour over km
= hours
```

Dividing does it. The kilometres cancel and the hours are left.

Put the numbers back.

```
12 divided by 40 = 0.3
```

That is 0.3 hours. Turn it into minutes with a conversion factor, as you did in the section before this one.

```
0.3 h times 60 min over 1 h
= 18 min
```

Eighteen minutes, not 480.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month. Somebody proposes a way of working out a household's grain for a year.

> *yearly grain = 5, divided by the number of people, times 12*

Strip the numbers out again and write only the units.

```
kilograms per person per month
divided by persons
times months
= kilograms per person squared
```

The months cancelled, which is right. You are left with kilograms per person squared, and there is no such quantity.

Ask what arrangement would leave kilograms standing instead.

```
kilograms per person per month
times persons
times months
= kilograms
```

So you multiply by the number of people. You do not divide by it. Now put the numbers in, for a household of four.

```
5 times 4 times 12 = 240
```

240 kilograms per household per year. The 240 is your arithmetic on it, and it is not a figure the Act carries.

A unit check throws out wrong arrangements. It never hands you a right formula.

Figures quoted in this illustration are from [10].

WHERE THIS PICTURE BREAKS. This check catches a formula of the wrong shape. It cannot catch a formula of the right shape and the wrong size.

Halving a quantity leaves every unit where it was. A formula can pass this check and still give you an answer twice the truth.

It also says nothing about the quantities themselves. Kilograms of grain and kilograms of anything else carry the same unit.

And the persons in the working above are not an SI unit.

MUST KNOW POINTS FOR YOU.

- Run the check in four steps when a formula you did not write lands in front of you.

- Matching units never make a formula right.
- The same letters are not the same units. Read the powers before you agree that two units match.
- You can reject a formula in a paper, a protocol or a spreadsheet without being able to derive the right one.
- Read the unit that falls out of any calculated dose before you act on it.
- This check finds an error in the shape of a formula and never an error in a plain number multiplying it.
- Teach a trainee who cannot remember a formula to check the units of the one in front of them.

EXERCISE 1 (critique). A colleague has checked somebody's formula and reports back in one line.

> *The units come out right on both sides, so the formula is correct.*

Say why that does not follow. Then give an example of a formula that passes a unit check and is still wrong.

EXERCISE 2 (teaching). A first-year resident says they can never remember formulas, and treats that as the end of the matter. You have ten minutes and a whiteboard.

Teach them the unit check so that they leave able to use it on the next formula they meet.

PRACTICE. Fourteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Simplify each of these unit expressions.

```
m/s times s
m times m times m
kg divided by m^3, times m^3
```

2. Which of these pairs can be added together as they stand, which can be added after a conversion, and which cannot be added at all?

```
3 m and 4 m
3 m and 4 s
3 kg and 4 g
3 m^2 and 4 m
```

3. Work out the unit of each of these quantities from what it is built out of. Use metres, kilograms and seconds.

```
a mass divided by a volume
a length divided by a time
a length divided by a time, and then divided by a time again
a number of people divided by an area
```

4. Cancel each of these down as far as it will go.

```
kg m² s⁻² divided by kg
kg m² s⁻² divided by s
m² divided by m
```

5. The SI states that J = kg m^2 s^{-2}, where J is the joule. Write out the unit of each of these in base units, cancelled as far as it will go.

```
J divided by kg
J times s², divided by m²
J divided by s
```

6. A protocol works out a total amount of a drug like this.

> *total amount = strength times volume*

The strength is recorded in milligrams per millilitre. The volume is recorded in millilitres.

Check the units of the right-hand side and say what unit the total amount comes out in.

7. A district note plans grain for a priority household of six people. It says the household should be counted for 360 kilograms of foodgrains in a year.

Work back to the rate for one person for one month, keeping the unit on the answer. Say which units you cancelled to get there. Then say whether the rate you land on is the one section 3(1) of the National Food Security Act, 2013 states.

8. A bus covers 18 kilometres in 30 minutes. A note says the speed is worked out as the time divided by the distance.

Say what unit that gives and what it is not. Then write the arrangement whose units come out as a speed, and work the speed out.

9. A block of material has a mass of 2,400 kilograms and a volume of 1.2 cubic metres. A note works out its density as the volume divided by the mass.

A density is a mass in every unit of volume. Check the note's units against that, fix the arrangement, and work the density out.

10. Here is a worked answer. Find the step that broke.

```
a rectangular plot measures 30 m by 20 m
the fencing needed is 30 times 20 = 600
so 600 m of fencing is needed
```

11. Here is a worked answer. Find the step that broke.

```
the entitlement is 5 kilograms per person per month
a year is 365 days
5 times 365 = 1,825
so the entitlement is 1,825 kilograms per person per year
```

12. Here is a worked answer. Find the step that broke.

```
energy in the SI has the unit kg m^2 s^-2
a note proposes that energy is a mass times a speed
a mass is in kg and a speed is in m/s
kg times m/s gives kg m/s
kg m/s contains kg, m and s, and so does kg m^2 s^-2
so the units match and the formula is fine
```

13. A press note crosses your desk with this line in it.

> *The plant treats 4.5 crore litres a day, which works out at 135 litres a person for the city's 30 lakh people.*

The plant, the city and both figures are made up for this problem. No real plant and no real population is being described.

Decide what to compute, compute it, and say what your answer does not establish.

14. A supplier's brochure makes this claim about a water treatment unit for a village tank.

> *It doses at 2 milligrams per litre. So a 5,000 litre tank needs 2 divided by 5,000, which is 0.0004 grams of chlorine a fill.*

The brochure, the unit and both figures are made up for this problem. Do not carry the 2 milligrams per litre away as a chlorine dose. What a tank should be dosed at is set by whoever sets the standard. This problem is about the arrangement of the units and nothing else.

Decide what to compute, compute it, and say what your answer does not establish.

B4 · Energy units: joule, kilojoule, calorie, kilocalorie

Checked against the sources as they stood on 2026-09-22.

DEFINITION. The joule is the SI unit of energy.

A kilojoule is one thousand joules. The prefix kilo means one thousand.

The calorie is not an SI unit. Its size in joules is fixed by agreement, not by measurement, and two agreements are in use at once. The thermochemical calorie is 4.184 joules exactly. The International Table calorie, settled at the Fifth International Conference on the Properties of Steam in July 1956, is 4.1868 joules.

A kilocalorie is one thousand calories. It is the calorie used to express the energy content of foods, and in food writing the prefix kilo is usually dropped.

In India, where a pre-packaged food has to carry nutritional information, the energy is declared in kilocalories. Regulation 5(3)(b) of the Food Safety and Standards (Labelling and Display) Regulations, 2020 requires the label to carry the energy value in kcal. Those regulations require no kilojoule figure anywhere.

Regulation 5(3)(e)(i) of the same regulations fixes the factors by which the declared energy is calculated rather than measured. [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]

IN PLAIN TERMS. Energy is measured, so energy has a unit. The SI unit is the joule.

A thousand joules is one kilojoule. Kilo means a thousand, here and everywhere else.

The calorie is not part of the SI. Its size was decided by committees, and two of them decided differently.

Both numbers are exact, because both are decisions. Nobody went to a laboratory and got 4.184.

A kilocalorie is a thousand calories. It is the unit used for the energy in food.

In food work people write calories when they mean kilocalories. So when a packet, a paper or a scheme document says calories, read kilocalories. Then check the size of the number to be sure.

ILLUSTRATION. Start with the mistake, because it is the one waiting for you.

Open the National Food Security Act, 2013, and go to Schedule II. Find the row for lower primary classes. The hot cooked meal is set at 450.

Take that 450 as calories and turn it into joules.

```
450 times 4.184 = 1,882.8
```

A meal of 1,882.8 joules. Something has gone wrong, and it is not the arithmetic.

Go back to the table and read the head of the column instead of the number under it. It says Calories, and then, in brackets, Kcal.

So the meal is 450 kilocalories. Do it again properly, one line at a time.

```
450 kilocalories, first into calories
450 times 1,000 = 4,50,000
now into joules
4,50,000 times 4.184 = 18,82,800
and into kilojoules
18,82,800 divided by 1,000 = 1,882.8
```

The digits are the same and the unit is not. Kilocalories to kilojoules uses the same 4.184 as calories to joules, because both sides of the sum were multiplied by a thousand.

Take the upper primary figure from the same table, 700 kilocalories, and convert it with each of the two calories in turn.

```
700 times 4.184 = 2,928.8
700 times 4.1868 = 2,930.76
```

Two answers, from one figure, both correct.

```
2,930.76 minus 2,928.8 = 1.96
```

Just under two kilojoules apart. The gap is in the fourth digit.

If their kilojoule figure and yours part company in the fourth digit, neither of you has slipped. You used different calories.

Figures quoted in this illustration are from [10, 23].

WHERE THIS PICTURE BREAKS. The two conversion factors are exact and they stay exact.

What a conversion cannot do is improve the number you started with. It renames an amount.

MUST KNOW POINTS FOR YOU.

- Nobody measured the calorie. Committees fixed it, and both of their values are exact by definition.
- One thermochemical calorie is 4.184 joules. That is the factor to use unless a paper tells you otherwise, and the same 4.184 turns kilocalories into kilojoules.
- A second calorie is in use. The International Table calorie is 4.1868 joules. Check which calorie each of them used before you tell either of them they are wrong.
- When food material says calories, read kilocalories, because the prefix is usually left off in practice. Then check the size. A meal in the hundreds is kilocalories.
- Say the unit out loud with every energy figure you quote, in clinic and in a meeting.
- Converting a figure never improves it. A two-digit figure written out to six digits after conversion has gained nothing but the look of precision.
- Schedule II of the National Food Security Act, 2013 heads its energy column Calories, with Kcal in brackets.
- An Indian food packet that must show nutritional information gives its energy in kilocalories. The rule is regulation 5(3)(b) of the labelling regulations of 2020. It asks for no kilojoule figure at all, so you must be able to convert.
- The energy printed on an Indian label is worked out, not measured. Regulation 5(3)(e)(i) of the same 2020 rules fixes the factors. Carbohydrate and protein are 4 kcal/g each, fat is 9 kcal/g and alcohol is 7 kcal/g. So take the grams printed on the label, multiply each by its factor, and add them up.

EXERCISE 1 (critique). Here is a line from a draft teaching note.

> *Careful measurement has established that one calorie is equal to 4.184 joules.*

Say what is wrong with it, and write the sentence you would put in its place.

EXERCISE 2 (teaching). A journalist has three minutes and asks you what the energy number on a food packet actually means. Explain it without jargon. Give one quotable sentence, and say plainly how much the two-calorie business matters to a reader.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Write each of these as the other unit asks for.

```
1,000 joules into kilojoules
2.5 kilojoules into joules
3 kilocalories into calories
7,000 calories into kilocalories
```

2. Turn each of these into joules, using the thermochemical calorie.

```
1    10    250    1,500
```

3. Now go the other way. Turn each of these joule figures into thermochemical calories.

```
8.368    2,092    4,184
```

4. Schedule II of the National Food Security Act, 2013 sets the hot cooked meal for lower primary classes at 450 kilocalories. Give that figure in kilojoules, using the thermochemical calorie, and keep the unit on the answer. Then say how many digits you would write down.

5. The same Schedule sets the upper primary hot cooked meal at 700 kilocalories. Convert it twice, once with the thermochemical calorie and once with the International Table calorie. Say how far apart the two answers are, in kilojoules and as a share of the figure.

6. A figure of 2,000 kilojoules is handed to you and the meeting wants it in kilocalories. Convert it, then say which of the two calorie definitions could have changed your answer at two digits.

7. A note gives a day's energy figure as 8,400 kilojoules. Turn it into kilocalories, then turn your answer back into kilojoules and say whether you get the number you started with.

8. A food packet gives, for every 100 g, 60 g of carbohydrate, 8 g of protein and 22 g of fat.

Regulation 5(3)(e)(i) of the labelling regulations of 2020 fixes the factors. Carbohydrate is 4 kcal/g. Protein is 4 kcal/g. Fat is 9 kcal/g.

Work out the energy the label should declare for every 100 g. Then give the same figure in kilojoules, using the thermochemical calorie.

9. Here is a worked answer. Find the step that broke.

```
a meal is set at 450 kilocalories
one calorie is 4.184 joules
450 divided by 4.184 = 107.6
so the meal is 107.6 kilojoules
```

10. Here is a worked answer. Find the step that broke.

```
a portion is 250 kilocalories
one calorie is 4.184 joules
250 times 4.184 = 1,046
so the portion is 1,046 joules
```

11. A colleague sends you this line.

> *The two papers disagree. One gives the energy content of the food as 2,510 kilojoules. The other gives 600 kilocalories. So at least one of them is wrong.*

Decide what to compute, compute it, and say what your answer does not establish.

12. In a planning meeting somebody says this.

> *Their dataset is in kilojoules and ours is in kilocalories, so the two cannot be compared. We would have to collect ours again.*

Decide what to compute, compute it, and say what your answer does not establish.

B5 · Concentration units: mg/dL, mmol/L, and converting between them

DEFINITION. A concentration is an amount of something held in a volume of something else.

A mass concentration is a mass in a volume. Milligrams per decilitre, written mg/dL, is a mass concentration.

An amount-of-substance concentration is an amount of substance in a volume. Millimoles per litre, written mmol/L, is an amount-of-substance concentration. A millimole is one thousandth of a mole.

The molar mass of a substance is the mass of one mole of it. Molar mass is a property of the substance and differs from one substance to another.

Converting a mass concentration into an amount-of-substance concentration therefore requires the identity of the substance.

A molar mass is measured and computed, not decided, so it can be revised. [24, 25, 26]

IN PLAIN TERMS. Milligrams per decilitre, written mg/dL, is a weight of something in a volume of blood. What mg/dL counts is weight.

Millimoles per litre, written mmol/L, counts something else. A mole is a fixed count of things, molecules here. What mmol/L counts is how many of those things there are.

Those are two different kinds of quantity. One says how heavy. The other says how many.

To get from how heavy to how many, you have to know how heavy one mole is. That weight is called the molar mass, and it is written in grams per mole, g/mol.

The molar mass belongs to the substance. Glucose has one. Cholesterol has a different one. So there is no single conversion factor between mg/dL and mmol/L, and there never can be.

So the job has four steps.

1. Find out what the substance is. Read the row label, not the number.
2. Get its molar mass in grams per mole. Look it up. Do not guess it.
3. Multiply the mg/dL figure by ten, because a litre holds ten decilitres. That gives milligrams per litre.
4. Divide by the molar mass. That gives millimoles per litre.

To go the other way, run the same steps backwards. Multiply by the molar mass, then divide by ten.

A conversion renames an amount. It does not make the amount better known.

ILLUSTRATION. Start with the sum you cannot do, because that is the one this section is really about.

Do the glucose one first.

The record is compound 5793. The page gives the molecular formula as C6H12O6 and the molecular weight as 180.16 g/mol.

Take a glucose value of 100 mg/dL and work it a line at a time.

```
100 mg in every decilitre
a litre holds ten decilitres
100 times 10 = 1,000
so 1,000 mg in every litre
one mole weighs 180.16 grams, so one millimole weighs 180.16 milligrams
1,000 divided by 180.16 = 5.5506
so 5.5506 mmol in every litre
```

Write 5.55 mmol/L. The calculator's 5.550621670 is a claim about the laboratory's work, not about yours.

Now the cholesterol, and watch it stop.

Go back to PubChem and search for cholesterol. The record is compound 5997. The page gives the molecular formula as C27H46O.

And you still cannot do the sum.

You cannot turn a mass into an amount of substance without the molar mass of the substance in front of you. Glucose's 180.16 is glucose's, and it says nothing about cholesterol.

You know the method. You know the substance. None of that is enough. The one number the method needs is a measured property of cholesterol, and nothing you know produces it.

Do not reuse glucose's divisor because both numbers came off the same report.

Write down that the conversion is pending. Name what you need, which is cholesterol's molar mass in grams per mole. Go and read it off PubChem compound 5997. Looking it up is not a detour around the method. It is part of the method.

Figures quoted in this illustration are from [27].

WHERE THIS PICTURE BREAKS. The 180.16 g/mol belongs to D-glucose and to nothing else. The working carries over to another substance only once that substance's own molar mass is in your hand.

And the conversion renames an amount. It cannot improve it.

MUST KNOW POINTS FOR YOU.

- There is no single number that turns mg/dL into mmol/L. The factor belongs to the substance, not to the pair of units. Dividing by 18 is a glucose move and it is wrong on everything else.
- One mole of D-glucose weighs 180.16 grams, as PubChem's record for compound 5793 gives it. That one figure moves a glucose value between mg/dL and mmol/L in either direction.
- Convert in four steps and say them out loud. Name the substance. Get its molar mass in grams per mole. Multiply the mg/dL figure by ten, because a litre holds ten decilitres. Divide by the molar mass, and the answer is in mmol/L.
- Two laboratory numbers in the same unit are not two numbers about the same thing. Read the row label before you convert, and carry the substance with every converted value you hand on.
- This conversion needs one substance with one molar mass. A figure that is a mixture reported as a single total has no molar mass, so it cannot be put into mmol/L at all.
- A conversion never adds a digit. A value the laboratory gave you as 100 mg/dL comes out as 5.55 mmol/L, not as 5.550621670 mmol/L.

- When you do not hold the molar mass, say so and stop. Quoting a converted value you cannot source costs you everything you say after it.

EXERCISE 1 (critique). Here is a line from a draft laboratory protocol.

> *To convert any laboratory value from mg/dL to mmol/L, divide by 18.*

Say what is wrong with it, and write the sentence you would put in its place.

EXERCISE 2 (teaching). A journalist has three minutes. They have two papers open. One reports a blood sugar figure as 92 and the other reports it as 5.1, and they want to know which paper is right.

Explain it without jargon. Give one quotable sentence, and say how confident you are.

PRACTICE. Thirteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A litre holds ten decilitres. Rewrite each of these.

```
5 per decilitre, as a figure per litre
0.7 per decilitre, as a figure per litre
120 per litre, as a figure per decilitre
96 per litre, as a figure per decilitre
```

2. Work each of these out and write the answer to four digits.

```
1,000 divided by 180.16
920 divided by 180.16
2,500 divided by 180.16
```

3. Take each number below, multiply it by ten, then divide the result by 180.16. Write each answer to four digits.

```
72    110    45
```

4. Now reverse the order. Take each number below, multiply it by 180.16, then divide the result by ten.

```
4    6.5    9
```

5. A laboratory reports a glucose value of 92 mg/dL. PubChem gives the molecular weight of D-glucose as 180.16 g/mol.

Give that value in mmol/L, keep the unit on the answer, and say how many digits you would write down.

6. A paper gives a glucose value as 7.2 mmol/L and your form needs it in mg/dL. PubChem gives the molecular weight of D-glucose as 180.16 g/mol.

Convert it, keep the unit on the answer, and say how many digits you would write down.

7. People often say that a glucose value in mg/dL becomes mmol/L if you divide it by 18. PubChem gives the molecular weight of D-glucose as 180.16 g/mol.

Work out the exact divisor. Then convert a glucose value of 180 mg/dL twice, once with the exact divisor and once with 18. Say how far apart the two answers are, as a share of the answer.

8. A report gives a glucose value of 110 mg/dL and a total cholesterol value of 200 mg/dL. Somebody asks you for both in mmol/L. PubChem gives the molecular weight of D-glucose as 180.16 g/mol, and its record for cholesterol gives the molecular formula as C27H46O.

Do what can be done. Then say plainly what cannot be done, and what you would do about it.

9. Here is a worked answer. Find the step that broke.

```
total cholesterol is 200 mg/dL
to turn a mg/dL figure into mmol/L you divide by 18
200 divided by 18 = 11.1
so the cholesterol is 11.1 mmol/L
```

10. Here is a worked answer. Find the step that broke.

```
a glucose value is 90 mg/dL
the molecular weight of D-glucose is 180.16 g/mol
90 divided by 180.16 = 0.4996
so the glucose is 0.4996 mmol/L
```

11. Here is a worked answer. Every line of arithmetic in it is correct. Find the step that broke anyway.

```
a glucose value is 100 mg/dL
the molecular weight of D-glucose is 180.16 g/mol
100 times 10 = 1,000
1,000 divided by 180.16 = 5.550621670
so the glucose is 5.550621670 mmol/L
```

12. A colleague sends you this line about a planned pooled analysis.

> *Their glucose figures are all in mmol/L and ours are all in mg/dL. The two sets cannot be pooled, so we would have to sample our participants again.*

Decide what to compute, compute it, and say what your answer does not establish.

13. In a meeting somebody says this.

> *The trial reports mean total cholesterol as 4.5 mmol/L. Our audit reports it as 210 mg/dL. Those are the same figure, near enough, so the two groups are comparable on this.*

Decide what to compute, compute it, and say what your answer does not establish.

B6 · Body-size units: kg, m, cm and kg/m^2

DEFINITION. A centimetre, written cm, is one hundredth of a metre, so one metre is 100 cm.

Dividing a mass in kilograms by the square of a length in metres gives a quantity whose unit is the kilogram per metre squared, written kg/m^2.

A person's mass in kilograms divided by the square of their height in metres is the body mass index, written BMI.

A height entered in centimetres rather than metres therefore divides the index by 10^4.

The index is a number together with its unit. Nothing in its construction states which values are high or low. That is a separate decision, made by a body rather than by the arithmetic. [28]

IN PLAIN TERMS. Mass is measured in kilograms, written kg. Length is measured in metres, written m. So 100 cm make one metre, and any height in centimetres becomes metres when you divide by 100.

Take a mass in kilograms. Take a height in metres and multiply it by itself, which is the squaring you met in the section on powers. Divide the first by the second.

What you get is called the body mass index, written BMI. It puts a mass and a height together into one number, and the short name for it here is the index.

It carries a unit, and the unit comes out of the division: kilograms on top, metres times metres underneath.

That is a derived unit.

The height must be in metres. Put centimetres in and the answer is not slightly off. It is ten thousand times too small, because the squaring squares the mistake as well.

And the index does not mean anything on its own. Whether a value counts as high or low is a decision somebody made, and this section does not make it.

ILLUSTRATION. Do the calculation the wrong way first, because the mistake is the thing worth seeing.

A person weighs 70 kg and stands 168 cm tall. The height is in centimetres, which is how a clinic form usually collects it. Square it as it stands, then divide the 70 by what you get.

```
168 times 168 = 28,224
70 divided by 28,224 = 0.00248 kg/cm²
```

Read the unit that fell out of that division. It is kilograms per centimetre squared, not kilograms per metre squared.

Now do it properly. Put the height into metres first.

```
168 divided by 100 = 1.68
1.68 times 1.68 = 2.8224
70 divided by 2.8224 = 24.8
```

24.8 kg/m^2. Say it out loud as kilograms per metre squared, because the unit is what tells you the first answer was not an answer at all.

One hundred centimetres in a metre, and the denominator is squared, so the mistake is squared too.

```
100 times 100 = 10,000
```

Take a shape and make every length twice as long, keeping everything else the same. Mass goes with volume, and volume has three lengths in it, so the mass goes up by 2 times 2 times 2.

```
2 times 2 times 2 = 8
```

The denominator has only two lengths in it.

```
2 times 2 = 4
```

So the index itself goes up.

```
8 divided by 4 = 2
```

Dividing by the square of a height does not remove size from the number. It removes some of it, and a squared denominator is a choice somebody made rather than a law you can check.

Figures quoted in this illustration are from [8].

WHERE THIS PICTURE BREAKS. The doubling argument holds only while the shape and the density stay the same. Real bodies are not scaled copies of each other.

The ten-thousand check holds only for centimetres against metres.

MUST KNOW POINTS FOR YOU.

- A height in centimetres does not give a slightly wrong index. It gives one ten thousand times too small, because the denominator squares the error.
- Convert the height to metres before anything else, by dividing by 100.
- Say the unit with the number: 24.8 kilograms per metre squared.
- In a dataset, find out which unit height was collected in before you compute a single row.
- Dividing by the square of a height does not take size out of the number. Scale a shape up and the index rises with it.
- The arithmetic stops at the number and its unit. What counts as a high value is not in it. Somebody decided that, and decisions like that get revised. Quote a cut-off only with whose it is and when it was set.

EXERCISE 1 (calculation). A person weighs 64 kg and is 155 cm tall. Work out the index, showing the height conversion as its own line, and state the unit. Then write one sentence saying what your answer does not tell you.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (teaching). A first-year resident has ten minutes and a whiteboard. They ask why the height is squared and why it matters that the answer is in kilograms per metre squared. Explain it so they could reconstruct it afterwards, and give them one error to watch for.

PRACTICE. Eight problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Work each of these out. Square the second number, then divide the first by it.

```
72 and 1.8
81 and 1.5
45 and 1.5
```

2. Turn each of these lengths into metres.

```
168 cm    155 cm    180 cm    96 cm
```

3. A quantity is divided by the square of a length. Somebody supplies the length in centimetres when metres were wanted. By what factor is the answer wrong, and in which direction? Show the step that produces the factor.

4. A person weighs 58 kg and is 1.52 m tall. Work out the index and give it with its unit, showing every line.

5. Two rows come out of a clinic register. The first records a mass of 61.5 kg and a height of 158 cm. The second records a mass of 73.4 kg and a height of 1.72 m.

Work out the index for each row, showing every line, and give each answer with its unit. Then say which row needed a conversion first, and how the row itself told you.

6. Here is a worked answer. Find the step that broke.

```
a person weighs 65 kg and is 170 cm tall
170 divided by 100 = 1.7
65 divided by 1.7 = 38.2
so the index is 38.2 kg/m²
```

7. Here is a worked answer. Find the step that broke.

```
a person weighs 62 kg and is 158 cm tall
158 times 158 = 24,964
62 divided by 24,964 = 0.00248
that is far too small, so multiply by 100
the index is 0.248 kg/m²
```

8. A colleague sends you this line about a survey.

> *Our height column turned out to be in centimetres, and the index column looks wrong because of it. It is only a scaling problem, so we can multiply the whole column by 100 and move on.*

Decide what to compute, compute it, and say what your answer does not establish.

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An entry marked outstanding has not yet been checked against the source itself. Nothing in these sections rests on one: the reasoning is a derivation you can check with a calculator, and the entry records an anchor still to be supplied.

Part C · Relationships and change

C1 · Variables and algebra: what a letter stands for

DEFINITION. A variable is a letter written in place of a number. It stands either for a number that is not yet known, or for a number that may take any of several values.

Two things written side by side are multiplied. So 2x means 2 times x, and xy means x times y. A letter multiplied by itself is written as a power: x^2 means x times x.

Brackets group a part of an expression so that it is treated as one quantity. A number multiplying a bracket multiplies every term inside it, so a(b + c) = ab + ac.

Substituting a number for a letter replaces that letter, everywhere it appears, with the number. The expression then becomes arithmetic. [1]

IN PLAIN TERMS. A letter in a sum is a number wearing a name.

```
2x
```

That is not a two sitting beside an x. It is 2 times x. In algebra, writing two things side by side means multiply them.

A letter obeys every rule a number obeys. It has to, because it is a number.

So you add letters the way you add numbers.

```
x plus x = 2x
```

Brackets do one job. They say treat what is inside as one quantity.

```
2(x + 5)
```

That is 2 times the whole of x + 5. It is not 2 times x with a 5 added on afterwards.

Without the brackets, 2x + 5 means multiply first and add second. That is the order the section on counting set out: multiplying comes before adding. Brackets are how you say otherwise.

When you do find out what the number is, you put it in. Putting a number in place of a letter is called substituting.

Say x turns out to be 7.

```
2x
2 times 7 = 14
```

Fourteen. The letter went away and arithmetic was left.

Now back to the brackets, because you will often want them gone. To take a bracket away, multiply every term inside it by what stands outside. This is called expanding the bracket. In words: 2 times the whole of x plus 5 is 2 times x, plus 2 times 5.

```
2(x + 5) = 2x + 10
```

Check it with x as 7, working each side on its own.

```
7 plus 5 = 12
2 times 12 = 24
2 times 7 = 14
14 plus 10 = 24
```

Both sides give 24. The same holds when the inside is a subtraction. Here 3 times the whole of 10 minus y becomes 3 times 10, minus 3 times y.

```
3(10 minus y) = 30 minus 3y
```

With a positive number outside, the subtraction stays a subtraction. Only the multiplying reaches inside.

If the number outside is negative, every sign inside flips. The same happens when the whole bracket is taken away. This is the rule for negatives from the section on counting: a negative times a negative is positive.

```
-3(10 minus y) = -30 plus 3y
(3x + 2y) minus (x minus y) = 3x + 2y minus x plus y = 2x + 3y
```

An expression is a piece of algebra that stands for a number, like 2x + 5. An equation is a statement that two expressions are the same number, like 2x + 5 = 19.

ILLUSTRATION. An Indian food label gets its energy figure by arithmetic, not by burning the food. The Food Safety and Standards (Labelling and Display) Regulations, 2020 fix the numbers to multiply by. Four kilocalories for every gram of carbohydrate. Four for every gram of protein. Nine for every gram of fat.

Take a packet whose panel says this for every 100 grams. Carbohydrate 60 grams, protein 12 grams, fat 20 grams. Those three gram figures are made up for this problem. The three factors are not.

Here is the working that comes back first. It is wrong.

```
60 plus 12 plus 20 = 92
92 times 4 = 368
```

The arithmetic is fine. What is wrong is using one factor for all three quantities.

Write the rule with letters instead. Let c stand for the number of grams of carbohydrate. Let p stand for the number of grams of protein. Let f stand for the number of grams of fat. And let E stand for the energy in kilocalories.

```
E = 4c + 4p + 9f
```

Read 9f as nine times f. And the f is not the symbol for gram.

Now substitute. Do the three multiplications first and the adding last. That is the order from the section on counting.

```
4 times 60 = 240
4 times 12 = 48
9 times 20 = 180
240 plus 48 plus 180 = 468
```

468 kilocalories.

A second packet comes along with 70 grams of carbohydrate, 5 grams of protein and 15 grams of fat. These are made up too.

```
4 times 70 = 280
4 times 5 = 20
9 times 15 = 135
280 plus 20 plus 135 = 435
```

435 kilocalories.

You wrote the same rule and put different numbers into it. That is what varies in a variable.

Figures quoted in this illustration are from [2].

WHERE THIS PICTURE BREAKS. The letters stand for numbers of grams. They do not stand for grams.

And the three factors here cover carbohydrate, protein and fat only.

MUST KNOW POINTS FOR YOU.

- Two things written side by side are multiplied. So 2x is two times x. Reading it as a two next to an x is the error that makes every later line unreadable. Say the multiplication out loud the first few times.
- A letter in a formula is a number, not a unit. In a line that ends 9f, the f is a count of grams and not the symbol for gram.
- Substitute, and the algebra turns into arithmetic you can check.
- One line with letters in it covers every case at once. So you check a whole column of a spreadsheet by checking the rule once and then testing two rows, rather than by reading the column.
- A letter standing for a number says nothing about which numbers are allowed. Read every answer back as a quantity before you use it.
- Name every letter in words when you put a formula on a board, in the same breath as you write it.

EXERCISE 1 (calculation). A rule is written like this.

```
T = 5a + 2b
```

Say the rule out loud in words. Then work T out for a equal to 6 and b equal to 4, showing every line.

Then work it out again for a equal to 6 and b equal to 9. Say in one sentence which part of your working you had to repeat, and which part you did not.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (teaching). A first-year resident reads 2x as a two and an x written together. They have ten minutes and a whiteboard, and they expect a mechanism rather than a rule to memorise.

Teach them why the two symbols side by side mean a multiplication, so that they can read the next formula they meet.

PRACTICE. Seven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Work each of these out for x equal to 4.

> 3x
> x + 9
> 2x + 5
> x^2

2. Write each of these as a line of algebra. Use n for the number.

- five times the number
- the number plus seven
- twice the number, less three
- the number multiplied by itself
- three times the whole of the number plus one

3. Expand each bracket. Then check each answer by putting 3 in place of the letter, on both sides.

> 4(x + 2)
> 5(10 minus y)
> 2(3n + 1)

4. Work each of these out for a equal to 5 and b equal to 2.

> ab
> 3a + 2b
> a^2 minus b
> 4(a + b)

5. A panel declares this for every 100 grams of a food. Carbohydrate 50 grams, protein 15 grams, fat 10 grams. The three gram figures are made up for this problem.

Using the factors the Food Safety and Standards (Labelling and Display) Regulations, 2020 set for calculating declared energy, work out the energy in kilocalories. Show every line and give the answer with its unit.

Then say in one sentence what your answer does not tell you.

6. Here is a worked answer. Find the step that broke.

```
a rule says E = 4c + 4p + 9f
a panel gives c as 55, p as 9 and f as 18
substitute the three numbers into the rule
4c becomes 455
4p becomes 49
9f becomes 918
455 plus 49 plus 918 = 1,422
so the energy is 1,422 kilocalories
```

7. In a meeting somebody says this about a packet.

> *The panel says 40 grams of carbohydrate, 5 grams of protein and 30 grams of fat in every 100 grams. It is mostly carbohydrate, so most of its energy comes from the carbohydrate.*

The three gram figures are made up for this problem. The factors for calculating declared energy are the ones the Food Safety and Standards (Labelling and Display) Regulations, 2020 set.

Decide what to compute, compute it, and say what your answer does not establish.

C2 · Rearranging an equation

DEFINITION. An equation is a statement that the expression on the left of the equals sign and the expression on the right stand for the same number. It is not an instruction to work the left side out.

If two expressions stand for the same number, then applying the same operation to both of them gives two expressions that still stand for the same number. Four operations therefore leave the solutions of an equation unchanged.

- adding the same quantity to both sides
- subtracting the same quantity from both sides
- multiplying both sides by the same quantity that is not zero
- dividing both sides by the same quantity that is not zero

To solve an equation for a letter is to reach an equation with that letter alone on one side, and no occurrence of it on the other.

An operation applied to a side that is a sum applies to the whole of that sum, not to one term of it. [3]

IN PLAIN TERMS. Start with what the equals sign is saying. It says the thing on its left and the thing on its right are the same number.

Picture a pair of scales with a pan on each side, sitting level. Take three kilograms off one pan and the scales tip. Take three off both and they stay level.

That is the whole rule.

> *Whatever you do to one side, do to the other.*

Now use it.

```
x + 7 = 12
```

The 7 is in the way. It got there by being added, so take 7 off both sides.

```
x + 7 minus 7 = 12 minus 7
x = 5
```

Check it by putting the 5 back where x was.

```
5 plus 7 = 12
```

That check costs you one line and catches almost everything.

Multiplying works the same way. Suppose you are told this.

```
3x = 21
```

The 3 got there by multiplying, so divide both sides by 3.

```
3x divided by 3 = 21 divided by 3
x = 7
```

Two things to hold on to from here on.

The first is order. When more than one thing is in the way, undo them in the reverse of the order they were done.

```
2x + 3 = 11
```

The x was multiplied by 2 first and had 3 added second. So undo the adding first and the multiplying second.

```
2x + 3 minus 3 = 11 minus 3
2x = 8
2x divided by 2 = 8 divided by 2
x = 4
```

The second is that a side which is a sum has to be treated whole.

There is one more use for the rule, and it is the one you will use most. Sometimes nothing is unknown. A formula ties several quantities together, and you want a different one of them on its own. Getting one letter on its own like that is also called making it the subject.

Take the rule that ties a distance, a speed and a time. The distance is the speed times the time.

```
d = s t
```

To get t by itself, divide both sides by s.

```
d divided by s = s t divided by s
d divided by s = t
```

The time is the distance divided by the speed. You did not measure anything new. You moved the same statement into the shape that answers your question.

One warning goes with that. Never divide both sides by a letter unless you know it is not zero, and the same goes for multiplying by one. Dividing by zero is not a move, and a letter can hide one.

ILLUSTRATION. You met the rule an Indian food label uses for its energy figure in the section before this one. Four kilocalories for every gram of carbohydrate, four for every gram of protein, nine for every gram of fat.

```
E = 4c + 4p + 9f
```

A packet reaches you with its fat line torn off. The panel still shows the energy as 468 kilocalories in every 100 grams, the carbohydrate as 60 grams and the protein as 12 grams. Those three figures are made up for this problem; the factors are not. Somebody wants the fat.

Here is the working that comes back first. It is wrong.

```
E = 4c + 4p + 9f
divide both sides by 9
f = E divided by 9, minus 4c, minus 4p
468 divided by 9 = 52
52 minus 240 minus 48 = -236
so the fat is -236 grams
```

Minus two hundred and thirty-six grams of fat. No packet holds that, and no packet holds a negative anything.

The division by 9 was applied to one term on the right and to nothing else. The right side is a sum of three terms, and dividing that side means dividing all of it.

Get the 9f on its own first, by taking the other two terms off both sides.

```
E minus 4c minus 4p = 9f
```

Dividing by 9 is safe, because there is nothing else on that side for the 9 to miss.

```
the whole of E minus 4c minus 4p, divided by 9 = f
```

The words “the whole of” do the job brackets do, which you met in the section before this one. In symbols the same line is this.

```
(E minus 4c minus 4p) divided by 9 = f
```

The brackets say that all three terms are divided by 9, not the last one alone. On other pages you will see it as a fraction instead. E minus 4c minus 4p sits above a line and 9 sits below it. The line does the same grouping the brackets do.

Put the numbers in, one line at a time.

```
4 times 60 = 240
4 times 12 = 48
240 plus 48 = 288
468 minus 288 = 180
180 divided by 9 = 20
```

Twenty grams of fat.

Now check it, the way you check every rearrangement.

```
4 times 60 = 240
4 times 12 = 48
9 times 20 = 180
240 plus 48 plus 180 = 468
```

The rearrangement held.

Figures quoted in this illustration are from [2].

WHERE THIS PICTURE BREAKS. Rearranging moves a quantity from one side to the other. It does not improve it.

The working here also assumes the packet holds nothing but carbohydrate, protein and fat that carries energy.

MUST KNOW POINTS FOR YOU.

- An equals sign states that two things are the same number.
- Do the same thing to both sides, and write both sides of that step on the same line.
- A side that is a sum is divided or multiplied as a whole.

- Check every rearrangement by putting your answer back into the equation you started from. One line, every time, before the number leaves your desk.
- Any one quantity in a formula can be recovered from the others, as long as every one of the others is known. Two more conditions apply. The rearranging must never divide by something that could be zero. And only one value may fit: where two values of the missing quantity give the same result, the formula cannot choose between them. So a register with one missing column is not always a register with missing information. Two gaps in the same row are a different matter, and one formula cannot fill both.
- Never divide both sides by a letter unless you know that letter is not zero.
- A recovered number carries the rounding of everything you put in. Multiplying or dividing by a rounded figure can make that rounding larger, and a squared figure brings its rounding in twice.
- Teach the order before the moves. Undo what was done last, first.

EXERCISE 1 (calculation). A formula says that a total cost T is a fixed charge F plus a rate r for each of n items.

```
T = F + rn
```

Rearrange it to give n on its own, showing every step and saying at each step what you did to both sides.

Then work n out for a total of 950, a fixed charge of 200 and a rate of 25.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A colleague sends you this line and asks you to look at it before it goes into a note.

> *The formula is T = F + rn, and we know T, F and r. So n is T divided by r, minus F.*

Say what has gone wrong and give the right rearrangement. Then say what you would have them change in how they work, rather than in this one line.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Solve each of these for x. Show the step you did to both sides.

```
x + 7 = 12
x minus 4 = 9
3x = 21
```

2. Solve each of these for x. Two steps each, and say which step you did first and why.

```
2x + 3 = 11
5x minus 4 = 26
```

3. Solve each of these for x.

```
5x minus 2 = 3x + 8
x divided by 4 = 6
```

4. Rearrange each of these to give the named letter on its own. There are no numbers to work out.

- A = L W, make W the subject
- y = m x + k, make x the subject
- v = u + a t, make t the subject

5. A clinic register records a person's body mass index as 27.0 kg/m^2 and their height as 1.60 m. The mass column has been left empty.

The index is the mass in kilograms divided by the square of the height in metres. Rearrange that to give the mass, then work the mass out and give it with its unit. Show every line.

Then say in one sentence what your answer does not tell you.

6. A panel gives the energy as 396 kilocalories in every 100 grams, the carbohydrate as 45 grams and the protein as 9 grams. The fat line is missing. These three figures are made up for this problem.

Using the factors the Food Safety and Standards (Labelling and Display) Regulations, 2020 set for calculating declared energy, work the fat out. Show every line and check your answer.

7. Schedule II of the National Food Security Act, 2013 sets a nutritional standard for the hot cooked meal in lower primary classes. It is 450 kilocalories and 12 grams of protein.

A kitchen plans a meal that meets that standard exactly, and its recipe supplies 78 grams of carbohydrate. The 78 grams is made up for this problem.

The Food Safety and Standards (Labelling and Display) Regulations, 2020 set the factors for calculating declared energy. Use them to work out how many grams of fat the meal must carry. Show every line. Then say in one line what your answer does not establish about the meal.

8. A store's register gives the energy in each gram of a packed food as 4.5 kilocalories. It gives the energy in one whole packet as 900 kilocalories. The weight column is empty. Both figures are made up for this problem.

The energy in each gram is the energy of the whole packet divided by the weight of the packet in grams. Rearrange that to give the weight, then work the weight out and give it with its unit. Show every line.

Then say what this needed that the same work does not need when the letter you want is already above the division line. Say why the direction of a rearrangement changes what you have to do.

9. Here is a worked answer. Find the step that broke.

```
a rule says the index is the mass in kilograms divided by the square of the height
in metres
a register gives the index as 25.0 and the height as 1.80 m
the mass is wanted, so multiply the index by the height
25.0 times 1.80 = 45
so the mass is 45 kg
```

10. Here is a worked answer. Find the step that broke.

```
a rule says E = 4c + 4p + 9f
a panel gives the energy as 500 kilocalories and the fat as 20 grams
the carbohydrate is wanted
9 times 20 = 180
500 minus 180 = 320
so 4c + 4p = 320
divide both sides by 4
c + p = 80
the panel also says the protein is 15 grams
so the carbohydrate is 80 grams and the panel holds together
```

11. A colleague writes this in a note about a clinic register.

> *The height column is in metres and the index column is filled in. We do not need to keep the mass column, because we can always work the mass back out.*

Take one row, with the height recorded as 1.65 m and the index recorded as 22.0 kg/m^2.

Decide what to compute, compute it, and say what your answer does not establish.

12. In a meeting about a school kitchen somebody says this.

> *The label rule counts nine kilocalories for every gram of fat. So if we take five grams of fat out of the recipe, the meal loses five kilocalories.*

Decide what to compute, compute it, and say what your answer does not establish.

C3 · Two equations at once

DEFINITION. Everything below concerns equations in which each unknown appears on its own, multiplied by a number and added — the only kind this book uses.

An equation may hold two unknowns. On its own such an equation does not fix either of them.

A system of two equations in two unknowns is a pair of equations required to hold at the same time. A solution of the system is a pair of values that satisfies both equations, not one.

Two methods solve such a system. Substitution rearranges one equation to give one unknown in terms of the other, and puts that expression into the second equation, which then holds one unknown. Elimination adds a multiple of one equation to the other, choosing the multiple so that one unknown cancels.

A system of two such equations has exactly one solution, or none, or infinitely many. It has none when the two equations make statements that cannot both be true. It has infinitely many when one equation is a multiple of the other, and so states nothing the first did not. [4]

IN PLAIN TERMS. The section before this one solved equations with one letter in them. This one starts where a line has two.

Here is such a line.

```
x + y = 10
```

Ask what x is. There is no single answer. Try some pairs and see.

x	y
1	9
4	6
7	3

One equation with two unknowns does not pin either of them down. It ties them together. Fix one and the other follows; fix neither and you have nothing.

Now add a second statement that has to be true at the same time.

```
x + y = 10
x minus y = 4
```

Now there is one pair that works, and only one. Two statements, two unknowns.

There are two ways to find it, and they are the same idea approached from different sides.

The first is substitution. You have substituted a number for a letter before now. This is the same move with an expression in place of the number, and what it leaves behind is algebra rather than arithmetic.

Rearrange one equation so that one letter stands alone, then put that into the other equation.

Rearrange the first line to give x on its own.

```
x = 10 minus y
```

Now put that in place of x in the second line.

```
10 minus y minus y = 4
10 minus 2y = 4
2y = 6
y = 3
```

Then go back and get x.

```
x = 10 minus 3
x = 7
```

The second way is elimination. Add the two equations together, left to left and right to right. The y and the minus y cancel.

```
2x = 10 plus 4
10 plus 4 = 14
14 divided by 2 = 7
```

So x is 7. Elimination has found one unknown, and it is not finished. Put the 7 back into either line to get the other. The first line is the simpler one.

```
7 + y = 10
10 minus 7 = 3
```

So y is 3, the same as substitution gave. Now check. Put both numbers into both statements, not into one of them.

```
7 plus 3 = 10
7 minus 3 = 4
```

Both hold, so the pair is the solution.

Adding worked because one line had plus y and the other had minus y. When the same letter carries the same number and the same sign in both lines, subtract instead. Take one line from the other, left from left and right from right.

```
x + y = 10
x + 3y = 16
```

Take the first line from the second. The x terms cancel, and 3y take away y leaves 2y.

```
16 minus 10 = 6
2y = 6
6 divided by 2 = 3
```

So y is 3. Put it back into the first line to get x.

```
10 minus 3 = 7
```

So x is 7. Check the pair in both lines.

```
7 plus 3 = 10
3 times 3 = 9
7 plus 9 = 16
```

And sometimes no letter carries matching numbers at all. Then make a match. Multiply every term of one line, on both sides, by the same number. That is the rule from the section before this: whatever you do to one side, do to the other.

```
x + y = 10
3x + 2y = 27
```

The second line has 2y and the first has only y. So multiply the whole of the first line by 2.

```
2 times 10 = 20
2x + 2y = 20
```

Now the y terms match. Take this new line from the second line. The 2y terms cancel, and 3x take away 2x leaves x.

```
27 minus 20 = 7
```

So x is 7. Put it back into the first line to get y.

```
10 minus 7 = 3
```

So y is 3. Check the pair in both lines.

```
7 plus 3 = 10
3 times 7 = 21
2 times 3 = 6
21 plus 6 = 27
```

The same pair sits behind all three examples, so you can watch each method land on it. Every elimination has the same four steps. Make one letter match, if it does not already. Add or subtract the lines so that it cancels. Solve for the letter that is left. Then put that value back into either line to get the other.

Sometimes no whole-number multiple of one line matches the other. Then multiply both lines, each by its own number.

```
2x + 3y = 12
3x + 2y = 13
```

Multiply the whole of the first line by 3 and the whole of the second by 2. Both x terms become 6x.

```
6x + 9y = 36
6x + 4y = 26
```

Take the second new line from the first. The 6x terms cancel, and 9y take away 4y leaves 5y.

```
36 minus 26 = 10
5y = 10
10 divided by 5 = 2
```

So y is 2. Put it back into the first line to get x.

```
3 times 2 = 6
12 minus 6 = 6
6 divided by 2 = 3
```

So x is 3. Check the pair in both lines.

```
2 times 3 = 6
3 times 2 = 6
6 plus 6 = 12
3 times 3 = 9
2 times 2 = 4
9 plus 4 = 13
```

Two more shapes to recognise, because they arrive without warning.

Sometimes the second statement repeats the first.

```
x + y = 10
2x + 2y = 20
```

The second line is the first line doubled. It is true whenever the first is true, so it adds nothing, and you are back to as many answers as you like.

And sometimes the two statements cannot both hold.

```
x + y = 10
x + y = 12
```

Two numbers cannot add to 10 and to 12. There is no pair at all, and the useful thing to say is not the answer but that one of the two statements is wrong.

ILLUSTRATION. A packet reaches you with two lines of its panel gone. The energy is still there, 468 kilocalories in every 100 grams, and so is the protein, 12 grams. The carbohydrate line and the fat line have torn away. Those figures are made up for this problem. The label rule is the real one you have used in the two sections before this.

```
E = 4c + 4p + 9f
```

Put in what the panel still gives you.

```
4 times 12 = 48
```

Take that 48 off both sides.

```
468 minus 48 = 420
```

So what you know is this.

```
4c + 9f = 420
```

One line, two unknowns.

Look at three pairs that all satisfy that line.

carbohydrate grams	fat grams	from carbohydrate	from fat	total
60	20	240	180	420
105	0	420	0	420
15	40	60	360	420

Every row is a genuine answer to the arithmetic. Not every row is a possible packet. The second row puts 105 grams of carbohydrate and 12 grams of protein into 100 grams of food. The section before this taught you to throw out a negative mass, and this row fails the same kind of test. Run that second check on any answer, because the arithmetic never runs it for you.

So what would fix them? A second statement that has to be true at the same time.

Suppose the maker tells you the carbohydrate is three times the fat. Write it beside the first.

```
4c + 9f = 420
c = 3f
```

Now substitute. Put 3f in place of c in the first line, so 4c becomes 4 times 3f.

```
4 times 3f, plus 9f = 420
12f + 9f = 420
21f = 420
```

What went in for c here was a single term, 3f. Often it is a difference instead, such as 75 minus f. Then it goes in inside brackets, and you expand them as the section on variables showed. Every term inside is multiplied by the 4.

```
4(75 minus f) = 300 minus 4f
```

Back to the packet. Divide both sides by 21.

```
420 divided by 21 = 20
```

Twenty grams of fat. Now go back for the carbohydrate.

```
3 times 20 = 60
```

Sixty grams of carbohydrate.

Check both numbers in both statements. Checking one is not checking the pair.

```
4 times 60 = 240
9 times 20 = 180
240 plus 180 = 420
```

The first statement holds. And 60 is three times 20, so the second holds as well.

Figures quoted in this illustration are from [2].

WHERE THIS PICTURE BREAKS. The second statement was handed to you here. In real work it has to come from somewhere, and where it came from is the part that decides whether your answer means anything.

MUST KNOW POINTS FOR YOU.

- One equation with two unknowns has many answers, not one. Anyone who reads a single relationship as fixing a single quantity is reading something that is not there.
- Count the unknowns and count the equations before you start.
- Check a solved pair in both equations.
- When somebody draws a conclusion about one quantity from a single relationship, ask what the second statement is and where it came from.
- Two statements pin two unknowns down only when they say different things. Two that repeat each other leave you with as many answers as before. Two that contradict each other leave you with none, which is itself worth reporting.
- Use substitution when one equation already has a letter standing nearly alone. Use elimination when the same letter carries matching numbers in both lines. Use it too when you can make them match by multiplying one whole line by a number.
- Solving a pair tells you what follows from the two statements, and nothing about whether either is true.

EXERCISE 1 (interpretation). A district officer shows you one line from a planning note.

> *Every meal served costs us the grain plus the cooking fuel, and the two together come to 18 rupees a meal.*

They then ask you how much the grain alone costs.

Say what you can and cannot answer from that line. Then say exactly what one further piece of information would let you answer it, and give two different forms that piece could take.

EXERCISE 2 (teaching). A journalist has three minutes and no appetite for algebra. They have been told two things about a scheme. Its total spend went up, and its total number of meals went up. They want to write that the cost of each meal went up.

Explain why that does not follow, in words they can use, with one sentence they could quote.

PRACTICE. Ten problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Two statements have to hold at the same time.

```
x + y = 7
x minus y = 1
```

Say for each of these pairs whether it is a solution.

x	y
4	3
5	2

x	y
6	1

2. Write down four different pairs of numbers that make this statement true.

```
x + y = 12
```

Then say how many such pairs there are, and what that means about what this one line can tell you.

3. Solve this pair by substitution.

```
y = 2x
x + y = 18
```

4. Solve this pair by elimination.

```
x + y = 15
x minus y = 3
```

5. A panel gives the energy as 440 kilocalories in every 100 grams and the protein as 10 grams. The carbohydrate line and the fat line are missing. A note attached to the packet says the carbohydrate and the fat together weigh 75 grams. All four figures are made up for this problem.

Using the factors the Food Safety and Standards (Labelling and Display) Regulations, 2020 set for calculating declared energy, work out the carbohydrate and the fat. Show every line and check the pair in both statements.

6. Schedule II of the National Food Security Act, 2013 sets two figures you need here. The hot cooked meal for lower primary classes is 450 kilocalories. The one for upper primary classes is 700 kilocalories.

A kitchen serves 50 meals in a day, some lower primary and some upper primary, and its plan totals 27,500 kilocalories for the day. The 50 and the 27,500 are made up for this problem.

Work out how many meals of each kind the plan is for. Show every line, and check the pair in both statements.

7. Schedule II of the National Food Security Act, 2013 sets four figures you need here. The hot cooked meal for lower primary classes is 450 kilocalories and 12 grams of protein. The one for upper primary classes is 700 kilocalories and 20 grams of protein.

A kitchen's day sheet gives only two totals: 27,500 kilocalories and 760 grams of protein. The meal count has not been written down. Both totals are made up for this problem.

Work out how many meals of each kind the sheet describes. Use elimination rather than substitution. Show every line, and check the pair in both statements.

8. Here is a worked answer. Find the step that broke.

```
two statements have to hold at the same time
x + y = 20
x minus y = 6
rearrange the first statement
x = 20 minus y
put that into the first statement
20 minus y + y = 20
20 = 20
so both statements hold whatever y is
and any pair adding to 20 will do
```

9. Here is a worked answer. Find the step that broke.

```
a note carries two statements about a kitchen's day
the lower primary and upper primary meals together number 50
L + U = 50
twice the lower primary meals and twice the upper primary meals together number 100
2L + 2U = 100
that is two statements and two unknowns, so the pair can be solved
from the first, L = 50 minus U
put it into the second
2 times the whole of 50 minus U, plus 2U = 100
100 minus 2U + 2U = 100
100 = 100
so L is 25 and U is 25
```

10. A programme note says this about a school kitchen.

> *Between last year and this year the kitchen's total number of meals went up, and its total kilocalories went up. So it served more lower primary meals and more upper primary meals than it did last year.*

Schedule II of the National Food Security Act, 2013 sets the lower primary hot cooked meal at 450 kilocalories. It sets the upper primary hot cooked meal at 700 kilocalories. Take last year as 30 lower primary meals and 20 upper primary meals a day. Those two counts are made up for this problem.

Decide what to compute, compute it, and say what your answer does not establish.

C4 · Functions: input, rule, output

DEFINITION. A function is a rule that attaches to every input it accepts exactly one output.

The collection of inputs the rule accepts is its domain. The collection of outputs it can return is its range.

Two different inputs may share one output, and the rule is still a function. The requirement runs in one direction only.

A function is written as a name with its input in brackets, so f(x) means the output that the rule named f gives for the input x. The brackets do not mean multiplication. [5]

IN PLAIN TERMS. A function is a machine with one slot in and one slot out.

You drop a number in. One number comes out. Drop the same number in tomorrow and the same number comes out, because the rule has not changed.

One input may give one output and no more. If the same input gives you 20 today and 35 tomorrow, you are not holding a rule at all. You are holding something else, and you cannot use it to answer a question.

There is a short way to write a rule down. Give it a name, say f. Write f(4), said out loud as “f of 4”, for the output the rule gives when you put 4 in. The brackets here do not mean times.

A rule can also be run backwards, from an output to the input that gave it. You undo what the rule did, the way the section on rearranging undid each step. That gives a single answer only where no two inputs share an output. Where two inputs do share one, going backwards from that output leads to both. Nothing in the output tells you which.

Two names, so you recognise them on somebody else’s page. The inputs a rule will accept are called its domain. The outputs it can give back are called its range. Ask a rule what it accepts and what it does not. That is often the quickest way to catch somebody running it outside what it was built for. A rule that counts people takes whole numbers from one upwards. Half a person is not an input it has an answer for.

Be careful with the second of those two words. In statistics, range means something else entirely, which is the largest value in a set of measurements minus the smallest. The two meanings have nothing to do with each other and you will meet both. Which one is meant depends on whether the thing being described is a rule or a set of measurements.

ILLUSTRATION. A block register lands on your desk. It lists four households, how many people are in each, and the foodgrains each is counted for in a month. The register is made up for this section.

household	people	kilograms a month
A	4	20
B	6	30
C	4	35
D	5	25

How many kilograms does it give for a household of four?

You cannot answer. Household A says 20 and household C says 35. One input has two outputs, so this table is not a function of the number of people.

That is a finding rather than a dead end. Something other than the number of people differs between A and C, and the register does not say what. Until somebody tells you what, you cannot work out a single row from the number of people alone.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month.

That sentence is a function. The input is the number of people. The rule is "multiply by five kilograms a month". The output is kilograms of foodgrains a month.

Put four in.

```
5 times 4 = 20
```

Twenty kilograms a month.

Write the rule short. Call it g. Let n stand for the number of people, in the way a letter stood for a number in the section on variables.

```
g(n) = 5 times n
```

Households A, B and D match it. Household C does not.

```
5 times 4 = 20
```

One input, one output, and anything that does not match is a question for whoever wrote the register.

Now run the rule backwards once. A priority household is counted for 30 kilograms a month. How many people are in it? The rule multiplied by five, so undo it by dividing by five.

```
30 divided by 5 = 6
```

Six people. Check it forwards.

```
5 times 6 = 30
```

That backwards step is safe here. Each extra person adds five kilograms, so no two household sizes share an output, and 30 points to one size only.

The section on ratios gives the likely reason for household C. A proviso just below section 3(1) covers households under the Antyodaya Anna Yojana. It entitles them to 35 kilograms a month. It does so "to such extent as may be specified by the Central Government for each State in the said scheme". So the Central Government sets how far the scheme reaches in each State. The 35 is per household, whatever its size.

If C is an Antyodaya household, the register holds two rules, and the row is right. The register does not say which scheme C is under. So treat this as the likely explanation and ask.

That rule also shows why going backwards can fail. Under it, a household of two and a household of nine both get 35 kilograms. Given 35, the rule cannot tell you the size.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. The one-slot machine holds while the rule takes one input. Many real rules take two.

The index you met in the section on body-size units needs a mass and a height together. That is still a function.

The machine says nothing about whether a rule is true of the world either. A rule can give exactly one output and still be the wrong rule.

MUST KNOW POINTS FOR YOU.

- When a table lands in front of you, look for one input appearing twice with two different outputs before you compute anything from it.
- Two inputs sharing one output is fine. One input with two outputs is not.
- Outputs that climb for a while and then fall again break nothing. So for this test, look for the repeated input, not for the shape of the outputs.
- When the same inputs give a different answer on a second run, something you did not notice changed. Find what changed before you trust either answer.
- Name the input, the output and the unit of each before you write anybody's rule on a whiteboard.
- A function says the output goes with the input. It never says the input produced it.
- Run a rule backwards only where no two inputs share an output. Where two do, an output points back to more than one input, and a single answer from it is a guess.

EXERCISE 1 (critique). A colleague sends you a two-column table of three hundred rows and calls it "the rule we use". You find that one value in the first column appears twice, with two different values beside it.

Say what that means for the table, and say what you would ask for next.

EXERCISE 2 (teaching). A first-year resident says they can never keep formulas straight. You have ten minutes and a whiteboard.

Teach them what a function is, so that they leave able to check whether something handed to them is one.

PRACTICE. Eight problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A rule multiplies the input by 3 and then adds 2.

Work out the output for the inputs 0, 1, 5 and 10.

2. Three tables are below. For each one, say whether it is a function of its first column, and say what told you.

first	second
1	4
2	8
3	12

first	second
1	4
2	8
1	9

first	second
1	4
2	4

first	second
3	4

3. A rule adds 7 to its input.

The outputs are 12, 19 and 40. Work out the input that produced each one.

4. A district office runs this rule to plan its grain lifting. Let n stand for the number of people in a household. The rule gives the kilograms of foodgrains a month that household is counted for.

```
g(n) = 5 times n
```

Say what the domain of that rule is, in words, and say what its range is. Then say what the office has done wrong if a row of its planning sheet reads n = 4.5.

5. Under section 3(1) of the National Food Security Act, 2013, a person in a priority household is entitled to five kilograms of foodgrains a month.

Work out the grain for a household of three, and for a household of seven. Give each answer with its unit. Then work out how many people are in a household counted for 45 kilograms a month. Say which way round you ran the rule each time.

6. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains a month. So a household of five is counted for 25 kilograms of foodgrains a month.

Treat the number of months as the input and the total kilograms as the output. Write the rule, work out the total for four months, and work out how many months it takes to reach 150 kilograms. Give every answer with its unit.

7. Here is a worked answer. Find the step that broke.

```
a register lists a household of four counted for 20 kilograms
it lists another household of four counted for 35 kilograms
the rule for four people is the average of the two
20 plus 35 = 55
55 divided by 2 = 27.5
so four people gives 27.5 kilograms a month
```

8. A colleague sends you this line about a block register.

> *Our register lists grain per household each month. Divide any row by five and you get the number of people. So we can drop the people column and save the data entry.*

Decide what to compute, compute it, and say what your answer does not establish.

C5 · Graphs: axes, scale, slope and intercept

DEFINITION. A graph is a drawing of a relationship between two quantities, made of points in a plane.

Two lines of reference cross at a point called the origin. The horizontal one is the x-axis and the upright one is the y-axis. Each axis carries one quantity, the unit that quantity is measured in, and a scale.

The scale is the rule that turns a distance along an axis into an amount of the quantity. Whoever draws the figure chooses it.

A point is fixed by two numbers, written (x, y), which are its position along the x-axis and its position along the y-axis.

The slope of a straight line is the change in the y quantity divided by the change in the x quantity, measured between any two points on it.

The intercept on the y-axis is the value the y quantity takes where the x quantity is zero. It is a property of the line as drawn. It states something about the world only where zero is a value the x quantity can take. [7]

IN PLAIN TERMS. Four things decide what the picture tells you. Read them in this order, every time. The first two come before you look at the line at all.

What is on each axis. Look at the flat line along the bottom. That is the x-axis. Look at the upright line up the side. That is the y-axis. Each should carry the name of a quantity and a unit. An axis with no unit on it is a picture you cannot read.

What the scale is. The scale is how much of the quantity one step along the axis stands for. Somebody chose it. They could have chosen another one. The same numbers on a taller axis look calm, and on a shorter axis look steep, and neither drawing is a lie. So read the numbers printed on the axis before you trust the shape.

How steep the line is. Steepness is called the slope. Pick two points on the line. Work out how far up it went between them, and how far across.

```
the slope = the change up, divided by the change across
```

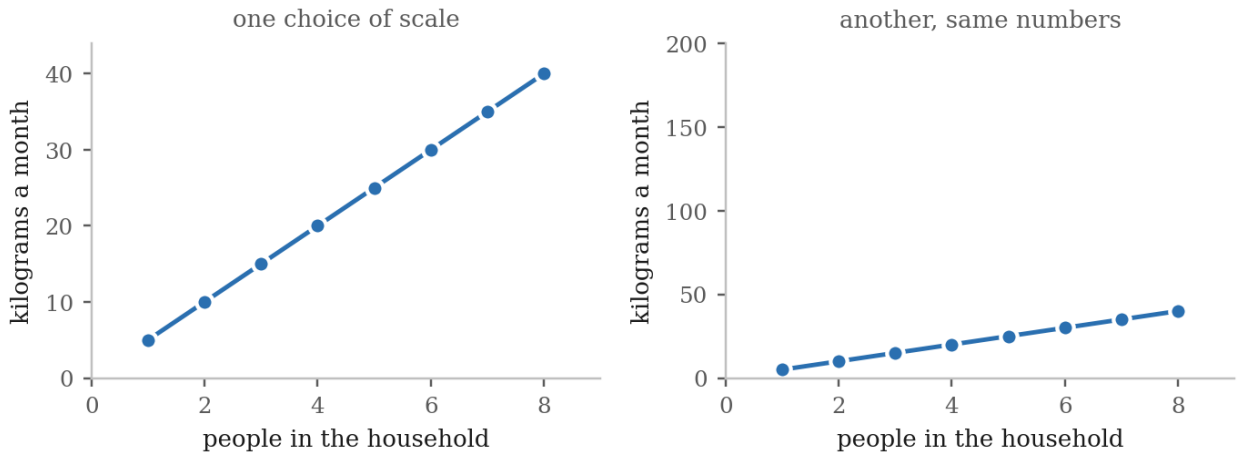
The change up is called the rise. The change across is called the run.

A slope is a rate, and it carries a unit like every rate you have met. The unit is the y unit divided by the x unit.

A slope can be negative. A line that falls as you go right has a rise below zero, so the slope comes out with a minus sign in front. The section on counting covers numbers like that. A slope of minus 3 means the y quantity drops by 3 for every one step across.

Where the line crosses. Follow the line to the place where the x quantity is zero. The height there is called the intercept. It answers one question: what was the y quantity when the x quantity was nothing at all?

Sometimes zero is outside anything anybody measured, and then the intercept is a number the drawing made.



The same eight entitlements drawn twice. Nothing about the numbers changed between the two panels; only the top of the side axis did. The left panel makes the rise look steep and the right makes it look flat, and neither is dishonest.



Slope read as a rise over a run, and the intercept read where the line meets the side axis. The rise and the run are marked between the months the text works through.

ILLUSTRATION. Take one rule and draw it twice.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of food-grains per person per month.

Work out eight points, one for each household size from one person to eight.

people in the household	kilograms a month
1	5
2	10
3	15
4	20
5	25

people in the household	kilograms a month
6	30
7	35
8	40

Now draw them. People go along the bottom. Kilograms a month go up the side.

Place one point by hand first. Take the household of three and its 15 kilograms. As a pair it is written (3, 15). The number along the bottom comes first and the number up the side second.

Start at the origin, where both axes read zero. Go along the bottom until you reach 3. From there go straight up until you are level with 15 on the side axis. Put the mark there. Every other row goes on the same way.

The brackets in (3, 15) are not the brackets in f(x) from the section on functions. There they held a rule's input. Here they hold a pair of numbers, and the comma between them is the sign.

Draw it once with the side axis running from 0 to a little above 40 kilograms. The line fills the picture and the rise is the first thing you see.

Draw it again with the side axis running from 0 to 200 kilograms. Every point is the same point as before. The line now sits in the bottom fifth of the picture and looks nearly flat.

Ask what changed between the two. Only the numbers printed up the side.

What differs is the impression, and the impression was set by a choice.

So here is the habit. Read the numbers on both axes first. Then decide whether a rise you can see is a rise that matters. Never take the steepness of a line as a measurement, because it is not one.

Now read a slope and an intercept off a line, which is the other thing a graph is for.

A household of four has 10 kilograms of grain in the house today. Every month it is counted for another 20 kilograms, because five kilograms a person times four people is twenty. The 10 kilograms is made up for this example.

```
5 times 4 = 20
```

Put months along the bottom, starting at today. Put kilograms in the house up the side.

months from today	kilograms in the house
0	10
1	30
2	50
3	70
4	90
5	110
6	130

The points sit on a straight line. Take two of them and read the slope off the picture. Use the point at one month and the point at four months.

Work out the rise, which is the change up the side.

```
90 minus 30 = 60
```

Work out the run, which is the change along the bottom.

```
4 minus 1 = 3
```

Divide the rise by the run.

```
60 divided by 3 = 20
```

Twenty. Now put a unit on it, and take the unit off the axes rather than out of your memory. The side axis is in kilograms and the bottom axis is in months.

```
kilograms divided by months
= kilograms per month
```

Twenty kilograms a month.

Pick a different pair of points and the slope comes out the same, because the line is straight.

Now the intercept. Follow the line back to month zero, where the bottom quantity is nothing at all. The line meets the side axis at 10 kilograms.

That number means something here. It is the grain that was in the house before any of these months began. Somebody can go and look in the house and check it.

Often nobody wrote down a reading at zero. Find the intercept anyway by stepping back from the nearest reading, using the slope. Suppose the table had started at two months, with 50 kilograms. Each month back takes off one slope's worth of grain.

```
20 times 2 = 40
50 minus 40 = 10
```

Ten kilograms, the same as the line gave.

The slope and the intercept together fix the whole line. In words, the output is the intercept, plus the slope times the input. Here that is 10 kilograms, plus 20 kilograms a month times the number of months. Write y for the side quantity and x for the bottom one. Write m for the slope and k for the intercept. Then the line is this.

```
y = m x + k
```

You met that line in the section on rearranging, as letters with no meaning attached. This is its meaning. For the grain in the house, m is 20 and k is 10. Try it at four months.

```
20 times 4 = 80
80 plus 10 = 90
```

Ninety kilograms, which is the row for four months in the table.

Now watch an intercept stop meaning anything. Go back to the first drawing, where the bottom axis was the number of people. Its intercept is the grain for a household of nobody. No such household exists, so that number is something the drawing produced and nothing anybody can check.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. Reading one slope off a line works while the line is straight.

An upright line has no slope, because the division has nothing to divide by.

The intercept means something only where zero is a value the bottom quantity can really take.

Two drawings of the same points are both honest, however different they look.

And a straight line through some points does not make the rule behind them true.

MUST KNOW POINTS FOR YOU.

- Read both axes, with their units, before you look at the shape of anything.
- A steep line is not a large effect.
- A side axis that does not start at zero makes every small wiggle on it look like a cliff.
- A slope carries the y unit divided by the x unit.
- An intercept is a fact about the world only where zero is a value the bottom quantity can actually take.
- When a figure alarms you, redraw it with the side axis starting at zero before you repeat it to a journalist.
- Teach a trainee to read a figure in one fixed order. What each axis measures and its unit come first, then the numbers on the scale. Only then the slope, and last the intercept.
- Before you read a value off any printed chart in clinic, find what one small division on each axis stands for.

EXERCISE 1 (interpretation). Two figures are printed side by side. Both plot the same seven monthly values of one quantity. The left figure has a side axis running from 0 to 100. The right figure has a side axis running from 58 to 66. The values run from 60 up to 64.

Say what each figure shows and what neither of them shows. Then say which you would print, and why.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A slide in a meeting shows a straight line rising across the whole picture. The speaker says:

> *You can see the effect is dramatic. Look at the slope.*

The side axis is labelled in kilograms and runs from 19.8 to 20.4. The bottom axis is labelled in months and runs from 0 to 12.

Say what is wrong with what the speaker said, and say what you would ask for.

EXERCISE 3 (teaching). A health secretary has one page and three minutes. They have been shown two figures of the same data by two different groups, and the figures look nothing alike. They want to know which one to believe.

Write what you would put on that page.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A straight line passes through the points (2, 7) and (6, 19).

Work out its rise, its run and its slope.

2. Work out the slope of the line through each of these pairs of points.

```
(0, 5) and (4, 13)
(1, 10) and (6, 10)
(2, 20) and (7, 5)
```

3. A straight line goes through (0, 12) and (5, 27).

Work out the slope, and write down the intercept on the y-axis. Then say what the intercept is the value of.

4. A straight line has a slope of 4 and an intercept of 6 on the y-axis.

Work out y at x equal to 0, 1, 3 and 10. Then work out the x at which y reaches 46.

5. A line is drawn with kilograms of grain in a house up the side and months along the bottom. It passes through the point at one month and 30 kilograms, and the point at five months and 110 kilograms.

Work out the slope, give it with its unit, and say where the line meets the side axis.

6. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month.

A figure plots kilograms a month up the side against the number of people along the bottom, for households of one to eight people. Work out the slope from the points for three people and seven people. Give it with its unit. Then say what the intercept is and whether it means anything.

7. A quantity is measured once a month for a year. The smallest value is 60 and the largest is 64.

One figure draws it with the side axis running from 0 to 100. Another draws it with the side axis running from 59 to 67.

Work out what fraction of the height of each figure the change takes up. Then say what that means for somebody looking at the two figures side by side.

8. Work out the unit of the slope of a line drawn on each of these pairs of axes.

up the side	along the bottom
kilograms	months
kilocalories	grams
rupees	kilograms
number of people	square kilometres

9. Here is a worked answer. Find the step that broke.

```
a line passes through (2, 30) and (8, 90)
the run is 8 minus 2, which is 6
the rise is 90 minus 30, which is 60
the slope is the run divided by the rise
6 divided by 60 = 0.1
so the slope is 0.1
```

10. Here is a worked answer. Find the step that broke.

```
figure A rises across the whole picture, from the bottom left to the top right
figure B rises only a little, staying near the bottom
figure A is steeper than figure B
so the quantity in figure A changed more than the quantity in figure B
figure A's side axis runs from 19.8 to 20.4
figure B's side axis runs from 0 to 200
```

11. A press line crosses your desk.

> *The chart is almost vertical. The average bag of grain leaving the depot has shot up over the last six months.*

You open the chart. The side axis runs from 4.90 to 5.10 kilograms. The line goes from 4.95 at the start to 5.02 at the end. All four figures are made up for this problem.

Decide what to compute, compute it, and say what your answer does not establish.

12. A note from a district office says this.

> *We plotted grain against household size for our blocks and fitted a straight line. The line crosses the side axis at 12 kilograms. So even a household with nobody in it is being counted for 12 kilograms a month, which is leakage we should recover.*

Decide what to compute, compute it, and say what your answer does not establish.

C6 · Recognising shapes: linear, exponential, saturating

DEFINITION. A relationship is linear when equal steps in the input add the same amount to the output every time.

A relationship is exponential when equal steps in the input multiply the output by the same factor every time.

A relationship is saturating when the output rises while the differences between successive outputs shrink towards zero, approaching a value it does not pass. That value is the ceiling, and the output may approach it without ever reaching it.

The tests are arithmetic, carried out on values taken at equal steps. Constant differences between successive outputs mean linear. Constant ratios mean exponential.

Differences that shrink while the output climbs do not on their own mean saturating, because an output can climb for ever by ever smaller amounts and approach no value at all. What settles it is a ratio taken on the differences themselves rather than on the outputs. Divide each difference by the difference before it. Three cases follow.

Where that ratio holds constant below one, each difference is a fixed share of the one before. The ceiling can then be computed. The gap still left above any value is the next difference divided by one minus the ratio.

Where the ratios vary but stay at or below some number under one, a ceiling exists. The differences still sum to a finite amount. The same division, using that number, gives the most the gap can be. It does not say exactly where the ceiling is.

Where the ratios creep up towards one, the test shows no ceiling. The differences may add up without limit, as a logarithm's steps do, and the shape has not been named.

Each case reads the steps observed. It holds beyond them only if the ratios keep behaving the same way. [8]

IN PLAIN TERMS. Three shapes turn up over and over.

Linear. Equal steps add. Take one step along the bottom and the value goes up by the same amount. Take another step and it goes up by that same amount again. Drawn out, it is a straight line.

Exponential. Equal steps multiply. Take one step along the bottom and the value is multiplied by the same number. Take another step and it is multiplied by that number again. Drawn out, it starts off looking lazy and then, if it is rising, leaves everything else behind.

Saturating. It rises, then flattens. It climbs quickly at first, then less, then hardly at all. It is heading for a level and it flattens against it. That level is called the ceiling.

Subtract each value from the next one. Are the differences all the same? Then it is linear.

Divide each value by the one before it. Are the ratios all the same? Then it is exponential.

A ratio below one is still exponential, and the values then fall towards zero. The test for saturating, which comes next, applies only to values that rise.

Do the differences keep shrinking while the values keep climbing? That is not yet enough to say saturating. A quantity can climb for ever by ever smaller amounts and head for no level at all.

A logarithm does exactly that. Take the multiplying ruler from the section on logarithms. Each number sits a little further along it than the one before, and each of those steps is shorter than the step before it. The ruler goes on for ever. Nothing on it is a level to flatten against.

So run one more division, and this time divide the differences rather than the values. Take each difference and divide it by the difference before it. Then look at what those ratios do.

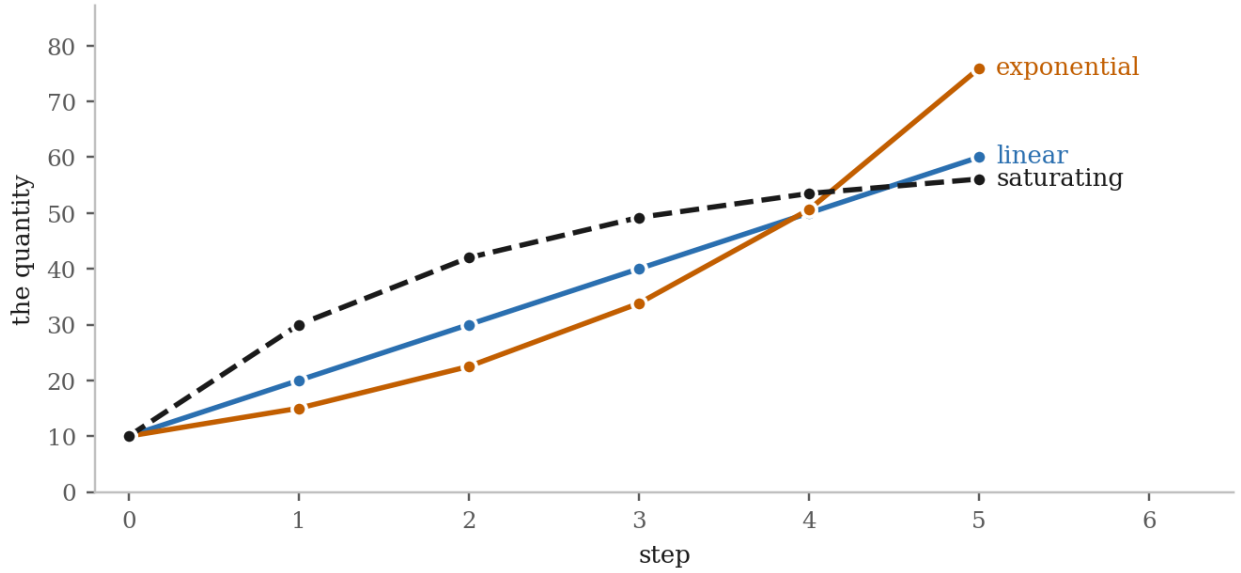
Do they hold at the same number, below one? Then each difference is a fixed share of the one before it. The differences add up to a finite amount, and you can work out where the ceiling is. That is saturating.

Do they wobble, but never go above some number below one, such as 0.5? Then the differences still add up to a finite amount, and there is a ceiling. You can say the most it can lie above where you stand. You cannot say exactly where it is.

Do they creep up towards one, say 0.5, then 0.75, then 0.83? Then the test has found you no ceiling. The steps may be shrinking the way a logarithm's do, heading nowhere. Do not go looking for a ceiling on the strength of the shrinking alone.

One more thing, and it is the one people get caught by. Look at the side axis before you name anything. Some side axes multiply by ten at every step, which is the ruler you met in the section on logarithms. On one of those, an exponential comes out as a straight line. So a straight line on that kind of axis means multiplying, not adding.

And a warning about how much you need to see. Three points do not have a shape. Any three rising points can be drawn as any of the three shapes here. You need enough values to see what the steps are doing.



The three shapes on one pair of axes, which is the only way the difference between them can be practised. All three start at the same place; by the last of the six readings they are nowhere near each other.

ILLUSTRATION. Start with three numbers and a wrong answer, because naming a shape too early is the mistake this section exists to stop.

Somebody shows you three values, taken one step apart: 10, 30, 42. They call the rise exponential and say it is roughly doubling.

Divide each value by the one before it.

```
30 divided by 10 = 3
42 divided by 30 = 1.4
```

The ratios are not the same, so nothing here is multiplying by a fixed factor. Exponential is the wrong word.

Now carry on to six values, and put two other shapes beside them for comparison.

step	linear	exponential	saturating
0	10	10	10

step	linear	exponential	saturating
1	20	15	30
2	30	22.5	42
3	40	33.8	49.2
4	50	50.6	53.5
5	60	75.9	56.1

Work down each column with the two tests.

Take the first column by subtraction.

```
20 minus 10 = 10
30 minus 20 = 10
60 minus 50 = 10
```

The same amount is added at every step, so the first column is linear.

Take the second column by division.

```
15 divided by 10 = 1.5
22.5 divided by 15 = 1.5
75.9 divided by 50.6 = 1.5
```

The same factor multiplies at every step, so the second column is exponential.

Take the third column by subtraction, and watch what the differences do.

```
30 minus 10 = 20
42 minus 30 = 12
49.2 minus 42 = 7.2
53.5 minus 49.2 = 4.3
56.1 minus 53.5 = 2.6
```

The steps shrink while the values keep climbing. Now divide each of those differences by the one before it, which is the test that says whether the shrinking is heading anywhere.

```
12 divided by 20 = 0.6
7.2 divided by 12 = 0.6
```

The ratio holds at about 0.6 all the way down the column, and 0.6 is below one. So if the ratio keeps holding, the shrinking differences add up to something finite, and a ceiling exists. Now find where it is.

Call the distance still left between a value and the ceiling the gap. Before the first step, the gap is the whole of what the differences will ever add up to.

Now look at the differences after the first one: 12, then 7.2, and so on. Each is 0.6 of the one before it in the full list. So together they add up to 0.6 of the full list's total. That means the first step leaves 0.6 of the gap still to go. It has closed the other 0.4.

The first step closed 20. So 20 is 0.4 of the gap, and the gap is 20 divided by 0.4. Add the gap to the starting value of 10.

```
1 minus 0.6 = 0.4
20 divided by 0.4 = 50
10 plus 50 = 60
```

If the ratio keeps holding, the ceiling is 60. Check it from a later value. At 42 the next difference is 7.2.

```
7.2 divided by 0.4 = 18
42 plus 18 = 60
```

The same ceiling. So the rule, in words: divide the next difference by one minus the ratio, and add the answer to the value you are standing on.

Now look at step one and step five, because that is where the trap is. At step one the exponential column is the lowest of the three. At step five it is the highest, and it has passed the ceiling the saturating column will never reach.

That is why you get more values before you name anything.

A rising exponential eventually passes any linear.

WHERE THIS PICTURE BREAKS. These three shapes are a starting kit and not a list of everything.

The two tests need equal steps along the bottom. Values taken at uneven intervals give differences and ratios that mean nothing. A table that does not say its step size cannot be tested at all.

Rounding hides small changes. A ratio that reads 1.5 three times running may be drifting in the digits you were not shown.

MUST KNOW POINTS FOR YOU.

- Run two tests on any set of values taken at equal steps. Subtract each from the next for a constant difference, and divide each by the one before for a constant ratio.
- Shrinking differences do not on their own mean a ceiling. A quantity can climb for ever by ever smaller amounts, as a logarithm does. So run one more test before you say saturating. Divide each difference by the difference before it. A ratio that holds below one lets you work out the ceiling. Ratios that vary but stay at or below some number under one show that a ceiling exists, and not exactly where. Ratios creeping up towards one show you no ceiling, so say you have none to quote rather than estimate one.
- Exponential does not mean fast. It means each equal step multiplies by the same factor.
- Three points have no shape. Ask for more values before you name anything.
- Read the side axis before you call a line linear. On an axis where each step multiplies by ten, a straight line is an exponential.
- Where a curve has flattened, a further equal step in the input buys much less than the step before it did.
- Naming a shape names a pattern and never a mechanism.
- When a journalist uses the word exponential, ask what the values were at equal times and whether the ratio held.

EXERCISE 1 (interpretation). A figure shows one curve, with time along the bottom and a count up the side. The curve rises steeply for the first part, then bends over and runs nearly flat for the rest.

Say which of the three shapes it is. Say what the flat part means in the arithmetic. Then say what the figure does not tell you.

EXERCISE 2 (teaching). A journalist has three minutes and no jargon. They have a chart and a press line saying the numbers are rising exponentially. They ask you whether that is right.

Say what you would tell them, and give them one quotable sentence.

PRACTICE. Ten problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Each of these is a set of values taken one step apart. Name the shape of each where the tests allow one, and say which test told you.

A	4	8	12	16	20
B	4	8	16	32	64
C	4	12	16	19	21.5

2. Each of these is a set of values taken one step apart. Name the shape of each.

D	100	50	25	12.5	6.25
E	100	130	160	190	220
F	2	6	18	54	162

3. Start at 8 and write five values for each of these rules, taking one step at a time.

```
add 6 at every step
multiply by 3 at every step
```

Then say which of the two is which shape.

4. Under section 3(1) of the National Food Security Act, 2013, a person in a priority household is entitled to five kilograms of foodgrains a month.

Write the total grain a household of four is counted for after one, two, three and four months. Name the shape. Give the slope of the line with its unit. Then say what would have to be true for this quantity to be exponential.

5. A quantity is measured at equal steps and comes out as 20, 44, 56.0, 62.0, 65.0, 66.5.

Name the shape, show the working that names it, and estimate the ceiling it is heading for.

6. Two quantities are recorded at equal steps.

G	5	35	65	95	125
H	5	10	20	40	80

Say which of the two will be the larger at step 12, and show the working that decides it.

7. Here is a worked answer. Find the step that broke.

```
the values at equal steps are 10, 30, 42, 49.2, 53.5, 56.1
the first step rose by 20, which is a big jump
the curve is clearly rising fast
a curve that rises fast is exponential
so the quantity is growing exponentially and will keep doing so
```

8. Here is a worked answer. Find the step that broke.

```
the figure has a side axis marked 10, 100, 1,000, 10,000
the points lie on a straight line across the figure
a straight line is the picture of a linear relationship
so the quantity is rising by a fixed amount each month
the rise from the first month to the last is steady and needs no further comment
```

9. A line in a meeting note says this.

> *Registrations are growing exponentially. We had 200 in the first month, 400 in the second and 600 in the third.*

The programme and all three figures are made up for this problem. No real programme is being described.

Decide what to compute, compute it, and say what your answer does not establish.

10. A district officer writes this in a review note.

> *The programme is running out of steam. Coverage went up 12 points in the first year, then 6, then 3, then 1.5. The gains are collapsing and we should move the money elsewhere.*

The programme and all four figures are made up for this problem. No real programme is being described.

Decide what to compute, compute it, and say what your answer does not establish.

C7 · Rate of change, average and instantaneous, without the machinery

DEFINITION. A rate of change is a change in a quantity divided by the time that change took. Its unit is the unit of the quantity divided by the unit of time.

The average rate of change over an interval is the change between the two ends of the interval divided by the length of the interval. Drawn on a graph with time along the bottom, it is the slope of the straight line joining the two end points. That joining line is a chord.

The rate at an instant is the rate of change at one moment rather than over a stretch of time. On a graph it is the steepness of the curve at that one point, which is the slope of the straight line that touches the curve there. That touching line is a tangent.

For a straight line the average rate over every interval is the same number, and it equals the rate at every instant on the line. For a curve the two differ, and an average over an interval fixes nothing about where inside that interval the change happened. [9]

IN PLAIN TERMS. A rate of change is a change divided by the time it took. Take the quantity at the start. Take it at the end. Subtract to get the change. Divide by how long it took.

```
the rate of change = the change in the quantity, divided by the time it took
```

The unit falls out of the division, exactly as it did for a slope. Kilograms divided by months is kilograms per month. A rate of change is never in kilograms on its own, and a rate quoted without its unit is a number you cannot check.

Now the part people get wrong, and it is the whole of this section.

A rate can be worked out over a stretch of time, or asked for at a single moment. Those are two different quantities and they usually have two different values.

The average rate over an interval. It uses the value at the start and the value at the end. It uses nothing in between. So it can tell you nothing about what happened in between.

The rate at one moment. This is how fast the quantity is changing right now. Its name is the instantaneous rate. On a graph it is the steepness of the line at that single point.

For a straight line the two are the same number. A straight line has one steepness and it has it everywhere.

For a curve they come apart. A curve is steep in some places and nearly flat in others. An average across the whole of it can sit far from its steepness at both ends.

Somewhere in between, though, the curve is exactly as steep as the average. It has to be. A curve that stayed steeper than the average all the way across would finish higher than it does. One that stayed flatter would finish lower. So the steepness passes through the average on the way. What the average will not tell you is where it passed. Nor will it tell you how far the curve got from the average on either side of that point.

Here is how you get each one off a drawn curve, with a ruler.

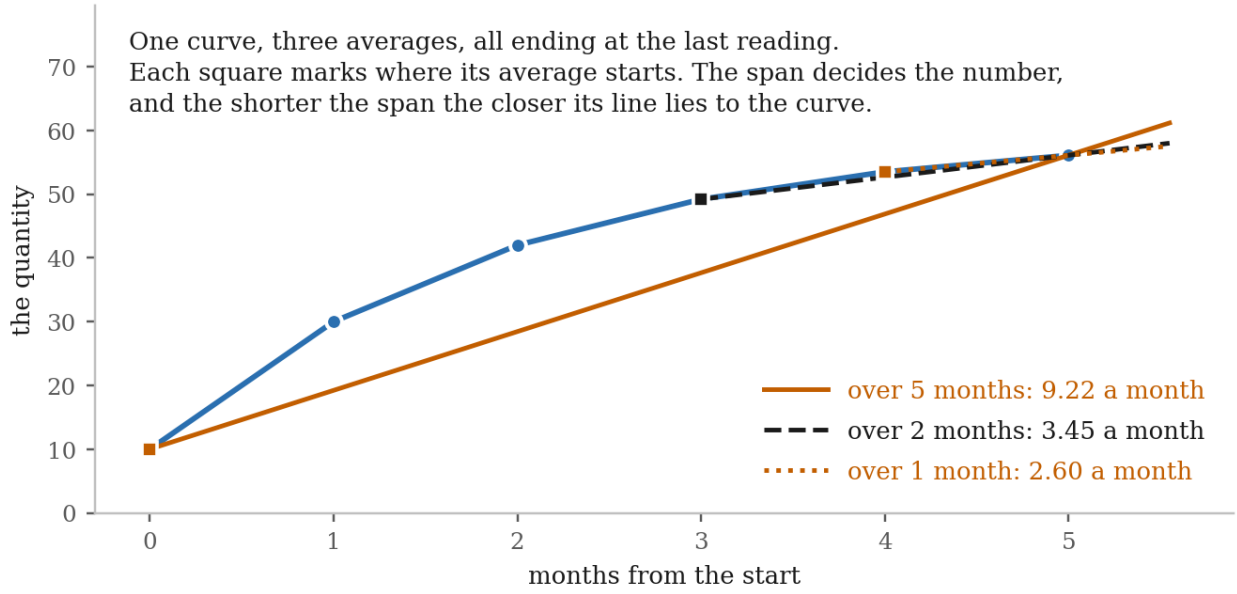
For the average rate between two points, lay the ruler across those two points. The line you have laid down has a name: it is a chord.

For the rate at one point, lay the ruler so that it touches the curve at that point and runs along it. Read its slope the same way. That touching line has a name too: it is a tangent.

A shorter interval gives you a closer answer. Take the average over a month and you get the month. Take it over a day around the same moment and you get much nearer to the moment itself.

One more thing, so that you recognise it on somebody else's page. The rate at a moment has a name in mathematics: the derivative. It is written dy/dx, said "dee y by dee x". The two letters d each mean "a change in". The whole symbol is the steepness at one point, which is what you have just been reading off a curve with a ruler. That is all you need. Nothing in this book asks you to work one out.

Last, run the unit check from the section on dimensional analysis before you believe any rate. A quantity in kilograms, changing over months, gives a rate in kilograms per month. If somebody hands you a rate whose unit is not the quantity divided by a time, it is not a rate of change.



One curve and three averages. Each straight line runs to the last reading from a different starting point, and each average is the slope of its own line: 9.22 a month taken over five months, 3.45 a month taken over the last two, 2.60 a month taken over the last one. Every one of the three is an average and none of them is the rate at a moment, which six monthly readings cannot give you.

ILLUSTRATION. Start with a sentence that is two thirds right, because the true part is what carries the false part past a reader.

Take the saturating column from the section before this one. A quantity was recorded six times, one month apart.

months from the start	the quantity
0	10
1	30
2	42
3	49.2
4	53.5
5	56.1

A note about it says this.

> *It rose by about 46 over five months. That is about 9 a month. Two more months take it past 70.*

Test the first claim. Subtract the first value from the last.

```
56.1 minus 10 = 46.1
```

Test the second. Divide that change by the time it took.

Count the time before you divide. Six readings a month apart span five months, not six. Each month runs from one reading to the next, so the intervals are always one fewer than the readings.

```
46.1 divided by 5 = 9.22
```

About 9.22 a month. That number is the average rate of change over the five months, and it is correct.

Now work out the rate over each single month. Each one is a change divided by one month.

```
30 minus 10 = 20
42 minus 30 = 12
49.2 minus 42 = 7.2
53.5 minus 49.2 = 4.3
56.1 minus 53.5 = 2.6
```

Put them beside the months they belong to.

the month	the rate over it
first	20 a month
second	12 a month
third	7.2 a month
fourth	4.3 a month
fifth	2.6 a month

No single month ran at 9.22. The average is a true statement about the five months together and a false statement about any one of them.

That is why the third sentence of the note fails. It takes an average over the past and uses it as the rate now. Use the most recent month's rate instead, which is 2.6 a month.

```
2.6 times 2 = 5.2
56.1 plus 5.2 = 61.3
```

About 61.3, not past 70. Even 61.3 is too high if the rate keeps falling, as it has every month so far. So treat it as an upper bound, not a forecast. The section before this one puts this column's ceiling at 60, if the ratio between its steps keeps holding.

The note is not wrong about its arithmetic. It is wrong about which interval its arithmetic describes.

Now the easy case, so that you can feel the difference.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month. A household of four is counted for twenty kilograms a month.

```
5 times 4 = 20
```

Say the house starts with 10 kilograms in it and nothing is taken out. Here is the grain in the house, month by month.

months from today	kilograms in the house
0	10
1	30
2	50
3	70
4	90

Take the average rate over the whole four months.

```
90 minus 10 = 80
80 divided by 4 = 20
```

Twenty kilograms a month. Now take it over the third month alone.

```
70 minus 50 = 20
20 divided by 1 = 20
```

Twenty again. Take any interval you like and you will get twenty, because the points sit on a straight line and a straight line has one steepness. Here the average rate and the rate at any moment are the same number, and there is nothing to argue about.

So the question to carry away is not "what is the rate". It is "over what interval".

Finish with the unit, because a rate without one is not checkable. Kilograms up the side, months along the bottom.

```
kilograms divided by months
= kilograms per month
```

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. The six values in the first table are bare numbers.

An average rate over an interval is exact for that interval.

A quantity measured once a month has no rate you can check between the measurements.

And time is not the only thing a quantity can change against. The grain a household is counted for changes against the number of people in it, at five kilograms a month for each extra person. Put people along the bottom and that is a slope. The chord, the tangent and the unit check carry over. The grain is in kilograms a month, so the slope is in kilograms a month per person: "per person" is added and "per month" stays. A dose along the bottom and a measured effect up the side works the same way.

MUST KNOW POINTS FOR YOU.

- Ask "over what interval" every time somebody hands you a rate.
- An average rate over a stretch of time is not the rate now. It is built from the first value and the last value alone.

- Quote a rate with its unit, taken from the quantity divided by the time. Twenty kilograms a month is a claim somebody can check.
- Two quantities can share an average rate and have nothing else in common. Ask for the values in between before you accept that two things are behaving alike.
- A rate you read off a curve by eye is an estimate and not a measurement. It is only as good as the drawing under your ruler.
- Suppose each year's gain has been smaller than the year before. Then the average over the whole programme is higher than the average over any shorter run of whole years ending today. Quoted as the rate now, it flatters the programme.
- When a press line says a quantity is rising at some figure a month, ask which months were used.
- Teach a trainee to write the interval down beside every rate they compute, before the number leaves their page.

EXERCISE 1 (interpretation). A figure has months along the bottom and a count up the side. The curve climbs steeply for the first three months, then bends over and runs almost flat for the next nine.

Say what the average rate over the twelve months describes, and what it does not. Then say which two numbers you would ask for instead, and why those two.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A programme note contains this line.

> *Coverage has grown at an average of 6 points a year for four years. On that trend we reach the target of 40 points above baseline in three more years.*

The yearly gains were 12 points, then 6, then 3, then 1.5.

Say what is wrong with the second sentence, and say what you would put in its place.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They keep saying "the rate" about a quantity that is changing, and they mean whichever rate is nearest to hand.

Teach them the difference between an average rate and a rate at a moment. They should leave asking the right question about the next figure they see.

PRACTICE. Fourteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A quantity is 14 at the start and 38 at the end. The change took 6 units of time.

Work out the average rate of change.

2. Work out the average rate of change for each of these.

```
from 50 to 90, over 8 units of time
from 90 to 50, over 8 units of time
from 7 to 7, over 5 units of time
```

3. A quantity is recorded at equal steps: 4, 10, 20, 34, 52.

Work out the rate over each single step. Then work out the average rate over the whole run.

4. A quantity changes at an average rate of 7 for each unit of time.

Work out the change over 9 units of time. Then work out how long it takes the quantity to change by 91.

5. Work out the unit of the rate of change in each of these cases.

the quantity	the time it is changing over
kilograms	months
kilocalories	days
number of people	years
rupees	weeks

6. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month.

A household of six collects its full entitlement every month and takes nothing out of the house. The house holds 12 kilograms today.

Write the grain in the house at months one, two and three. Work out the average rate of change over those three months, with its unit. Then work out the rate over the second month alone and say why the two agree.

7. A quantity is recorded five times, a month apart: 200, 320, 380, 410, 425.

Work out the average rate of change from the first reading to the last. Then work out the rate over the first month and the rate over the last month. Say which of the three you would quote to somebody who asks how fast it is growing now, and why.

8. A quantity falls from 900 to 300 over 12 months.

Work out the average rate of change, with its sign. Then say what the sign means, and say what this number does not tell you about the twelve months.

9. Two quantities are recorded at the same five monthly times.

month	quantity P	quantity Q
0	100	100
1	130	190
2	160	230
3	190	248
4	220	256

Work out the average rate of change of each over the four months. Then work out the rate over the last month for each. Say what the pair of comparisons shows.

10. Here is a worked answer. Find the step that broke.

```
a quantity goes from 40 to 100 over 5 months
the change is 100 minus 40, which is 60
the rate is the time divided by the change
5 divided by 60 = 0.083
so the quantity is rising at 0.083 a month
```

11. Here is a worked answer. Find the step that broke.

```
the monthly values are 10, 30, 42, 49.2, 53.5, 56.1
the total rise is 56.1 minus 10, which is 46.1
that took 5 months, so the average rate is 46.1 divided by 5
46.1 divided by 5 = 9.22
so the quantity is currently rising at 9.22 a month
and in five more months it will be about 56.1 plus 46.1
```

12. Here is a worked answer. Find the step that broke.

```
a curve is drawn with months along the bottom and kilograms up the side
at month 2 the curve passes through 30 kilograms
at month 8 the curve passes through 66 kilograms
the rate at month 8 is the rise divided by the run
66 minus 30 = 36
36 divided by 6 = 6
so at month 8 the quantity is rising at 6 kilograms a month
```

13. A district note crosses your desk with this line in it.

> *The waiting list has come down from 900 to 300 over the last twelve months. That is a steady clearance rate, so the list will be empty in six more months.*

The office, the list and both figures are made up for this problem. No real service is being described.

Decide what to compute, compute it, and say what your answer does not establish.

14. A colleague sends a one-line summary of a programme review.

> *Gains were 12 points, then 6, then 3, then 1.5, over four years. That averages about 5.6 points a year, so budget for the same again next year.*

The programme and all four figures are made up for this problem. No real programme is being described.

Decide what to compute, compute it, and say what your answer does not establish.

C8 · Accumulation, a running total, without the machinery

DEFINITION. An amount is a rate multiplied by the length of time over which that rate held.

A running total is the sum of those amounts across successive intervals.

Drawn with time along the bottom and the rate up the side, each amount is the area of a rectangle whose width is the interval and whose height is the rate over it. The running total up to any time is the area under the rate line up to that time.

The total is certain to be exact only where the rate held steady across each interval. Where the rate moved inside an interval, the rectangle over-counts or under-counts. Take the largest gap between the height used and the rate at any moment inside that interval. That gap, multiplied by the length of the interval, is the most the rectangle can be wrong by. The error is an amount and carries the quantity's unit.

Added across the intervals, those amounts are the most the total can be wrong by. That most is zero only where the rate held steady inside every interval.

Take a rate that only falls, or only rises, with each height read at one end of its interval. Halving the intervals then halves the most the total can be wrong by, and never raises the actual error. Where the rate rises and falls, halving usually lowers the most, but not always. The actual error can then grow or shrink, though it never passes the most for the intervals used.

A rate and a total are different quantities with different units, and neither can be read off the other without the time it applied over. [10]

IN PLAIN TERMS. The section before this one took a quantity and got a rate out of it. This one goes the other way. You start with a rate and you get an amount back.

A rate multiplied by a time gives an amount.

```
the amount = the rate, times the length of time the rate held for
```

Twenty kilograms a month, held for three months, gives sixty kilograms.

```
20 times 3 = 60
```

Check the unit, because the unit is what tells you the step was done the right way round. Kilograms per month, multiplied by months, leaves kilograms. The months cancel, exactly as they did in the section on dimensional analysis.

They cancel only when the two time units match. Say water is drawn at 40 litres a day for 2 months, both figures made up. Litres a day times months is not litres. So turn the months into days first. Count a month as 30 days, and say that you did.

```
2 times 30 = 60
40 times 60 = 2400
```

Sixty days at 40 litres a day is 2,400 litres. Now the days cancel.

Rates change. So you cut the time into intervals short enough that the rate can be treated as steady inside each one. Work out the amount for each interval. Then add them up as you go.

That sum is called a running total.

Draw time along the bottom and the rate up the side. Mark the rate over each interval at the height it reached. The line those marks make across the figure is the rate line, and everything below it is what you are about to measure.

Now take one interval and draw a rectangle on it. Its height is the rate over that interval, so its top sits on the rate line.

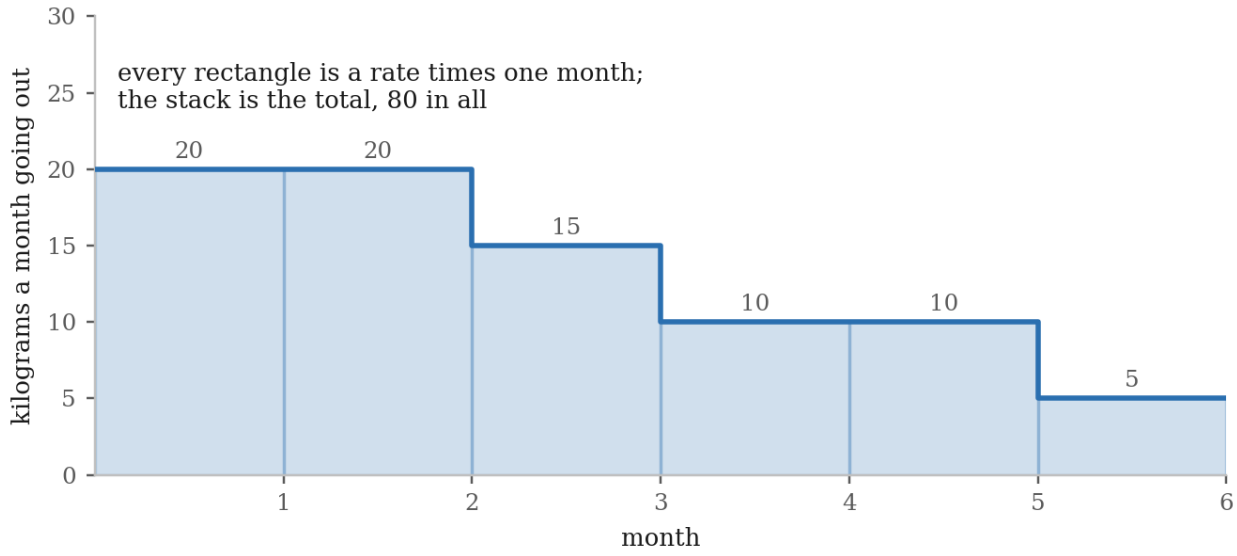
The area of that rectangle is its height times its width. The area of the rectangle is the amount.

Stack the rectangles side by side across the whole stretch of time. The total area is the running total.

A rectangle assumes the rate held steady across its whole width. Where the rate was falling, a rectangle drawn at the starting rate counts too much. Where the rate was rising, it counts too little.

Now cut each interval in half. Where the rate only falls, or only rises, the error does not grow, and it usually shrinks. Where the rate rises and falls, the over-counts and under-counts can partly cancel. Then halving can leave the error larger. Only a rate that held steady inside every interval gives a total you know to be exact.

One name, so you recognise it on somebody else's page. The total under a rate line is called an integral. It is written with the sign $\int$, which is a stretched letter S for sum. That is all you need. Nothing in this book asks you to work one out.



Each rectangle is one month's rate multiplied by one month, so its area is that month's amount. Stacking them is the running total, and the total is the area under the whole rate line.

ILLUSTRATION. A store issues grain. Somebody records the rate at which it goes out, month by month, for six months.

month	kilograms a month going out
1	20
2	20
3	15
4	10
5	10
6	5

A note on the file says this.

> *The store issues 20 kilograms a month. Over six months that is 120 kilograms.*

The multiplication is right.

The error is that 20 was the rate in two of the six months and not in the other four. One month's rate was used as though it held for all six.

Do it properly. Each month is one interval, and each interval's amount is its rate times one month.

Add them as a running total, which is the amount so far after each month.

```
20 plus 20 = 40
40 plus 15 = 55
55 plus 10 = 65
65 plus 10 = 75
75 plus 5 = 80
```

Eighty kilograms over the six months, against the 120 in the note. The note is out by half again.

Now draw it, because the picture is how everybody else will show you this.

Put months along the bottom and kilograms a month up the side. Mark each month's rate at the height it reached, and the six marks make one stepped line across the figure. That line is the rate line.

Now over month one, draw a rectangle one month wide and 20 kilograms a month tall, so that its top sits on the rate line. Its area is its height times its width.

```
20 times 1 = 20
```

Twenty kilograms. Do the same for each month and you get six rectangles standing side by side. Their heights are 20, 20, 15, 10, 10 and 5, and every width is one month. The whole shaded area under the rate line is 80 kilograms, which is the answer you already have.

Check the unit before you go on.

```
kilograms per month, times months
= kilograms
```

A total is in the unit of the quantity, never in the unit of the rate.

Those 80 kilograms went out of the store, so for the store itself they are subtracted. Say it held 200 kilograms at the start and nothing came in. That 200 is made up for this line.

```
200 minus 80 = 120
```

One hundred and twenty kilograms left. Subtract month by month instead. The running total of grain issued still rises, 20, 40, 55, 65, 75, 80. What falls is the stock left in the store.

```
200 minus 20 = 180
180 minus 20 = 160
160 minus 15 = 145
145 minus 10 = 135
135 minus 10 = 125
125 minus 5 = 120
```

A flow out is an amount taken away rather than added.

Last, the part that decides how much your total is worth.

Take a rate that is falling steadily, from 20 kilograms a month down to nothing over four months. Read it at the start of each month and you get 20, 15, 10 and 5.

This rate line is a different drawing from the one for the store. There we took each month's rate as holding for the whole month, so the line was a staircase and the rectangles sat exactly on it. Here the rate falls all the time, so its line is a straight slope from 20 down to nothing. Rectangles always make a staircase. The gap between that staircase and the slope is what the table below measures.

Build the total from four rectangles, each one month wide.

```
20 plus 15 plus 10 plus 5 = 50
```

Fifty kilograms.

Now cut the intervals in half. Read the rate every half month and you get 20, 17.5, 15, 12.5, 10, 7.5, 5 and 2.5. Each rectangle is half a month wide, so add the eight heights and multiply by half a month.

```
20 plus 17.5 plus 15 plus 12.5 plus 10 plus 7.5 plus 5 plus 2.5 = 90
90 times 0.5 = 45
```

Forty-five kilograms.

The rate falls in a straight line from 20 to nothing, so its average over the four months is halfway between the two ends.

```
20 plus 0 = 20
20 divided by 2 = 10
```

Ten kilograms a month, for four months.

```
10 times 4 = 40
```

Forty kilograms, and that one is exact.

Put the three answers beside each other and watch the error close.

how the total was built	answer	how far above the exact answer
four whole-month blocks	50 kg	10 kg
eight half-month blocks	45 kg	5 kg
the exact total	40 kg	none

Halving the interval halved the error, and it did that because this rate falls in a straight line. On a rate like this one, halving again would halve the error again. Read that as a property of a rate that changes steadily, not as a rule about intervals.

Where a rate bends but keeps falling, halving the intervals still shrinks the error, but by no fixed share you can count on. Where a rate rises over part of the span and falls over the rest, two rectangle errors can point opposite ways and partly cancel. Halving can then leave the total error larger than it was. It still never passes the most from the definition: each interval's largest gap times its width, added across the intervals.

In this worked case the error never reaches zero, and every rectangle counts too much, because the rate was falling inside each one of them.

So a total is not one number. It is one number and one assumption, and the assumption is that the rate held across each interval you used.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. The six monthly rates in the first table are bare numbers.

Area is a total only when time is along the bottom and a rate is up the side. Read both axis labels before you shade anything.

A rectangle built on the rate at the start of an interval counts too much while the rate is falling. While it keeps falling, halving the intervals shrinks that error and never removes it. Where the rate rises and falls, halving can leave the error larger.

And every amount here is an amount going one way. A flow out of something is an amount subtracted rather than added, and the amount left then falls.

MUST KNOW POINTS FOR YOU.

- Multiply a rate by the time it held for before you quote any total.
- Check the unit of every total. If the unit of your answer still has a time on the bottom, you have added rates instead of accumulating them.
- Taking one measurement of a rate and multiplying it by a whole year is the commonest way a total goes wrong.
- The area under a rate line is a total and not a rate.
- A total carries its interval assumption with it. Halving the intervals usually shrinks the error, but where the rate rises and falls it can make it larger. Only a rate that held steady inside every interval gives a total you know to be exact.
- Before accepting an annual figure built from one measurement, ask how many times the rate was measured and when. A yearly total from a single reading is a forecast wearing the clothes of a count.
- A total says how much has piled up, a rate says how fast it is piling up now, and the two answer different questions.
- Teach a trainee to draw the rectangles before reaching for a formula.

EXERCISE 1 (interpretation). A figure has months along the bottom and litres a day up the side. The line starts high, falls for six months, and then runs flat for six more.

Say what the area under the line stands for, and give its unit. Then say what the figure does not tell you.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A district file carries this line.

> *The depot issued 400 bags in the first week of April. That is 400 a week, so it will issue about 20,800 bags this year.*

Take a year as 52 weeks.

Say what has been assumed, say what you would ask for, and say what you would write in its place.

EXERCISE 3 (teaching). A health secretary has one page and three minutes. Two reports have reached them about the same programme. One gives a monthly rate and one gives a yearly total, and the two do not look like they can both be true.

Write what you would put on that page.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A quantity grows at a steady rate of 7 for each unit of time.

Work out how much is added over 9 units of time, over 20 units of time, and over half a unit of time.

2. A quantity changes at different rates over three successive stretches of time.

stretch	the rate	how long it held
first	12	4 units of time
second	5	6 units of time
third	20	2 units of time

Work out the amount over each stretch, and the total over all three.

3. A quantity is added at these rates over six successive intervals, each one unit of time long: 9, 9, 4, 4, 4, 2.

Write the running total after each interval.

4. A quantity accumulates over three stretches. Over the first it grows at 10 for 5 units of time. Over the second it grows at 6 for 5 units of time. Over the third it grows at 6 for 10 units of time.

Work out the total. Then work out the average rate over the whole twenty units of time.

5. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month.

Work out the grain a household of seven is counted for over a full year. Show the rate you used and the time you multiplied it by, and give the unit at every line. Then say which figure in your working comes from the Act and which are yours.

6. A rate is recorded once a month for six months, in litres a month: 300, 300, 450, 450, 200, 100.

Work out the running total after each month, and the total over the six months. Then state the assumption the total rests on.

7. A rate is measured every three months and comes out at 40 units a month, then 60, then 60, then 20. Each reading is the rate for the three months that start on the day it was taken, and the four together cover the year.

Work out the total across the year. Then work out what you would get by adding the four readings, and say what unit that sum is in.

8. A quantity accumulates over 8 months and the total comes to 96 kilograms. Over the first 2 months the rate was 18 kilograms a month.

Work out the average rate over the whole 8 months. Then work out the total over the remaining 6 months, and the average rate over those 6 months.

9. Here is a worked answer. Find the step that broke.

```
a tank is filling and the rate is measured at four times
the readings are 12, 9, 6 and 3 litres a minute
the total is 12 plus 9 plus 6 plus 3
12 plus 9 plus 6 plus 3 = 30
so 30 litres went in
```

10. Here is a worked answer. Find the step that broke.

```
a figure has months along the bottom and kilograms in the store up the side
the line runs at about 300 kilograms for all twelve months
the area under the line is 300 times 12
300 times 12 = 3600
so 3,600 kilograms passed through the store during the year
```

11. A note reaches you from a block office.

> *Our vehicle used 8 litres of fuel an hour on the day we measured it. It runs about 6 hours a day. So budget for about 17,500 litres this year.*

The office, the vehicle and all three figures are made up for this problem. No real service is being described.

Decide what to compute, compute it, and say what your answer does not establish.

12. A line in a review meeting goes like this.

> *The scheme distributed 24 lakh kilograms of grain last year. The year before it was 20 lakh. It is clearly reaching more people than ever.*

The scheme and both figures are made up for this problem. No real scheme is being described.

Decide what to compute, compute it, and say what your answer does not establish.

C9 · Stocks and flows

DEFINITION. A stock is a quantity that exists at a moment and can be measured at that moment. A flow is a rate that changes a stock and, like any rate, it is quoted over a stretch of time or at a moment.

The stock at any time is its starting value, plus everything that flowed in up to that time, minus everything that flowed out. Each of those two totals is a rate multiplied by a time and added across intervals, which is the running total of the previous section.

The net flow is the inflow minus the outflow. A stock rises while the net flow is positive, falls while it is negative, and holds steady while it is zero.

A stock and a flow carry different units. A stock carries the unit of the quantity. A flow carries that same unit divided by a unit of time. The two are never added to each other.

A stock that holds steady is evidence that the inflow and the outflow matched over the period measured, and is no evidence at all that either one was small. [11]

IN PLAIN TERMS. Picture a water tank with a tap running into it and a drain running out of it.

The stock is what is in the tank. It is an amount, it exists right now, and you could measure it this minute by looking. Litres in the tank is a stock. Kilograms of grain in a store is a stock. A number of people on a waiting list is a stock.

A flow is what goes in or what goes out. It is a rate, so it has a time on the bottom of its unit. Litres a minute is a flow. Kilograms a month is a flow. People joining the list each week is a flow.

Look at the unit. If it has a “per” or an “a” with a time after it, you are holding a flow. If it does not, you are holding a stock.

Now the one line this whole section is for. Say it in words first.

The amount in the tank now is the amount you started with, plus everything that came in, minus everything that went out.

```
the stock now = the starting stock, plus the total in, minus the total out
```

Each of those two totals is built the way the last section built one. Take the flow, multiply by how long it ran, and add the pieces up.

The difference between the inflow and the outflow has a name: the net flow.

- More in than out, so the net flow is positive, and the stock rises.
- More out than in, so the net flow is negative, and the stock falls.
- The same in as out, so the net flow is zero, and the stock does not move.

Read that third line again, because it is the one people get wrong.

A stock that does not move tells you the two flows matched. It tells you nothing whatever about how big they were.

A stock can be large, steady and busy all at once. That is the sentence to carry out of this section.

A stock and a flow are never added together.

ILLUSTRATION. Start where the mistake gets made, which is an inspection.

A fair price shop keeps its grain in a store room behind the counter. An inspector weighs what is on the shelves at the end of April and finds 2,000 kilograms. She comes back at the end of May and finds 2,000 kilograms again.

Her note says this.

> *The stock has not moved in a month. Nothing is being distributed here.*

The first sentence is a measurement. The second is a conclusion, and it does not follow.

Work out what was going through the shop. This one serves 500 priority households and each of them has four people in it. The shop, the 500 households and the 2,000 kilograms are made up for this example. The rate below is not.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of food-grains per person per month. So each household is counted for twenty kilograms a month.

```
5 times 4 = 20
```

Five hundred households at twenty kilograms a month each.

```
20 times 500 = 10000
```

Ten thousand kilograms a month go out of the shop to households. The same ten thousand have to come in from the depot, or the shelves would empty.

Now put those two flows against the stock on the shelves.

```
10000 divided by 2000 = 5
```

Five times the whole contents of the store room passed through it during the month. The shelves held 2,000 kilograms on both days the inspector looked, and they held different grain.

So write the balance line out for May and watch the two flows cancel.

```
2000 plus 10000 minus 10000 = 2000
```

Two thousand kilograms at the end of May, which is what she measured. Nothing in that number could have told her the size of either flow, because the two flows left it exactly where they found it.

Now make the flows differ, so that you can see the stock move.

month	stock at the start	came in	went out	stock at the end
1	2,000	10,000	10,000	2,000
2	2,000	9,000	10,000	1,000
3	1,000	11,000	10,000	2,000

Check each row with the same line.

```
2000 plus 10000 minus 10000 = 2000
```

Month two, where the depot sent a thousand kilograms short.

```
2000 plus 9000 minus 10000 = 1000
```

The stock ends where it began, three months later. Read the first row and the last row alone and you would say nothing happened at all.

Check the units before you go on, because this is where the two kinds of quantity get mixed. The stock is in kilograms. The flows are in kilograms a month. Adding one to the other gives nothing, and multiplying a flow by a month is what turns it into something you can add.

```
kilograms per month, times months
= kilograms
```

A person's body weight is a stock, in kilograms. Its flows are in the same unit: the mass going into the body, and the mass leaving it. A body gains or loses mass only by mass crossing its edge, so this balance line is as exact as the store room's.

The weight now is the weight you started with, plus everything that came in, minus everything that went out.

So take a weight that has not changed over a year. Its mass in and its mass out came to the same total across that year. It is not a person in whom nothing is happening. Both flows may be large, and the arithmetic says only that they matched.

There is a second balance in the same body, and people run the two together. The energy stored in a body is a stock too. Food energy taken in is a flow into it and energy spent is a flow out of it. Both are rates, in the kilojoules or kilocalories a day you met in the section on energy units.

Now run the unit check you just ran on the store room. It says these are two balances and not one. The weight is in kilograms. A flow in kilocalories a day, multiplied by days, gives kilocalories and never kilograms. The energy flows cannot go into the balance line for weight.

Joining the two balances means saying how much energy a kilogram of change in a body holds. That depends on what the change is made of, which is a claim about the body. Book 0 does not carry that claim, and the subject booklet on energy balance begins from it. The line stops here on purpose, before any number goes into it.

So be exact about what a steady weight tells you from this book. It tells you the mass going in and the mass going out matched. It does not, on its own, tell you that energy in matched energy out. That is the step this book does not take, and it is the step most arguments about weight take without noticing.

This section gives you no figure for any of these flows and you should not take one from here. Three questions follow. How large is each flow, how is each one measured, and what happens to one when the other changes? All three are the work of the subject booklet on energy balance. Every one of those needs evidence that this section does not carry.

What you have is the shape, and it holds for both balances. A level that has not moved is not a level with nothing going through it. That is what stops a steady weight being read as an absence of movement.

Figures quoted in this illustration are from [6].

WHERE THIS PICTURE BREAKS. The shop, the 500 households, the four people in each and the 2,000 kilograms on the shelves are all made up for the example. Only the five kilograms per person per month comes from the Act.

The store room has one door in and one door out, and every kilogram through either is counted. Count only some of the doors and the traffic you missed turns up in the arithmetic as an error in a flow you did count.

A store room's two flows are set from outside it. Grain arrives because a depot sent it. Grain leaves because shops drew it. Neither answers to what is on the shelves.

That is not true of every stock. In some systems one flow moves when the other moves, or moves with the level of the stock. Change the inflow in a system like that and the balance does not shift by the amount you changed. The balance line still holds. What fails is the assumption that you can alter one term and leave the others where they were.

Whether a person's two flows work that way is a question about the body, not about arithmetic. This section does not answer it. That is why you get the shape here and no numbers.

Two readings a month apart do not make a steady stock.

MUST KNOW POINTS FOR YOU.

- A stock that is not moving is not a stock with nothing going through it. The level on its own cannot tell the two cases apart.
- A time on the bottom means a flow. No time means a stock, and the two answer different questions.
- Never add or subtract a stock and a flow. Kilograms and kilograms a month are different kinds of quantity.
- Write the balance line out before you argue about any of its parts. Starting stock, plus everything in, minus everything out.
- The balance line tells you what must be true of the numbers and never what caused a flow to change. Do not use it to explain why anything moved.
- A person whose weight has been steady for a year is not a person in whom nothing is happening. Steadiness means the mass going in and the mass going out matched over the year, not that either was small. It does not, on its own, mean energy in matched energy out. That needs the step from kilojoules to kilograms, which this book does not take.
- Auditing a stock cannot find a matched pair of wrong flows.
- When a journalist asks whether something is getting worse, separate the level from the traffic through it.

EXERCISE 1 (interpretation). A report gives two figures about a district scheme. One is 4,000 kilograms of grain held in the block store on 31 March. The other is 12,000 kilograms a month moving through that store.

Say which is a stock and which is a flow, and how you can tell. Then say what each one answers and what neither answers.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A monitoring note contains this line.

> *We counted the store on the first of every month for six months. It held about 2,000 kilograms each time. The scheme is therefore static and no grain is reaching households.*

Say what is wrong, and say what you would ask for.

EXERCISE 3 (teaching). A journalist has three minutes and no jargon. They ask why a person's weight can stay the same for a year, when, as they put it, "so much is going in and out".

Say what you would tell them, and give them one quotable sentence.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A stock starts at 500. Over one period, 120 flows in and 90 flows out.

Work out the stock at the end of the period, and the net flow over it.

2. Work out the stock at the end of each of these periods, and say which way the stock moved.

stock at the start	came in	went out
800	200	200
800	200	350
800	400	150

3. Say which of these is a stock and which is a flow, and give the reason from the unit.

the quantity	its unit
grain in the block store	kilograms
grain arriving at the store	kilograms a month
people on a waiting list	people
people joining the list	people a week
energy spent	kilocalories a day

4. A stock stood at 1,400 at the start of a month and 1,100 at the end of it. Over that month 2,600 flowed in.

Work out how much flowed out.

5. Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month.

A fair price shop serves 300 priority households of five people each. Its store room holds 1,500 kilograms today. Every month it issues the full entitlement and receives exactly the same amount from the depot.

Work out the monthly outflow, then the stock after one month and after six months. Say which figures are yours and which comes from the Act.

6. A tank holds 6,000 litres and holds 6,000 litres again a day later. Water runs in at 15 litres a minute all day, and the drain runs at the same rate. A day is 24 hours.

Work out how many litres went through the tank in the day. Then work out how many times the contents of the tank were replaced.

7. A block store held 4,000 kilograms on 1 January and 4,000 kilograms on 1 July. Over those six months it issued 9,000 kilograms a month to shops.

Work out the total issued, the total received, and the monthly inflow. Then say what the two January and July counts would have looked like if the depot had sent 8,500 kilograms a month instead.

8. Here is a worked answer. Find the step that broke.

```
a store held 2,000 kilograms in April and 2,000 kilograms in May
the stock did not change over the month
a stock that does not change has no flows
so nothing came into the store and nothing left it
so the shop distributed no grain in May
```

9. Here is a worked answer. Find the step that broke.

```
a store holds 3,000 kilograms today
it receives 800 kilograms a month and issues 950 kilograms a month
the net flow is 800 minus 950, which is minus 150
3,000 minus 150 = 2,850
so the store will be empty in 2,850 months
```

10. A note from a district office says this.

> *The waiting list stood at 1,200 in April and 1,200 in October. Six months and no movement, so the clinic is not clearing anybody.*

The office, the clinic and both figures are made up for this problem. No real service is being described.

Decide what to compute, compute it, and say what your answer does not establish.

11. A colleague says this in a meeting about a patient.

> *Her weight is the same as it was a year ago. So whatever she eats and whatever she spends must both be small. Nothing is going on.*

Decide what to compute, compute it, and say what your answer does not establish.

References · Part C

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An entry marked outstanding has not yet been checked against the source itself. Nothing in these sections rests on one: the reasoning is a derivation you can check with a calculator, and the entry records an anchor still to be supplied.

Part D · Uncertainty

D1 · What a probability is

DEFINITION. A probability is a number from 0 to 1 that states how often an outcome would turn up if the same chance process were repeated a great many times. The complete list of everything that could happen is the sample space. One member of that list is an outcome, and any collection of outcomes from it is an event.

The complement of an event is every outcome in the sample space that is not in it. An event's probability and its complement's probability always add to 1.

Call the probability of an event p, the letter p standing for probability. The odds are how many times more likely the event is than its opposite: p divided by (1 minus p). [1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 10]

IN PLAIN TERMS. A probability is a proportion.

It always sits between 0, which means never, and 1, which means every time.

When every outcome is exactly as likely as every other, working out a probability is counting. Count the outcomes that make your event true. Count every outcome there is. Divide the first by the second.

The complement of an event is everything that is not it.

Odds are how many times more likely the event is than its opposite, not how likely it is. To get them, divide the probability by 1 minus the probability — that is, by the complement.

A probability of 0.5 gives odds of 1, usually said as "evens" or "one to one". A probability of 0.75 gives odds of 3, said "three to one". Learn to move between the two now.

ILLUSTRATION. Go back to section 3(1) of the National Food Security Act, 2013. It set five kilograms of foodgrains per person per month for a priority household. Read on to section 3(2), which sets a ceiling on how many people that entitlement can reach. It shall extend up to seventy-five per cent of the rural population, and up to fifty per cent of the urban population. Section 3(2) does not say it bounds any one State or district. Section 9 leaves "the percentage coverage ... for each State" to the Central Government.

Pick one rural person, anywhere the Act covers, entirely at random. At random means every rural person has exactly the same chance of being the one you pick. So the whole rural population is your sample space, and the person you land on is your outcome.

Let A be the event that the person you pick is covered by the entitlement. Ask for P(A), said "P of A", meaning the probability of A.

You are not told how many people are actually covered, only the largest share the Act allows. So work with 0.75 as a ceiling on P(A), not a measured figure, and carry that qualification into every answer below.

```
P(A) is at most 0.75
```

Now the complement. Not being covered is everything else in the sample space.

```
1 minus 0.75 = 0.25
```

At least one rural person in four, on this reading of the Act, is not covered.

Now the trap. Turn 0.75 into odds: the probability divided by its complement.

```
0.75 divided by 0.25 = 3
```

The odds a random rural person is covered are, at most, three to one. Here that happens to read the same as "three in four". That is exactly what makes the trap dangerous. It works once and then stops working.

Watch it stop. The urban ceiling is fifty per cent.

```
0.5 divided by 0.5 = 1
```

Odds of one to one, usually said "evens". Odds of one mean half the time.

Suppose you read that the odds are two to one against — two to one that the event does not happen. Odds "against" are the odds of the complement, so first flip them round: odds of two to one against is odds of one-half in favour.

```
odds in favour = 1 divided by 2 = 0.5
```

Turn odds back into a probability by counting parts. Odds of 0.5 mean 0.5 parts for the event to 1 part against it. Add those parts and there is one and a half parts in all.

```
0.5 divided by 1.5 = 0.3333…
```

About a third.

Figures quoted in this illustration are from [11].

WHERE THIS PICTURE BREAKS. Every probability above is a bound, not a measurement. The Act bounds how far the entitlement can extend; it does not say how many people are actually receiving it in any one district. Treat 0.75 as the largest the chance of being covered could be, and 0.25 as the smallest the chance of not being covered could be. Say so whenever you quote a figure built from them.

Odds and probability are almost the same number near 0: 0.01 gives odds of 0.0101. Near 1 they pull apart: 0.99 gives 99, and 0.999 gives 999.

MUST KNOW POINTS FOR YOU.

- Odds of a figure are not the same as a probability of that figure. Odds of 0.25 convert to a probability of 0.25 divided by 1.25, which is 0.2 — a one-in-five chance. Convert before you speak.
- Near 0, odds and probability are almost the same number. Above one half they pull apart, and near 1 they pull apart fast: 0.99 to 0.999 multiplies the odds by ten.
- A probability describes a group, or a process repeated many times. It does not describe the one person in front of you. Knowing that 0.75 of a population is covered by an enti-

tlement tells you nothing about whether this particular person is one of them. Use the figure to plan for a group. Then go and check the individual case on its own.

- A probability or odds computed from a ceiling is itself a ceiling. Its complement is a floor: at most 0.75 covered means at least 0.25 not covered. Say which in the same sentence you quote the number, or the reader will treat it as a count.
- "Odds of three to one" means three chances in four. Say "three in four" and the journalist hears it right.

EXERCISE 1 (teaching). A journalist calls about the National Food Security Act's rural ceiling. She says, "so the odds of being covered are seventy-five per cent." You have three minutes and no jargon.

Correct the sentence, explain the difference between the two figures, and give the journalist one sentence they can print.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A briefing paper contains this line: "The odds of coverage in this block are 60%."

Say what is wrong with the sentence, and rewrite it so the figure means something.

PRACTICE. Nine problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A bag holds 5 red balls and 3 blue balls, and nothing else. One ball is drawn at random.

Work out the probability of drawing red, and the probability of drawing blue. Check that they add to 1.

2. An event has a probability of 0.2. Work out its odds.

Separately, an event has odds of three to two — that is, 1.5. Work out its probability.

3. An event has a probability of 0.4. Work out its complement, and then work out the odds.

4. Section 3(2) of the National Food Security Act, 2013 sets the entitlement's reach at up to 75 per cent of the rural population. Pick one rural person at random.

Work out the largest that the probability they are covered can be, and the smallest that the probability they are not covered can be.

5. Use the rural ceiling of 0.75 from section 3(2) of the National Food Security Act, 2013. Work out the odds, at most, that a randomly picked rural person is covered.

6. Section 3(2) of the National Food Security Act, 2013 sets the urban ceiling at up to 50 per cent. Work out the odds, at most, of coverage in the urban population, and compare them with the rural odds of three to one. Say what the comparison shows about how odds behave.

7. Here is a worked answer. Find the step that broke.

```
a report gives the odds of household coverage as 0.25
odds of 0.25 mean a probability of 0.25
so a quarter of households are covered
```

8. Here is a worked answer. Find the step that broke.

```
a probability of coverage is 0.6
odds equal probability divided by 1 plus probability
0.6 divided by 1.6 = 0.375
so the odds of coverage are 0.375
```

9. A briefing note says: “Under the food security law, three in four rural people here are covered.”

Decide what to compute, compute it, and say what your answer does not establish.

D2 · Counting outcomes; independence

DEFINITION. Tossing a coin n times has 2 to the power n outcomes, one factor of 2 for every toss.

Independence is checked, not assumed: A and B are independent exactly when P(A and B) — the probability that A and B both happen — equals P(A) times P(B), and dependent otherwise. Where no joint figure exists, the check is an argument that knowing one tells you nothing about the other. If you can make neither, treat them as dependent. When they are independent, multiply to get P(A and B).

When A and B cannot both happen — they are mutually exclusive — P(A or B), the probability that at least one of A or B happens, equals P(A) plus P(B): add. [12, 13, 14, 15]

IN PLAIN TERMS. Toss a coin twice and write out every pair: heads-heads, heads-tails, tails-heads, tails-tails. Four outcomes, because each toss has two and there are two tosses. Every extra toss doubles the list. Two tosses give 2 squared outcomes. Three tosses give 2 cubed, which is 2 to the power 3, and n tosses give 2 to the power n.

The word “and” points at multiplying, but only on one condition. The two events have to be independent — meaning that finding out one of them happened tells you nothing new about the other. Multiply their probabilities to get the probability of both.

The word “or” points at adding, and it too has a condition: the two events have to be mutually exclusive, meaning they cannot both happen. If they can both happen, plain adding counts the overlap twice, and the fix is to take the overlap back out once.

Never multiply two probabilities together just because a sentence has the word “and” in it. Check first: does knowing one change the chance of the other?

ILLUSTRATION. Section 3(1) of the National Food Security Act, 2013 entitles every person belonging to a priority household to five kilograms of foodgrains a month. Section 3(2) caps how far that entitlement reaches: up to 75 per cent of the rural population. Treat 0.75 as the probability, at most, that a rural person picked at random is covered — the figure from the section before this one.

Pick two rural people entirely at random, from two different, unrelated households. Knowing whether the first is covered tells you nothing about the second, so treat the two as independent. Multiply.

```
0.75 times 0.75 = 0.5625
```

At most 0.5625, so a little over half, that both are covered.

Now pick two people from the same household instead. If the household is covered, every person belonging to it is covered along with it. The Act gives the entitlement to “every person belonging to priority households”. It is households that the State Government identifies. The two outcomes are not independent, and multiplying is the wrong move.

```
P(both covered, same household) is at most 0.75
```

Not 0.5625.

Pick three rural people, each from a different household, and ask for the probability that at least one of them is covered. Go by the complement instead: work out the probability that none of the three is covered, and subtract that from 1. If the three “covered” events are independent, the three “not covered” events are too. So multiply the three “not covered” chances. Each “1 minus” step turns a ceiling into a floor, or a floor into a ceiling.

```
1 minus 0.75 = 0.25
```

P(not covered) is at least 0.25.

```
0.25 times 0.25 = 0.0625
```

```
0.0625 times 0.25 = 0.015625
```

P(none covered) is at least 0.015625.

```
1 minus 0.015625 = 0.984375
```

P(at least one covered) is at most 0.984375. At most, about 98.4 per cent that at least one of the three is covered.

Figures quoted in this illustration are from [11].

WHERE THIS PICTURE BREAKS. Two people can look like two separate draws and actually be one draw, counted twice. Before multiplying, name what would have to be true for the two events to be independent. Then check whether it is.

"At most 0.5625" and "at most 98.4 per cent" are bounds built from a bound, not measured figures. Every extra multiplication carries that same qualification forward. It does not sharpen it.

MUST KNOW POINTS FOR YOU.

- Independence is a checkable claim, not a default. Test it by asking whether P(A and B) equals P(A) times P(B).
- Two people drawn from the same unit that a decision is made about are almost never independent. Think of the same household, the same clinic, the same litter. This holds even when the underlying probability looks identical to two people drawn from different units. Ask what the decision was actually made about, before you multiply.
- The word "and" in a sentence is not permission to multiply, and the word "or" is not permission to add.
- "At least one of several things happens" is answered fastest through the complement. Work out the probability that none of them happen, and subtract from 1.
- Multiplying ceilings gives a ceiling, and multiplying floors gives a floor. Every "1 minus" step turns one into the other. Label each line "at most" or "at least" as you go.
- When you teach this to a trainee, put the independence check before the arithmetic every time, even in a case where it is obviously true.

EXERCISE 1 (critique). A colleague claims that the chance two randomly chosen rural residents are both covered is 0.75 times 0.75, which is 0.5625. No further detail about the two residents is given.

Say what is missing from the claim before it can be checked. Then say what changes if it turns out the two residents live in the same household.

EXERCISE 2 (teaching). A first-year resident keeps multiplying probabilities together whenever a question uses the word “and”. You have ten minutes and a whiteboard.

Teach them the actual test for independence, so they leave able to catch themselves the next time they reach for multiplication.

PRACTICE. Thirteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section’s name.

1. A coin is tossed 3 times. List every outcome, and say how many there are.

2. Event A has a probability of 0.6 and event B has a probability of 0.5, and A and B are independent. Work out P(A and B).

Separately, event C has a probability of 0.3 and event D has a probability of 0.4, and C and D are mutually exclusive. Work out P(C or D).

3. Two independent events each have a probability of 0.2. Work out the probability that at least one of them happens.

4. You are told P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.2. Check whether A and B are independent.

5. Two fair coins, each equally likely to land heads or tails, are tossed. Let A be the event that the first toss is heads, and B be the event that the second toss is heads. List the sample space, then check whether A and B are independent by counting outcomes.

6. Under section 3(2) of the National Food Security Act, 2013, the rural coverage ceiling is 0.75. Pick two rural people at random from two different, unrelated households.

Work out the probability, at most, that both are covered.

7. Now pick two rural people from the same household. Using the same 0.75 ceiling, work out the probability, at most, that both are covered. Then compare your answer with the previous problem’s.

8. Pick three rural people at random, each from a different household. Use the 0.75 coverage ceiling from section 3(2) of the National Food Security Act, 2013.

Work out the probability, at most, that at least one of the three is covered.

9. Here is a worked answer. Find the step that broke.

```
event A has a probability of 0.7, event B has a probability of 0.6, and they are independent
the probability of A or B happening is P(A) plus P(B)
0.7 plus 0.6 = 1.3
so P(A or B) is 1.3
```

10. Here is a worked answer. Find the step that broke.

```
two rural people are picked from the same household
the coverage ceiling is 0.75 for each person
P(both covered) = 0.75 times 0.75
0.75 times 0.75 = 0.5625
so at most 0.5625 of same-household pairs have both people covered
```

11. Here is a worked answer. Find the step that broke.

```
three rural people are picked, each from a different household, coverage ceiling
0.75 each
the probability that at least one is covered is found by adding the three probabil-
ities
0.75 plus 0.75 plus 0.75 = 2.25
so at most 2.25 of the three are covered, meaning it is virtually certain at least
one is
```

12. A note says: “Two rural residents are picked at random; the chance both are covered is 0.5625.”

Decide what has to be true for that figure to be correct, and say what your answer does not establish.

13. A district report says coverage is up to 75 per cent of the rural population. Pick one rural person at random. They live in a household of four. The report concludes that the chance at least one member of that household is not covered is high.

Decide what to compute, compute it, and say what your answer does not establish.

D3 · Conditional probability through the two-way table

DEFINITION. P(A given B), written P(A|B), is the count of people who are both A and B, divided by the count of people who are B. P(A|B) and P(B|A) divide the same top count by two different totals and are not generally equal.

Sensitivity is the share of people with the condition that the test catches, written P(test positive given condition present).

Specificity is the share of people without the condition that the test correctly clears, written P(test negative given condition absent).

The positive predictive value is the share of positive results that are right, written P(condition present given test positive).

Prevalence is how many people in a group have the condition, written P(condition present), with no test involved. [16, 17]

IN PLAIN TERMS. Given a positive result, how likely is the condition actually there?

A two-way table sorts the same people twice. Sort them one way down the columns, by whether they truly have the thing you care about. Sort them the other way across the rows, by what a quick measurement says. Where a row meets a column is a count of people who are both — that count is a cell.

Add up everything and you get the grand total.

A conditional probability is the same move with a chosen denominator. “Of those who test positive, how many truly have it” divides by the row of positive results. “Of those who truly have it, how many test positive” divides by the column of people who truly have it. Ask which one you have been handed before you read anything into it.

The positive predictive value is the share of positive results that are right. Its mirror is the negative predictive value, the share of negative results that are right. Prevalence — how many people in a group have the condition — needs no test at all. It is a fact about the group before anyone is measured.

The positive predictive value belongs to the room the test is used in, not to the test by itself.

ILLUSTRATION. Kiran, Harshitha and Bhargava measured 282 medical students in Mangaluru, including their body-mass index and the circumference of the upper arm. This section uses only the 131 men.

The study’s Table 3 reports a 31.3 centimetre arm cut-off for men, labelled “overweight, body-mass index 25 or more”. The arm is measured with a tape, and a man whose arm measures 31.3 centimetres or more counts as a positive result. It gives sensitivity 86 per cent and specificity 74 per cent. Table 2 gives 42 men with an index from 25 to 29.9, and 10 more at 30 or more.

```
42 plus 10 = 52
```

```
131 minus 52 = 79
```

Try to rebuild the table the label describes: 52 men against 79. Sensitivity of 86 per cent should mean some whole number of the 52 caught by the tape. Check it yourself: work out 0.86 times 52, then try the whole numbers either side of it.

```
0.86 times 52 = 44.72
44 divided by 52 = 0.8461…
45 divided by 52 = 0.8653…
```

Rounded, 44 gives 85 per cent and 45 gives 87 per cent. Neither is 86. No whole number of 52 rounds to exactly 86 per cent. Try the 79 the same way, against the specificity of 74 per cent.

```
0.74 times 79 = 58.46
58 divided by 79 = 0.7341…
59 divided by 79 = 0.7468…
```

Neither is 74 either. The label's own group sizes, 52 and 79, cannot have produced the paper's printed percentages.

This is a denominator question, the kind this whole section teaches. Somewhere else in Table 2 is a group of men that does divide back cleanly. Try the 42 men in the 25-to-29.9 band on their own, against the other 89 men in the study.

```
131 minus 42 = 89
```

```
0.86 times 42 = 36.12
36 divided by 42 = 0.8571…
```

Thirty-six of the 42 rounds to exactly 86 per cent.

```
0.74 times 89 = 65.86
66 divided by 89 = 0.7415…
```

Sixty-six of the 89 rounds to exactly 74 per cent. Both round back to the printed figure.

On that rounding this row fits the 42 and not the 52.

The rest of each row and column follow by subtraction.

```
42 minus 36 = 6
89 minus 66 = 23
36 plus 23 = 59
6 plus 66 = 72
```

	in the 25-29.9 band	not in the band	total
arm 31.3 cm or more	36	23	59
arm under 31.3 cm	6	66	72
total	42	89	131

This table is rebuilt, not printed by the paper in this form.

Take the row of men the tape flags — arm 31.3 centimetres or more — 59 of them in total.

```
36 divided by 59 = 0.6101…
```

About 61 per cent. This is the positive predictive value: of the men the arm measurement flags, the share who are actually in the 25-to-29.9 band.

The 89 include the 10 men at 30 or more, so a flagged man at 30 or more counts here as a wrong result. For the question ‘is his index 25 or more?’, 61 per cent is the least the share could be.

Now reverse the question. Take the column of men in the 25-to-29.9 band — 42 of them.

```
36 divided by 42 = 0.8571…
```

About 86 per cent. This second ratio is the sensitivity, and it is the number the paper itself reports. P(in the band given arm 31.3 or more) is not P(arm 31.3 or more given in the band).

Specificity is the share of the 89 men outside the band whom the measurement correctly clears.

```
66 divided by 89 = 0.7415…
```

Prevalence is how many of the 131 men are in the band in the first place, no measurement involved. Table 2 already gives it directly: 42 of 131.

```
42 divided by 131 = 0.3206…
```

About 32 per cent, among these 131 men.

Keep the same cut-off, the same 86 per cent sensitivity and the same 74 per cent specificity. Apply them to a stated group instead: 1,000 young men in which one in five is in the band.

```
0.20 times 1,000 = 200
1,000 minus 200 = 800
```

```
0.86 times 200 = 172
0.74 times 800 = 592
```

```
200 minus 172 = 28
800 minus 592 = 208
172 plus 208 = 380
```

	in the band	not in the band	total
arm 31.3 cm or more	172	208	380
arm under 31.3 cm	28	592	620
total	200	800	1,000

```
172 divided by 380 = 0.4526…
```

About 45 per cent, against about 61 per cent among the medical students themselves.

Figures quoted in this illustration are from [18].

WHERE THIS PICTURE BREAKS. The 36-23-6-66 table is rebuilt, not measured. It was built by applying two rounded percentages to two counts. Of the two group sizes tried, only one rounds back to both 86 and 74. Wherever you see a sensitivity and a specificity with no counts behind them, run the same check before you trust which group they describe.

The positive predictive value of 61 per cent describes these 131 medical students and nobody else. Sensitivity and specificity stayed the same. Never quote a positive predictive value without saying whose prevalence produced it.

Whether the measurement is worth taking in a clinic is a separate question.

MUST KNOW POINTS FOR YOU.

- P(A given B) is not P(B given A). Sensitivity and the positive predictive value are built from the same top count. They divide it by different totals.
- Run the same test where the condition is common, and most positive results are right. Run it where the condition is rare, and most positive results are wrong.
- Before dividing anything, say out loud which row or column you are dividing by.
- Carrying sensitivity and specificity forward to a new group is itself an assumption. Say so whenever you do it.
- Divide back. Try every whole number of the stated group. A row's own label is not proof that it matches the count behind it.
- A headline that reports a test as some per cent "accurate" tells you nothing on its own. It names none of the sensitivity, the specificity or the positive predictive value. Ask which one was meant, then ask for the prevalence the third one needs.
- When two published figures for the same measurement disagree, check whether either one conflicts with something else printed alongside it. A shared total, or rows that must add up, can settle which figure the paper actually computed from. Do this before you assume the more prominent figure is correct.

EXERCISE 1 (teaching). A health secretary has one page and wants an answer first. The briefing says an arm-measurement test for a weight-related condition is "86 per cent accurate". Write the page.

EXERCISE 2 (critique). The paper defines higher percent body fat (PBF) separately for men and women. It measures body fat with a machine that passes a small current through the body. Its abstract states the cut-off in one place as "(men: ≥20%; women: ≥30%)". Its methods section, Table 2 and Table 3 all use "≥28" per cent for women instead.

Say how you would check which of the two women's figures, 30 or 28 per cent, the paper's own numbers are actually built on.

PRACTICE. Thirteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Of 100 people, 20 truly have a condition and 80 do not. A test is positive for 15 of the 20 who have it, and positive for 10 of the 80 who do not.

Build the two-way table, with row and column totals.

2. Using the table you built in the previous problem, work out P(test positive given condition present) and P(condition present given test positive). Say which total each one divides by.

3. Using the same invented table, work out P(test negative given condition absent).

4. Still using the invented table, name which cell and which total each of these divides by: sensitivity, specificity, and the positive predictive value. Give the value of each.

5. Kiran, Harshitha and Bhargava's 2022 study found that 52 of 131 men had a body-mass index of 25 or more. Work out this share as a percentage, and the percentage of men under 25.

6. Table 3 also reports a women's arm-circumference cut-off. Sensitivity is 88 per cent and specificity 83 per cent, labelled "body-mass index 25 or more" the same way the men's row was. Table 2 gives 41 women in the 25-to-29.9 band and 5 more at 30 or more. Together that is 46 women carrying the label, out of 151 women in the study.

Try to reproduce 88 per cent from whole numbers of 46, and 83 per cent from whole numbers of 105 (151 minus 46). Say what the check shows.

7. Table 2's own women's row gives 41 women in the 25-to-29.9 band on its own, and 151 minus 41 women outside it. Use these two totals with the same 88 per cent sensitivity and 83 per cent specificity, and work out the positive predictive value.

8. Using the men's rebuilt table — 36, 6, 23 and 66 — work out the negative predictive value. Of the men the arm measurement clears, what share truly are outside the 25-to-29.9 band?

9. Here is a worked answer. Find the step that broke.

```
the abstract says 113 had abdominal obesity — the paper's label for a waist of 90
centimetres or more in men and 80 or more in women
113 divided by 282 = 0.4007…
0.4007 rounds to 40.1 per cent, close enough to the abstract's own 38.7 per cent
so 113 is confirmed as the paper's count for abdominal obesity
```

The results text and Table 2 give the same group as 109.

10. Here is a worked answer. Find the step that broke.

```
the abstract reports 186 (66%) had higher percent body fat (PBF), out of 282
Table 2 reports the same measure as 185 (65.6%)
185 divided by 282 = 0.6560…, which rounds to 66 per cent
186 divided by 282 = 0.6596…, which also rounds to 66 per cent
both numbers are consistent with their own printed percentages, so nothing in the
paper
can decide which count is right
```

Table 2 also gives 71 men and 114 women in the high row, and 97 students in the normal-range row.

11. Here is a worked answer. Find the step that broke.

```
Table 3 gives its 31.2 cm arm cut-off's sensitivity as 85 per cent for spotting the
41 men
whose waist is 90 centimetres or more (Table 2)
0.85 times 41 = 34.85
so 34 of the 41 men were caught by the tape
34 divided by 41 rounds to 85 per cent, matching the paper
```

12. A district officer says: "The tape correctly identified men in the 25-to-29.9 body-mass-index band most of the time among these medical students. It will be right about as often wherever I use it."

Suppose one in three men in the district were in the band. Decide what to compute, compute it, and say what your answer does not establish.

13. A note says: "Neck circumference correctly identifies men with a body-mass index of 25 or more, most of the time. It should replace body-mass index screening everywhere."

Table 3 gives a neck-circumference cut-off for men: sensitivity 81 per cent, specificity 71 per cent.

Decide what to compute, compute it, and say what the claim does not show.

D4 · Variation: what differs and by how much

DEFINITION. A distribution is the set of values a quantity takes, across the people, occasions or measurements counted, together with how often each value turns up.

A frequency table lists each value, or each band of values, against the count of times it occurred. Dividing a count by the total number counted gives the relative frequency.

The range of a distribution is its largest value minus its smallest.

The shape of a distribution describes where its values sit relative to each other. They can be bunched together near the middle, or spread evenly across it. Or they can thin out slowly on one side and quickly on the other, which is a long tail.

A spread of measured values comes from one or more of three sources. It can be a real difference between the people or things measured. It can be a real difference within one person or thing from one occasion to another. Or it can be a difference between repeated measurements of the same person or thing at the same moment. [19, 20, 21]

IN PLAIN TERMS. Write down every value you get. That list, together with how often each value shows up, is a distribution.

One column lists the values, or bands of values. The other column counts how many times each one turned up.

Divide a count by the total number of values in the table and you have the relative frequency. It is the proportion from the section on ratios, rates and proportions, under a longer name.

The range is the largest value minus the smallest. It uses two numbers out of the whole table and says nothing about the rest.

Many distributions bunch up somewhere in the middle. Others thin out slowly on one side and quickly on the other. The side that thins out slowly is called a long tail.

Count how many values sit above some line and how many sit below it. That tells you about shape only when the line sits in the true middle, with equal room on each side. Watch that fail on eleven bare numbers, as symmetric a set as exists: five below the middle number, five above it, one dead centre.

```
1   2   3   4   5   6   7   8   9   10   11
```

band	count
1 to 3	3
4 to 8	5
9 to 11	3

Three below the middle band, three above it.

band	count
1	1
2 to 8	7
9 to 11	3

One below the new middle band, three above it. Nothing about the eleven numbers changed. Only where the band edges sat changed, and the count on each side changed with it. A set that is perfectly symmetric can be made to look like it has a long tail just by moving the bands. Count how many values sit above a band and how many sit below it. That tells you about shape only once you know the band is centred and equally wide on both sides. Otherwise it tells you about the band.

A set of numbers can look spread out for three completely different reasons, and telling them apart is the main use of this section.

The first is a real difference between people, or between things. Two students really do have different heights.

The second is a real difference within one person or thing, from one time to another. The same student, weighed in the morning and again in the evening, gives two different numbers.

The third has nothing to do with the person at all. The scale, the tape, or the hand holding it produced the difference, not the person. An error of this kind can also hide completely. Two readings agreeing is not proof that either one is right.

Before you explain a spread, or act on it, decide which one you are looking at.

ILLUSTRATION. Open the paper by Kiran, Harshitha and Bhargava, published in Heliyon in 2022, on arm and neck measurements in 282 medical students in Mangaluru. In its abstract, the paper defines overweight by body-mass index, written BMI.

> *to screen overweight (BMI ≥25 kg/m2)*

The sign ≥ means “at or above”.

> *Of the 282 participants, 83 (29.4%) were overweight*

Run the check a frequency table always deserves: divide the count by the total.

```
83 divided by 282 = 0.294326241…
0.294326241 times 100 = 29.4326241
```

That rounds to the 29.4 per cent printed, so the division check passes. Does 83 actually match the definition just quoted?

Table 2 of the same paper sorts all 282 students into four bands of body-mass index.

band of body-mass index (kg/m^2)	count	relative frequency
under 18.5	24	8.5 per cent
18.5 to 24.9	160	56.7 per cent
25 to 29.9	83	29.4 per cent
30 or more	15	5.3 per cent

Now look at which band the reported 83 actually comes from. It is the count beside the band “25 to 29.9”, not beside “25 or more”. The paper’s own bands stop at 29.9 and start a new one at 30.

```
83 plus 15 = 98
```

Ninety-eight students, not 83, have a body-mass index of 25 or more.

```
98 divided by 282 = 0.347517730…
0.347517730 times 100 = 34.7517730
```

That rounds to 34.8 per cent, not the 29.4 per cent printed beside the reported 83. The paper's own definition and its own headline count describe two different groups of students. This is not two counts disagreeing with each other. It is a label attached to the wrong slice of its own table. Dividing the count by the total could not catch it, because 83 out of 282 really is 29.4 per cent. Only matching the label to the band it covers did.

Table 2 also tempts a claim about shape. Ninety-eight students sit in the two bands above the middle one, against 24 in the band below it. Resist calling that a tail. The section above showed why a comparison like this proves nothing on its own.

These four bands are the World Health Organization's (WHO), which the paper adopted. They were not drawn around these students. They are not equally wide: 18.5 to 24.9 spans about 6.5 units, 25 to 29.9 about 5. The two outer bands have no outer edge.

Notice what the table cannot give you, either way. It cannot give you the range of body-mass index among these 282 students. The range needs the single largest and the single smallest value in the whole group. Once every value has been sorted into one of four bands, those two individual numbers are gone. The table can tell you which band held them and nothing about where inside it they sat.

Two students landing in different bands need not be a real difference between people. Two students either side of 25.0 can be separated by the measurement alone.

Table 2 gives you only one reading per student. It cannot tell you which of the three you would be looking at if you went back and measured again.

Figures quoted in this illustration are from [18].

WHERE THIS PICTURE BREAKS. The check above only catches a label attached to the wrong band.

A frequency table built from bands, like this one, can never give you back the range. It cannot give you the exact position of a value inside its own band either. Both are lost the moment individual numbers are grouped.

MUST KNOW POINTS FOR YOU.

- Two distributions can share exactly the same range and look nothing alike. The range cannot tell the two apart, because it only ever looks at the two most extreme values.
- Before you trust a frequency table's percentages, divide each count by the printed total and check you get back the percentage beside it. Then match each label to the band it covers. A count can pass the first check and still sit under the wrong label.
- A frequency table built from bands has already thrown away the individual values inside each band. Ask for the raw values when any of those is what you actually need.
- Before you explain why two readings differ, decide which of three sources produced it. Call a scale's own error a difference between people, and you invent a health difference that is not there. Call a real change within a person a measurement glitch, and you miss it.
- An instrument that is wrong by the same amount every time gives two matching readings, and both are wrong. Two readings agreeing is not, by itself, a reason to trust either one.

- A single figure, such as the share of a group above one band's edge, says nothing about the rest of the group. The edge itself may not sit where you assume it does. Do not let one band, or one comparison between two bands, stand in for a whole distribution.

EXERCISE 1 (critique). A colleague sends you two facts about the same set of 500 patients' waiting times at a clinic.

> *The shortest wait was 2 minutes and the longest 340, a range of 338 minutes. The average patient waits about 45 minutes.*

They conclude: "So most patients wait somewhere between 2 and 340 minutes." Say what is wrong with that conclusion. Say what you would ask for instead.

EXERCISE 2 (teaching). A first-year resident keeps saying "the readings don't match" whenever two measurements on the same patient disagree, and treats every mismatch the same way.

You have ten minutes and a whiteboard. Teach them the three sources of difference, so they leave able to ask the right next question about a real disagreement.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A count of 50 events is sorted into four categories with these counts.

```
10    25    10    5
```

Work out the relative frequency of each category, and check that the four add to 1.

2. Work out the range of this list of six numbers.

```
3    17    9    42    8    15
```

3. Two sets of six numbers are given below.

set A	10	11	12	13	14	50
set B	10	20	30	40	45	50

Work out the range of each set. Say what you notice, and say what it does not tell you about the two sets.

4. Three pairs of readings are listed below, each under stated conditions. For each pair, say which source of difference mainly produced the gap. The three are: between people, within a person, and between measurements.

1. Two different patients are weighed once each, on the same scale. The readings are 58 kilograms and 71 kilograms.
2. The same patient is weighed on Monday morning and again on Friday evening. The readings are 64.0 kilograms and 65.2 kilograms.
3. The same patient is weighed twice on the same scale, one minute apart. Nothing about them changes in between. The readings are 70.4 kilograms and 70.6 kilograms.

5. Table 2 of Kiran et al. (2022) sorts 282 medical students into four bands of body-mass index. The counts are 24 under 18.5, 160 from 18.5 to 24.9, 83 from 25 to 29.9, and 15 at 30 or more.

Work out the relative frequency of the band at 30 or more, and give it as a percentage.

6. The same table, split by sex, gives the men's counts across the four bands as 5, 74, 42 and 10. There are 131 men in the study.

Check that the four counts add up to 131. Then work out the relative frequency, among the men only, of the band at 30 or more.

7. The same paper, describing what it counts as its own strength, says this.

> *The strength of the study was single observer measurements with minimum inter-observer variation.*

Inter-observer means between different people taking the measurement.

Say which of the three sources of difference having one observer take every measurement reduces, and which two it leaves untouched.

8. Among the 151 women in the same study, 41 have a body-mass index from 25 to 29.9, and 5 more are at 30 or more.

Work out what fraction of the 151 women have a body-mass index of 25 or more, as a percentage.

9. Here is a worked answer to this question, using Table 2 of Kiran et al. (2022): what share of the men are in the band from 25 to 29.9?

```
42 divided by 282 = 0.148936170
0.148936170 times 100 = 14.8936170
so about 14.9 per cent of the men are in the band from 25 to 29.9
```

Find the step that broke.

10. A colleague looks at Table 2 of Kiran et al. (2022) and says this.

"The band under 18.5 holds 8.5 per cent of the 282 students, and the band at 30 or more holds 5.3 per cent. Those two are close in size, so the distribution of body-mass index in this sample must be roughly symmetric."

Find the flaw in that reasoning.

11. A note about the same study says this.

"In our sample of 282 students, body-mass index ranges from underweight to 30 or more. So weight varies enormously in this group."

Underweight is the paper's name for its band under 18.5.

Decide what to compute, compute it, and say what your answer does not establish.

D5 · Average and spread, and why the average is not the person

DEFINITION. This section turns a distribution into two single numbers: a typical value, and how spread out the rest are around it.

The mean of a set of values is their sum divided by how many there are. The median is the middle value of the same set once it is ordered from smallest to largest; where the count is even, it is the mean of the two middle values.

The first quartile is the median of the lower half of the ordered values, and the third quartile is the median of the upper half. The second quartile is the median of the whole set. When the count itself is odd, leave the middle value out of both halves before taking their medians. The interquartile range is the third quartile minus the first, and it describes the spread of the middle half of the values, ignoring anything beyond it.

The population is the whole group a question is about. The sample is the part of that group that was actually measured.

The deviation of a value from the mean is that value minus the mean. The variance is worked out from the squared deviations: add them, then divide by the count for a whole population, or by one less than the count for a sample. The standard deviation is the square root of the variance, expressed in the same unit as the original values, because taking a square root undoes the squaring. To say how far one value sits from the mean in standard deviations, divide its deviation by the standard deviation.

The mean of a set of values says nothing on its own about where any one member of the set actually sits. [22, 23, 24, 21, 25, 26, 27, 21]

IN PLAIN TERMS. Take six numbers: 5, 8, 12, 15, 20, 40. Add them and divide by how many there are. That is the mean, which everyday speech calls the average.

```
5 plus 8 plus 12 plus 15 plus 20 plus 40 = 100
100 divided by 6 = 16.6666666667…
```

The mean is about 16.7.

For the median, put the same six numbers in order, which they already are, and find the middle. With six values there is no single middle one, so take the two in the middle and average them.

```
12 plus 15 = 27
27 divided by 2 = 13.5
```

The median is 13.5, noticeably below the mean of 16.7. One number in this set, the 40, sits a long way above the rest. It drags the mean up with it and leaves the median almost where it would have been without it.

Take eight ordered numbers: 2, 4, 4, 6, 8, 8, 10, 12. The middle value, the second quartile, is the average of the two central values.

```
6 plus 8 = 14
14 divided by 2 = 7
```

Now split the set into its lower half and its upper half, four numbers each, and find the middle of each half the same way. The lower half is 2, 4, 4, 6.

```
4 plus 4 = 8
8 divided by 2 = 4
```

That gives the first quartile, 4. Now the upper half, 8, 8, 10, 12.

```
8 plus 10 = 18
18 divided by 2 = 9
```

That gives the third quartile, 9. The interquartile range is the third quartile minus the first.

```
9 minus 4 = 5
```

It says the middle half of these eight values is spread across five units.

Now the standard deviation, step by step, on six more bare numbers: 5, 7, 3, 7, 10, 4.

First, the mean.

```
5 plus 7 plus 3 plus 7 plus 10 plus 4 = 36
36 divided by 6 = 6
```

The mean is 6. Now the deviation of every value: how far it sits from that mean, kept with its sign.

```
5 minus 6 = -1
7 minus 6 = 1
3 minus 6 = -3
7 minus 6 = 1
10 minus 6 = 4
4 minus 6 = -2
```

Square every deviation next, which means multiplying each one by itself. A negative number times a negative number gives a positive number.

```
-1 times -1 = 1
1 times 1 = 1
-3 times -3 = 9
1 times 1 = 1
4 times 4 = 16
-2 times -2 = 4
```

Add the squares.

```
1 plus 1 = 2
2 plus 9 = 11
11 plus 1 = 12
12 plus 16 = 28
28 plus 4 = 32
```

Divide that by one less than the count of values, which is five here, not six. That division is the variance.

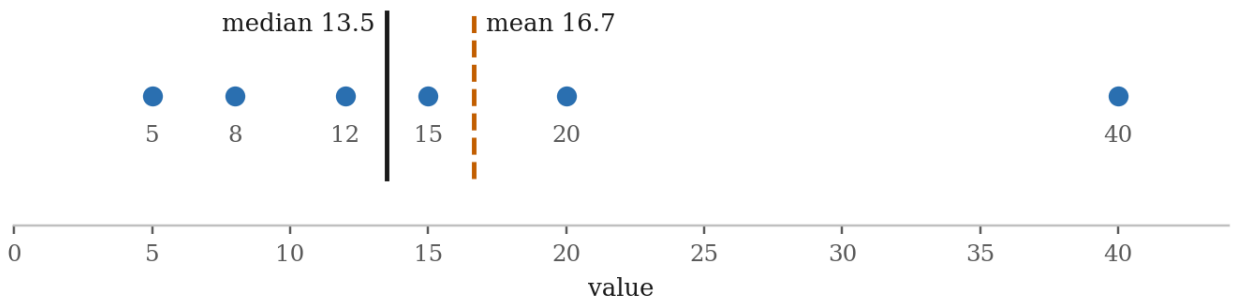
```
32 divided by 5 = 6.4
```

The variance is 6.4, and it is in the wrong unit. Squaring every deviation squared the unit too, so a variance worked out from kilograms is really in kilograms squared, which nobody can picture. Take the square root and the unit comes back to the one you started in. That last step is the standard deviation.

```
square root of 6.4 = 2.529822128
```

The standard deviation of these six numbers is about 2.53.

Go back to the division by five rather than six. Most papers do the same thing: a sample's variance is divided by one less than the count. Distances from the sample's own mean come out a little small on average, because that mean sits in the middle of these particular numbers. Dividing by one less than the count makes the variance come out right on average. The textbook this book follows puts it briefly: dividing by one less 'gives a better estimate of the population variance'.



The six numbers from this section on one line. The median, 13.5, sits in the middle of the crowd. The mean, 16.7, has been pulled towards the single value of 40. Without it, both would be 12.

ILLUSTRATION. Open Table 1 of the paper used in the section on variation: Kiran, Harshitha and Bhargava, 282 medical students in Mangaluru. It gives weight, height, body-mass index and more, each as a mean with its standard deviation beside a median with its first and third quartiles. Papers write the standard deviation (SD) by its two initials. Read the first two rows for the 131 men.

indicator	mean (SD)	median (first and third quartiles)
weight (kg)	72.0 (13.1)	70.9 (167.7, 177.8)
height (cm)	172.2 (6.8)	173 (62.6, 80.9)

Stop on the weight row's two quartiles, and check them yourself. The median has to sit between the two quartiles printed beside it: the first quartile below it, the third above it. Here the median is 70.9, and the two numbers printed beside it are 167.7 and 177.8. 70.9 sits below both of them, not between them.

Try the same check on the height row underneath it. The median there is 173, and the two numbers printed beside it are 62.6 and 80.9. It sits above both of them, not between them either.

Both rows fail, in opposite directions. Each passes against the other row's numbers: 70.9 sits between 62.6 and 80.9, and 173 between 167.7 and 177.8. The numbers fit if the two rows' quartiles were printed in each other's places.

Men's mean body-mass index is 24.2 kilograms per square metre, with a standard deviation of 3.9.

Forty-two of the men are in the band from 25 to 29.9. Ten more are at 30 or more.

```
42 plus 10 = 52
```

Fifty-two of the 131 men have a body-mass index of 25 or more.

```
52 divided by 131 = 0.396946565…
0.396946565 times 100 = 39.6946565
```

That rounds to about 39.7 per cent, close to two in every five. The mean of the group, 24.2, sits under the line at 25. Close to two in five of the individual men in that same group sit at it or above it. The mean describes the group. It was never a description of any one of the 131 men. Deciding whether one particular man is above or below 25 needs his own reading, not the group's average.

Men's mean body fat is 22.1 per cent and their median is 20.3 per cent. The mean sits above the median.

```
22.1 minus 20.3 = 1.8
```

By the median's own definition, at least half of the 131 men have a body fat percentage at or below 20.3. That is also below the mean of 22.1. The men above the median must reach far enough up to pull the mean past it. The quartiles printed beside the median show it: 16.7 and 28.8.

```
20.3 minus 16.7 = 3.6
28.8 minus 20.3 = 8.5
```

The first quartile sits 3.6 below the median, and the third sits 8.5 above it. The quarter of men just above the median stretches more than twice as far as the quarter just below it. That is a longer stretch running upward than downward. It is exactly the shape the section on variation described in words.

Figures quoted in this illustration are from [18].

WHERE THIS PICTURE BREAKS. Checking a median against its own two quartiles catches a pair of quartiles that sits nowhere near the median.

The mean sitting below 25 while many men sit above it is a fact about this one group. It is 131 men, measured this way, in this college, in 2019. It says nothing about any other group. It says nothing about any one of these men beyond what his own reading gives you.

The standard deviation and the interquartile range answer different questions about the same spread. Neither can be worked out from the other.

MUST KNOW POINTS FOR YOU.

- A long tail on one side of a distribution pulls the mean toward it. Reading this backwards, "the mean is higher, so most values are higher", is an error.
- Compute the median alongside the mean whenever a handful of extreme values might be present. Compare the two before you quote either one on its own. A gap between them is itself a finding, not a rounding difference to ignore.
- A mean, on its own, says nothing about any one person in the group it was computed from.
- Divide by one less than the count whenever you compute a standard deviation from a sample.
- A mean sitting below a cut-off does not mean most of a group is below it.
- Check a printed median against the two quartiles printed beside it before you trust any of the three.

- In a lopsided distribution, the standard deviation hides that the two sides spread out differently. Where the mean and median disagree by much, look at the quartiles instead of trusting a single standard deviation to describe both sides.

EXERCISE 1 (critique). A report states this. “Average daily step count in our clinic’s patients is 6,200, comfortably above the report’s own target of 5,000 steps. So inactivity is not a major concern here.” (The figures are made up for this exercise.)

The same report’s raw data shows a median of 4,100 steps. Say what the gap between the two numbers tells you. Say what you would ask for before agreeing with the conclusion.

EXERCISE 2 (teaching). A first-year resident reads a paper’s abstract, sees a mean below a clinical cut-off, and concludes “so this group is fine”. You have ten minutes and a whiteboard.

Teach them why a mean sitting below a cut-off does not mean the group is.

PRACTICE. Sixteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section’s name.

1. Find the mean and the median of these seven numbers.

```
4    9    2    15    7    9    3
```

2. Find the mean and the median of these six numbers.

```
6    9    11    14    18    50
```

3. These eight numbers are already ordered.

```
3    5    5    9    11    11    15    19
```

Find the first quartile, the second quartile, the third quartile, and the interquartile range.

4. These seven numbers are already ordered.

```
2    5    7    9    12    15    20
```

Find the first quartile, the second quartile, the third quartile, and the interquartile range.

5. Work out the sample standard deviation of these six numbers, step by step.

```
8    6    10    4    12    8
```

6. Give the sample variance and the sample standard deviation of a sample whose deviations from its own mean are the five values below.

```
-2    4    -1    3    -4
```

7. Table 1 of Kiran et al. (2022) gives men’s mean body-mass index as 24.2 kg/m^2 and their median as 23.8 kg/m^2.

Work out the gap between the two, and say which side of the distribution its tail is likely on.

8. Men’s body-mass index in the same table has a median of 23.8 kg/m^2, with the two quartiles printed beside it as 21.2 and 26.8.

Work out the interquartile range as a single number, and check whether the median sits between the two quartiles it is printed beside.

9. The same table prints men's body-mass index as a median of 23.8 kg/m^2, with a first quartile of 21.2 and a third quartile of 26.8.

Work out how far each quartile sits from the median. Say what the two distances suggest about which side of the distribution has the longer tail.

10. Suppose a man in this group had a body-mass index of 30 kg/m^2. The men's mean is 24.2 with a standard deviation of 3.9.

Work out how many standard deviations above the mean his reading would sit.

11. Table 1 of Kiran et al. (2022) gives the 151 women's body fat as a mean of 33.8 per cent. The median is 34.0 per cent, with the two quartiles printed beside it as 28.1 and 39.2.

Work out the interquartile range as a single number. Check whether the median sits between the two quartiles. Then say which way the mean sitting below the median points.

12. Five patients were picked from a clinic's much larger patient list. Here is someone's worked sample standard deviation of their ages, in years. Find the step that broke.

```
the numbers are 12, 15, 9, 21, 18
mean = (12 plus 15 plus 9 plus 21 plus 18) divided by 5 = 15
deviations: -3, 0, -6, 6, 3
squares: 9, 0, 36, 36, 9
sum of squares = 90
variance = 90 divided by 5 = 18
standard deviation = square root of 18 = 4.242640687
```

13. A teammate is about to quote this line from Table 1 of Kiran et al. (2022) in a talk.

"Median weight among the women was 57.4 kilograms, with quartiles of 155.0 and 163.0."

Find what is wrong with the line. Say what you would check before quoting any pair of quartiles taken from a table you have not read closely.

14. Somebody makes this claim from the body fat figures in Table 1 of Kiran et al. (2022).

"The mean body fat among the men is 22.1 per cent and the median is 20.3 per cent. Since the mean is higher, most men must have a body fat percentage above 22.1."

Find the error in the reasoning.

15. A note about the study says this. "Women's average body-mass index was 23.0, under the cut-off of 25, so weight was not a concern in this group of students."

Decide what to compute, compute it, and say what your answer does not establish.

16. A colleague says this about the same study. "Body fat in this group of men averages 22 per cent, so most men have a body fat percentage somewhere between 20 and 24."

Decide what to compute, compute it, and say what your answer does not establish.

D6 · Sampling: how a part can tell you about a whole

DEFINITION. A parameter is a number that describes the population, if every member of it could be measured. A statistic is the same kind of number, computed instead from a sample.

A random sample is picked at random: every member of the population has exactly the same chance of being the one picked, and an actual chance process decides it, not the person picking. A convenience sample is built the other way, out of whoever happens to be available, and the chance that any one member of the population ends up in it is neither equal nor known.

Draw a second random sample of the same size from the same population and its statistic will not match the first sample's exactly. This is sampling variation.

Take the average of a random sample. Over many samples of the same size, that average has its own spread, meaning its standard deviation as the section on average and spread taught it. The square-root law states that this spread equals the spread of the individual values in the population, divided by the square root of the sample size. Multiplying the sample size by four divides that spread by two. This spread does not depend on how big the population is, only on how big the sample is. Papers name this spread the standard error of the mean.

The square-root law describes how far a random sample's average is likely to sit from the average of the population it was actually drawn from. It says nothing about whether that population is the one the sample was meant to stand for. Enlarging the sample does not close that gap. [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38]

IN PLAIN TERMS. Call the whole group a question is about the population. Call the part of that group that was actually measured the sample.

A number that describes the whole population exactly is called a parameter. You could only get it by measuring everyone, which is usually too slow, too costly or simply impossible. A number worked out the same way from your sample is called a statistic. Since a sample is not the whole population, a statistic is not the parameter. It is an estimate of it, a number made from part of the group and used to stand in for the whole.

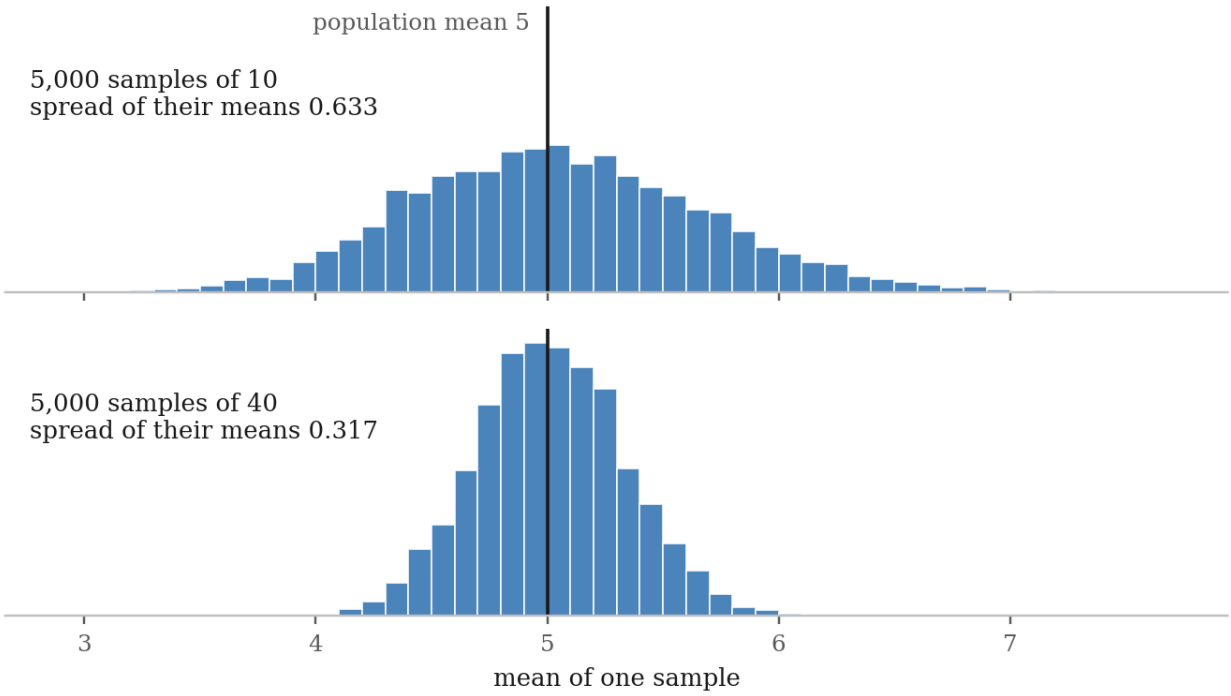
A random sample is picked at random: every member of the population has exactly the same chance of being the one picked. A convenience sample is whoever happens to be available: volunteers, people who answered an advertisement, the first hundred people through a door. Nothing forces a convenience sample to look like the population it gets used to describe.

Draw one random sample of a hundred people and work out its average. Draw a second random sample of a hundred different people, from the same population, the same way, and work out its average too. The two averages will not be the same number. This bouncing around from sample to sample is called sampling variation, and every sample has it.

Take a sample four times the size, and the bounce in its average shrinks to half of what it was. This is the square-root law. A paper that reports this spread usually calls it the standard error of the mean.

Suppose a convenience sample of volunteers is not representative of the group it is meant to describe. No amount of enlarging it brings it any closer.

Sample size protects you against sampling variation. It buys you nothing against a sample that was never random to begin with.



Each bar counts how many of 5,000 sample means landed there. Top: samples of 10. Bottom: samples of 40, four times bigger. Both crowd around the population mean of 5, and the bottom crowd is half as wide, 0.317 against 0.633, which is the square-root law happening rather than being stated. The draw was made by computer from the eight numbers 2, 4, 4, 4, 5, 5, 7 and 9, each put back after it was drawn.

ILLUSTRATION. Kiran and colleagues did the study you have already met three times.

Before recruiting anyone, they set their target using a figure from the National Family Health Survey, NFHS-4, run in 2015-16. The study itself ran in July to September 2019. The paper gives the prevalence of overweight as 19 per cent in men and 21 per cent in women. It reports these as NFHS-4's figures for India, without saying which ages or which people they cover. The authors rounded them to a planning figure of 20 per cent to size their study. NFHS-4's figures are themselves a survey result that reaches this book second-hand, through the paper, not a fixed truth.

They measured a sample: 282 medical students at one college in Mangaluru who came forward after the study was announced on campus. Nobody drew these 282 by chance.

They chose to come forward, from the one college that was convenient to the researchers.

Table 2 puts 83 of the 282 students at a body-mass index of 25 to 29.9, and another 15 at 30 or more. Add the two bands together.

```
83 plus 15 = 98
98 divided by 282 = 0.3475177305
```

Ninety-eight of the 282 students, about 34.8 per cent, had a body-mass index of 25 or more. The gap between that and the 20 per cent planning figure is about 14.8 percentage points.

Take the population made of these eight numbers: 2, 4, 4, 4, 5, 5, 7, 9. Work out its mean the way the section on average and spread taught.

```
2 plus 4 plus 4 plus 4 plus 5 plus 5 plus 7 plus 9 = 40
40 divided by 8 = 5
```

Now its spread, the same way: each value's distance from 5, squared, then averaged, then square-rooted.

value	distance from 5	squared
2	-3	9
4	-1	1
4	-1	1
4	-1	1
5	0	0
5	0	0
7	2	4
9	4	16

```
9 plus 1 plus 1 plus 1 plus 0 plus 0 plus 4 plus 16 = 32
32 divided by 8 = 4
square root of 4 = 2
```

This is the whole population, not a sample from it, so the division is by 8, the full count, not by 7. This population's spread is 2.

Draw a sample of four numbers from it, one at a time, putting each one back before the next draw. Here are three such samples.

sample	numbers	sum	mean
first sample of four	5, 4, 7, 2	18	4.5
second sample of four	4, 4, 5, 2	15	3.75
third sample of four	4, 2, 4, 7	17	4.25

That bouncing around is sampling variation. The square-root law says how big it typically is: divide the population's spread, 2, by the square root of the sample size, which is 2.

```
2 divided by 2 = 1
```

For samples of four, the law predicts a spread of 1 around the true mean of 5.

Now return to the medical students. Suppose the 282 had instead been drawn at random from young adults, the group the paper set out to study. A share is itself an average: count each yes as 1 and each no as 0. So the square-root law would then say how far 34.8 per cent was likely to sit from that group's own figure.

They were not drawn at random. They were students at one college who chose to come forward.

Nothing in this arithmetic can say which of several things produced the 14.8-point gap. It could be that these are medical students at one college. It could be the years between when NFHS-4 was measured and when the study ran. It could simply be who chose to volunteer. Size cannot settle that question.

Figures quoted in this illustration are from [18, 39].

WHERE THIS PICTURE BREAKS. A real population's spread is usually not known in advance. Papers estimate it from the same sample they are using. That estimate adds its own extra uncertainty, which this demonstration does not show.

Real surveys never ask the same person twice, so they sample without replacement. That matters only when the sample is a large share of a small population, such as one village. There the scatter is a little less than the law predicts.

MUST KNOW POINTS FOR YOU.

- A bigger sample is not a repair for a biased one. Enlarging a convenience sample tightens its estimate of whatever population it actually came from.
- Before trusting a report's sample size, ask how the people in it were found. A small random sample can outperform a huge convenience sample.
- Kiran and colleagues' 282 medical students gave 34.8 per cent with a body-mass index of 25 or more, worked out from Table 2. The planning figure the authors rounded from NFHS-4 was 20 per cent. Size alone cannot tell you what made the two figures differ.
- The square-root law describes only how a random sample's answer scatters around the population it was actually drawn from. Quoting it does not rescue a sample that was never random.
- Halving the scatter of a sample's average costs four times as many people, every time.
- Teach a trainee to name the intended population before looking at a study's sample size. If they cannot say who a convenience sample leaves out, they have not learned the part of this that matters.

EXERCISE 1 (critique). A hospital's social media post reads: "We surveyed 4,000 followers and found 60 per cent support a new clinic timing. With a sample this large, the result is extremely reliable."

Say what the sample size claim gets right and what it leaves out.

EXERCISE 2 (teaching). A first-year resident says a colleague's finding "must be right because they had a huge sample size." You have ten minutes and a whiteboard.

Teach them the difference between a sample being big and a sample being random.

PRACTICE. Eleven problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A population has a spread (standard deviation) of 10. Work out the spread of the average of a sample of size 4 from it.

2. A population has a spread of 8. Work out the spread of the average of a sample of size 16. Then work out the spread of the average of a sample of size 64, and say what happened to it.

3. A population has a spread of 15. What sample size gives a sample average with a spread of 3?

4. Kiran and colleagues report a standard deviation of 3.9 kg/m² for body-mass index among the 131 men in their sample (Table 1). Suppose a fresh sample of 25 men is drawn from a population with that same spread. Work out the spread of that sample's average body-mass index.

5. Kiran and colleagues report a body-mass index spread of 3.9 kg/m² for the men in their own sample. Suppose a random sample of 131 men is drawn from a population with that same spread. Work out the spread of the sample average. Then say what the spread would be for a fresh sample of 524 men from the same population.

6. Suppose a population's spread for body-mass index is 3.9 kg/m². What sample size would bring the spread of the sample average down to 0.1 kg/m²?

7. A colleague writes in a report: "Kiran and colleagues' men have a mean body-mass index of 24.2 kg/m². The standard deviation is 3.9. So the true mean for the population lies between 20.3 and 28.1."

Find the step that broke.

8. Here is a worked answer. Find the step that broke.

```
a population has a spread of 50
a sample of 100 has a spread of 50 divided by square root of 100, which is 5
doubling the sample to 200 should halve the spread
so the new spread is 5 divided by 2, which is 2.5
```

9. Here is a note passed round a department.

```
Kiran and colleagues found 98 of 282 medical students, 34.8%, at a body-mass index
of 25 or more
NFHS-4 gives 20% for young adults nationally
a sample four times the size, 1,128, would have half the sampling spread by the
square-root law
so a bigger sample would bring the 34.8% much closer to the true national 20%
```

Find the step that broke.

10. A colleague sends you this line for a newsletter.

> *Kiran and colleagues found that medical students are far more overweight than India's young adults generally.*

Decide what to compute, compute it, and say what your answer does not establish.

11. A colleague says: "A random sample of 2,000 people can't possibly tell us anything reliable about a whole country. You would need several million to get a trustworthy number."

Decide what to compute, compute it, and say what your answer does not establish.

D7 · Random error and systematic error as different things

DEFINITION. A measurement's error is the value it returns minus the value that is actually true, or, since the true value is usually unknown, minus a trusted reference value it is checked against. The International Vocabulary of Metrology (VIM), 3rd edition, 2012, entry 2.16, gives this in its own words: a measurement error is "measured quantity value minus a reference quantity value".

An error can behave in two different ways when the same thing is measured again and again.

A systematic error is the part of the error that, in the VIM's own words (entry 2.17), "remains constant or varies in a predictable manner". It pushes every reading the same way, by close to the same amount, every time.

A measurement bias is, in the VIM's own words (entry 2.18), an "estimate of a systematic measurement error". In plain words, it is an error that points one way and does not shrink when you collect more.

A random error is the part that, in the VIM's own words (entry 2.19), "varies in an unpredictable manner" from one reading to the next.

In plain words, accuracy is how close a measured value comes to the true value.

In plain words, trueness is how close the average of many repeated readings sits to the true value. That makes trueness about the size of the systematic error alone.

Precision, in the VIM's sense, is how closely repeated readings agree with each other. That makes it about the size of the random error alone. An instrument can be precise and wrong at the same time.

Resolution (VIM 4.14) is a different thing again: the "smallest change in a quantity being measured that causes a perceptible change in the corresponding indication". It says nothing about whether those digits sit close to the truth. [40, 41, 42, 43, 44, 45, 46, 47, 48, 49]

IN PLAIN TERMS. Take any measurement. Its error is what it says minus what is actually true. If a scale says 61.2 kilograms and the true weight is 60.5, the error is 0.7 kilograms. An error can be positive or negative. Stating it needs you to already know the true value, which in real life usually comes from a trusted reference instrument, not from guessing.

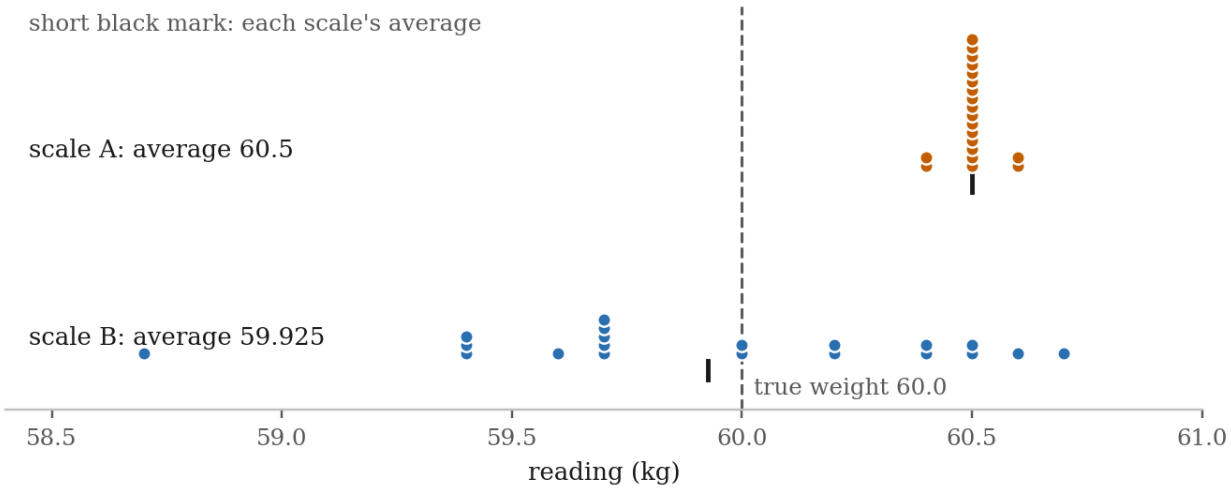
A systematic error pushes every reading off in the same direction by roughly the same amount. Say a kitchen scale reads 0.5 kilograms with nothing on the pan, instead of nothing. That is a systematic error: everything you weigh on it afterward comes out half a kilogram heavy. Take a hundred readings on this scale and average them. The half-kilogram is still there, because every single reading carried it.

A random error pushes readings around in a way that is not predictable from one reading to the next. Take a hundred readings from an instrument with only random error, and average them. The average comes out close to the truth.

Averaging more readings fixes random error. It does nothing at all for systematic error. A very careful, very repeated measurement on an instrument with a systematic error is not a truer measurement. It is a more confident wrong one.

A scale that reads 61.4, 61.4, 61.4 and 61.4 kilograms on the same object agrees with itself to the last digit it shows. If the true weight is 60.0 kilograms, it is also wrong every time. Trueness is how close the average of many repeated readings sits to the true value. Accuracy needs both: small scatter and no push off target.

Resolution is the smallest step an instrument's display can move by. A scale reading to the nearest 100 grams has a resolution of 100 grams. It tells you nothing about whether those digits are close to being true.



Twenty readings from each scale against a weight known to be 60.0 kilograms, the dashed line. Scale A's readings sit in a tight pile, about half a kilogram too high. Scale B's scatter widely on both sides of the truth, and their average lands close to it. How closely repeated readings agree with each other and how close they sit to the truth are two different things.

ILLUSTRATION. Kiran and colleagues' methods section states a figure worth testing before it is used.

> *Weight and height were recorded with accuracy of 100 g and 0.1 cm.*

Read that sentence the way the definition above teaches you to. A hundred grams is the step the weights were written down in. No reading can show a change smaller than that. That is a statement about resolution: the "smallest change in a quantity being measured that causes a perceptible change in the corresponding indication" (VIM 4.14). It is not a measured closeness to a true value. That would need the readings checked against a known reference weight, and nothing in the paper's methods reports such a check for this scale.

This mix-up is common enough to be worth seeing worked through with numbers on a stated case. No real scale's repeated calibration readings are held in this book's sources. Say a calibration weight — a weight already checked and trusted to be correct — is known to be exactly 60.0 kilograms. Each of two stated scales weighs it twenty times.

```
scale A readings (kg), tight and shifted
60.5 60.5 60.5 60.5 60.5 60.4 60.5 60.5 60.5 60.5
60.5 60.6 60.5 60.5 60.5 60.4 60.5 60.6 60.5 60.5
```

```
scale B readings (kg), centred and wide
59.7 60.2 60.2 60.4 60.0 59.6 59.4 60.6 58.7 59.7
59.4 60.0 59.7 59.7 60.4 59.7 59.4 60.5 60.5 60.7
```

Scale A's twenty readings barely move: they sit within a tenth of a kilogram of their own average. That is precision: how closely repeated readings agree with each other. Average them.

```
604.9 plus 605.1 = 1210.0
1210.0 divided by 20 = 60.5
```

Against the known 60.0 kilograms, every reading is 0.4 to 0.6 kilograms high, and the average is 0.5 high.

```
60.5 minus 60.0 = 0.5
```

No amount of repeating the weighing removes it, because this part of the error does not vary from reading to reading. It is systematic.

Scale B's twenty readings scatter far more widely, from 58.7 up to 60.7, including two readings of exactly 60.0. That is wide, so this scale is much less precise than scale A. Average them anyway.

```
598.5 plus 600.0 = 1198.5
1198.5 divided by 20 = 59.925
60.0 minus 59.925 = 0.075
```

Its average sits 0.075 kilograms below the truth.

This scale's error is random: it wobbles both above and below the truth from reading to reading. The wobbles partly cancel when averaged, and a few more readings would tend to bring the average closer still.

Scale A looks better reading by reading: tighter, more repeatable, a more confident-looking number each time. Yet its average sits further from the truth than scale B's average does: half a kilogram out against seven and a half hundredths. Precision and trueness are not the same property, and a fine resolution tells you about neither one.

Figures quoted in this illustration are from [18].

WHERE THIS PICTURE BREAKS. In most real measurement, the true value is exactly what you are trying to find out, so you cannot compute an error this way. Instead, you check an instrument's trueness once, in advance, against a trusted reference standard, using exactly this arithmetic.

An instrument that reads true at 60 kilograms can drift at 120.

MUST KNOW POINTS FOR YOU.

- Averaging more readings on a biased instrument does not get you a truer answer.
- Measurement bias is the VIM's own word (entry 2.18) for the same shape of problem as a biased sample. It is an error that points one way and does not shrink when you collect more.
- An instrument's stated "accuracy" is often a number of grams, millimetres or decimal places. It is not a closeness to a known true value. Check which one a paper means.
- Knowing an instrument's precision (VIM 2.15), how closely repeated readings agree with each other, tells you nothing about whether they agree with the truth.
- Part of the difference between observers is each observer's own systematic habit. Mixed across many observers, those habits scatter like random error. A single observer taking every measurement removes that scatter, but not the one observer's own habit. That habit then reaches every reading in the study, and no amount of data from that study alone can reveal it.
- Once a systematic error's size is known, subtract it. Finding the size needs an independent check against a known value, not simply more readings from the same instrument.
- Teach a trainee to ask two separate questions of any instrument. How closely do repeated readings agree with each other? That is precision. How close does the average of many repeated readings sit to the true value? That is trueness. Accuracy needs both together.

EXERCISE 1 (critique). mmHg means millimetres of mercury. It is the unit blood pressure is read in. A device maker says: "Our new blood-pressure monitor is accurate to within 1 mmHg." Here is where that number comes

from. Used again and again on the same person inside one minute, the monitor gives the same reading each time, to within 1 mmHg.

Say what word this number really describes. Say what question it leaves unanswered.

EXERCISE 2 (teaching). A trainee says a colleague's instrument "must be fine because it gives the same number every time." You have ten minutes and a whiteboard.

Teach them why that is not enough.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A device is checked against a known true value of 200. On one occasion it reads 194. Work out the error, and say what its sign tells you.

2. A device is checked five times against a known true value of 75. It reads 78, 77, 79, 76 and 80. Work out the error for each reading.

3. Two instruments are each checked five times against a known true value. Their errors are listed below. For each set, work out the average error and the range of the errors. Say which set looks more systematic and which looks more random.

```
set P: 3, 2, 4, 1, 5
set Q: -3, 4, -5, 2, -1
```

4. A scale is checked against a calibration weight of 20.0 kg. It gives five readings: 20.3, 20.2, 20.4, 20.3 and 20.3 kg. Work out the error for each reading and the average error, with its unit.

5. A thermometer is checked against a reference temperature of 37.0 °C (degrees Celsius). It gives five readings: 36.4, 37.6, 36.9, 37.3 and 37.0 °C. Work out the error for each reading, the average error, and the range of the errors. Say whether this instrument looks more like the scale in the previous problem or not.

6. Kiran and colleagues' scale recorded weight in steps of 100 g. Suppose one student's weight is recorded as 68.3 kg on this scale. A reference scale, checked against a known weight, gives 68.0 kg for the same student. Work out the error. Can knowing how the scale records its weights tell you whether this error is systematic or random?

7. Here is a worked answer. Find the step that broke.

```
a thermometer has a zero error: with its probe in melting ice, it reads 0.3 °C in-
stead of 0.0
twenty readings of a pan of boiling water are taken and averaged, giving 100.3 °C
averaging twenty readings reduces error, since averaging is how error is reduced
so 100.3 °C can be treated as an accurate reading
```

8. Here is a worked answer. Find the step that broke.

```
a scale is checked twenty times against a known 40.0 kg weight
the readings average to 40.05 kg
the average is close to the true weight, so the scale is accurate
each individual reading is therefore also close to the true weight
```

9. Here is a worked answer. Find the step that broke.

```
a paper states: neck circumference was measured with 1 mm precision
precision means how closely repeated readings agree with each other
so the paper has shown its neck-circumference readings agree with each other to
within 1 mm
this supports treating the reported neck-circumference figures as free of measure-
ment error
```

10. Here is a worked answer. Find the step that broke.

```
a scale has a known zero error: with nothing on it, it reads 0.4 kg
a reading of 62.1 kg is taken on this scale
to correct for the zero error, add it back: 62.1 plus 0.4 = 62.5
so the corrected weight is 62.5 kg
```

11. A colleague says: "Our new digital scale must be more accurate than the old one — it shows an extra decimal place."

Here are five readings from each scale, weighing the same known 50.00 kg calibration weight.

old scale (kg)	50.1	49.9	50.2	49.9	50.0
new scale (kg)	50.63	50.57	50.71	50.55	50.64

Decide what to compute, compute it, and say what your answer does not establish.

12. A methods section says: "One trained observer took every measurement, so this study has no measurement error to consider."

Kiran and colleagues also used one observer. They call it a strength: no two people's readings can then differ.

Now say a second person also measures the necks of the same five people. Each writes down the size round the neck in cm.

observer 1 (cm)	34.2	35.6	33.8	36.1	34.9
observer 2 (cm)	34.6	36.0	34.3	36.4	35.2

Decide what to compute, compute it, and say what the claim does not establish.

References · Part D

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Part E · The physical and living world

E1 · Atoms, molecules and bonds — enough to see where energy is stored

DEFINITION. The electrons occupy almost all of an atom's volume.

A chemical bond is an attraction between atoms or ions that holds them together in a molecule or a crystal.

A covalent bond results from the mutual attraction of two atoms for a shared pair of electrons.

Energy is the capacity to supply heat or do work. Work, in this sense, is a force moving something through a distance; lifting a weight is work. Potential energy is the energy an object has because of its relative position, composition or condition. Kinetic energy is the energy an object possesses because of its motion. The energy held in a food is potential energy of composition. It is held in which atoms are bonded to which, and not in the atoms themselves.

Separating any pair of bonded atoms requires energy. The stronger a bond, the greater the energy required to break it. Forming a bond releases energy.

Breaking every bond in the starting materials takes energy in. Forming every bond in the products gives energy out. The net energy change of the reaction is approximately the difference between those two totals.

Where the bonds in the products are stronger than the bonds in the starting materials, the reaction releases heat and is called exothermic. Where the bonds in the products are weaker, the reaction absorbs heat and is called endothermic. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]

IN PLAIN TERMS. Everything you eat is made of atoms.

An atom has a tiny heavy centre, and a cloud of much lighter particles around it, called electrons.

An atom is mostly space.

A bond is an attraction between two atoms that holds them together.

A group of atoms held together by bonds is a molecule. Sugar is a molecule.

In an ionic bond, one atom hands an electron over to the other. One is left positive, the other negative, and opposite charges pull. An atom left carrying a charge this way is called an ion.

In a covalent bond, the two atoms share a pair of electrons instead of handing them over.

The energy in a food is held in the arrangement of its atoms. It is in which atom is bonded to which, and not in the atoms themselves.

Pulling a bond apart costs energy. Making a bond gives energy back.

Most people have that the wrong way round, so say it out loud. Breaking costs. Making pays.

Strong means it takes a lot of energy to break, and it means nothing else.

When a food burns, old bonds break and new bonds form. The reaction hands you the difference between the two.

If the new bonds are stronger than the old ones, more comes out than went in. That is heat you can measure, and it is what anyone means by the energy in a food.

A reaction that gives heat out is exothermic. A reaction that takes heat in is endothermic.

So the energy in a food is not a stuff sitting in it, waiting to be poured out. It is a difference between two sums. It shows up only when the arrangement changes.

ILLUSTRATION. Open OpenStax Chemistry 2e, section 2.3, Atomic Structure and Symbolism, at https://openstax.org/books/chemistry-2e/pages/2-3-atomic-structure-and-symbolism. Open your browser's find box. Type the words An atom is and press enter.

Nothing.

Open section 2.4, Chemical Formulas, at https://openstax.org/books/chemistry-2e/pages/2-4-chemical-formulas, and search inside it for A molecule is.

Nothing again.

So search that page for the word bond instead.

> *(A chemical bond is an attraction between atoms or ions that holds them together in a molecule or a crystal.)*

Search for the claim, not for the heading.

Open section 7.5, Strengths of Ionic and Covalent Bonds, at https://openstax.org/books/chemistry-2e/pages/7-5-strengths-of-ionic-and-covalent-bonds. Search inside it for the word Separating.

> *Separating any pair of bonded atoms requires energy.*

Pulling a bond apart costs you energy.

Search for the word released.

> *...the energy released when all bonds are formed in the products.*

Two energies, opposite in direction, inside one reaction. Whether the reaction gives you anything depends on which of the two is bigger.

Hydrogen gas is molecules of two hydrogen atoms, joined by one bond each. Chlorine gas is molecules of two chlorine atoms, joined by one bond each. Put them together and they make hydrogen chloride, in which each hydrogen atom is joined to one chlorine atom.

Count the bonds with a pencil. Take one hydrogen molecule and one chlorine molecule. Two bonds break: the hydrogen-to-hydrogen bond, and the chlorine-to-chlorine bond. Two bonds are then made, each one hydrogen to chlorine.

The page gives you all three numbers, in kilojoules per mole of bonds. A mole is the fixed count of things from the section on concentration units (B5).

bond	energy in kilojoules, per mole of bonds
hydrogen to hydrogen	436
chlorine to chlorine	243
hydrogen to chlorine	432

None of them is the energy of a single pair of atoms.

Energy in, to break the two bonds.

```
436 plus 243 = 679
```

Energy out, from making the two new bonds.

```
432 times 2 = 864
```

In, minus out.

```
679 minus 864 = -185
```

Read the sign before you read the digits. The answer is negative, which means more came out than went in. One hundred and eighty-five kilojoules left the reaction as heat.

A food is molecules. Burning it breaks the bonds inside those molecules, and makes new bonds with oxygen from the air. The new bonds are stronger than the old ones. So the sum runs the way it just ran for hydrogen and chlorine, and the difference comes out as heat.

That difference is what anybody means when they say a food contains energy.

There is no energy sitting in the food the way tea sits in a cup.

Change the arrangement and you collect the difference. Leave it alone and there is nothing there to collect.

When somebody tells you that a substance releases energy, ask two questions. What gets broken, and what gets made. If the answer names only one of the two, you have been handed half a sum.

Figures quoted in this illustration are from [17].

WHERE THIS PICTURE BREAKS. The three bond energies are quoted for a mole of each, not for one pair of atoms. So 185 kilojoules is not what one molecule gives up. No figure here is the energy of a single bond you could point at.

The sum is approximate and the book says so. It gives the difference as approximately the net change, not as the net change. Two bonds with the same name in two different molecules are not quite the same bond, and the tables carry averages.

Hydrogen and chlorine is not a food. Nothing about the shape of the sum changes.

The sum gives you the size of an energy change and tells you nothing about whether the change happens, or how fast. Sugar can sit in a jar, in air, for years, unchanged, though burning it releases a great deal. Something, such as a flame, has to start it.

And it tells you nothing about where the energy goes after it leaves.

MUST KNOW POINTS FOR YOU.

- Energy is not a substance sitting inside a food. So refuse any claim that something can release a food's energy without changing what its molecules are.

- A strong bond is one that takes a lot of energy to break, and that is all the word means. It does not mean the bond is holding more energy for you.
- When someone gives you a figure for what a reaction releases, ask what was broken and what was made. A figure that names only one side of that is half a sum.
- Bond energies give you the size of an energy change and nothing else. They cannot tell you whether the reaction happens, or how fast, or what starts it.
- Teach the two-sided sum before you teach a single step of metabolism. Break, then make, then subtract.
- A substance that goes into a person and comes out unchanged has supplied nothing, whatever the bonds inside it are worth.

EXERCISE 1 (retrieval). Without looking back, write down what happens to the energy when a bond breaks, and what happens when a bond forms.

Then write the rule for working out whether a reaction gives heat out or takes heat in.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A wellness brochure says this. The brochure is made up for this exercise.

> *Our drink is built from molecules with unusually strong chemical bonds. More energy is locked inside each bond, so more energy is released into your body when you drink it.*

Say what is wrong, and say what you would ask for.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They can say that food gives you energy and they cannot say where the energy is.

Teach them where it sits, so that they leave able to check a claim about any substance they have not met.

E2 · Chemical energy, combustion, and the bomb calorimeter

Checked against the sources as they stood on 2026-09-23.

DEFINITION. Combustion is burning: a substance combines vigorously with oxygen.

A bomb calorimeter operates at constant volume and is used for reactions that release large amounts of energy and produce gases, which is what combustion does. It is a strong steel container, the bomb, which holds the sample and is itself submerged in water. The bomb is filled with oxygen at high pressure and a small electrical spark ignites the sample. The energy produced is taken up by the steel bomb and the water around it, and a thermometer records the rise.

A substance whose energy is already known is burned in it, and the rise that produces is measured. The known energy divided by that rise is the calorimeter's energy equivalent, in joules for each degree. Afterwards any rise measured in that calorimeter, multiplied by that same energy equivalent, gives the energy the reaction released.

What that measures is the total combustible energy content of the food, also called its theoretical maximum energy content. The Food and Agriculture Organization of the United Nations calls this the ingested energy, or the gross energy.

Not all of it is available to a person. Incomplete digestion in the small intestine, sometimes with fermentation of unabsorbed carbohydrate in the colon, loses energy as faecal energy and as gaseous energy in combustible gases. Some of the energy that is absorbed is then lost as urinary energy, mainly as nitrogenous waste compounds from incomplete breakdown of protein. A small amount is lost from the body surface. What remains after those losses is metabolizable energy.

Below that the Organization's 2003 report carries one further step, net metabolizable energy. It takes off the energy the body itself spends on digesting the food and on handling what is absorbed. The report's cascade has no step called digestible energy. The phrase is not in chapter 3, where the report sets out the cascade, or in chapter 4.

The energy figure on a food label is not a burning measurement. It is derived using the Atwater system, which uses the average caloric content of protein, carbohydrate and fat. Those averages are the Atwater general factors. There is one value for each of the three, used regardless of the food it is found in. They are based on the heats of combustion of protein, fat and carbohydrate, corrected for losses in digestion, absorption and urinary excretion of urea.

The three values are 17 kilojoules for each gram of protein, 37 for each gram of fat and 17 for each gram of carbohydrate. In kilocalories for each gram the same three values are 4.0, 9.0 and 4.0.

One thing the general system does not do is take fibre out. Some labelling systems do discount fibre, as OpenStax Chemistry describes. Atwater's general system does not. The Indian labelling regulations give dietary fibre its own factor, 2 kilocalories a gram.

To work out the energy content of a food, the quantities of carbohydrate, protein and fat are each multiplied by the factor for that one, and the products are added. [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]

IN PLAIN TERMS. You cannot weigh energy. What you can do is burn the food and watch a thermometer.

The machine for it is called a bomb calorimeter. The name is unhelpful and the thing is straightforward. It is a thick steel pot, sealed, sitting in a tank of water. You put a weighed scrap of dried food inside, fill the pot with oxygen, seal it, and set it off with an electric spark. The food burns completely. The warmth spreads into the steel and the water. A thermometer in the water reads higher than it did before.

Nothing about a temperature rise tells you how many joules it took. That depends on the machine: how much steel, how much water, how it is built.

You burn something whose energy you already know. You watch how far the thermometer goes up. Then you divide.

```
the energy equivalent = the known energy, divided by the rise it caused
```

Call it the calorimeter's energy equivalent. Doing this is called calibrating the machine, and it is done again before each set of measurements rather than once when the machine is bought.

After that the work is one multiplication.

```
the energy released = the rise, times the energy equivalent
```

Burn a known thing to get the constant, then burn the unknown thing and multiply.

What you have measured is all the energy the food will give up when it is burnt to nothing. The Food and Agriculture Organization of the United Nations calls it the gross energy. It is the most the food could ever supply.

A person is not a furnace.

Some food is not digested and absorbed at all, and leaves in the faeces, carrying its energy with it. Some unabsorbed carbohydrate is fermented in the large intestine and leaves as gas, which also carries energy. Some of what is absorbed leaves in the urine, mostly as the waste left when protein is broken down and not finished off. And a small amount is lost from the body surface.

What is left after those losses has a name. It is called metabolizable energy, spelt with a z in the report that names it.

So the cascade is gross energy, then metabolizable energy. Below that the report carries one more step, called net metabolizable energy. It takes off the energy the body itself spends on digesting the food and handling it. The report's cascade has no step called digestible energy. The phrase is not in chapter 3, where the report sets out the cascade, or in chapter 4. You may hear somebody put one in the middle and say the report has it. If they do, they have not read that chapter.

The number on a packet was almost certainly never measured by burning anything. It was calculated.

The fixed figures are called the Atwater general factors.

what it is	kilojoules a gram	kilocalories a gram
protein	17	4.0
fat	37	9.0
carbohydrate	17	4.0

Those factors already have the losses taken out of them. They were built from burning measurements and then corrected for what is lost in digestion, in absorption and in the urine.

So the figure on the packet is an estimate of metabolizable energy for an average, arrived at by arithmetic on three weights. It is a good estimate and it is not a measurement of that packet.

Burning tells you what is in the food. The factors tell you what an average person would get out of it. Neither one tells you anything about the person in front of you.

ILLUSTRATION. Open OpenStax Chemistry 2e, section 5.2, Calorimetry, at https://openstax.org/books/chemistry-2e/pages/5-2-calorimetry. Open your browser's find box and search inside the page for the word bomb.

> *The sample is placed in the bomb, which is then filled with oxygen at high pressure. A small electrical spark is used to ignite the sample.*

Now search the same page for the word calibration, because that is the word this whole section turns on.

> *The calibration is generally performed each time before the calorimeter is used to gather research data.*

Each time. Not once when the machine arrives.

How many joules one degree is worth depends on this machine.

Every figure in this calorimeter example is made up.

The known sample releases 26,400 joules. The thermometer goes up by 2.2 degrees Celsius, which is the ordinary degree marked on a kitchen thermometer.

Divide the energy by the rise.

```
26,400 divided by 2.2 = 12,000
```

That is the calorimeter's energy equivalent, and it is the only thing you needed.

A dried pellet of food weighing 1.50 grams goes in, and the thermometer goes up by 1.85 degrees.

```
12,000 times 1.85 = 22,200
```

Put it in kilojoules, because nobody quotes food in joules.

```
22,200 divided by 1,000 = 22.2
```

Now for each gram, since the pellet weighed 1.50 grams.

```
22.2 divided by 1.50 = 14.8
```

Turn it into the unit food is quoted in, using the 4.184 from the section on energy units.

```
14.8 divided by 4.184 = 3.5373
```

About 3.5 kilocalories in each gram.

The question nobody asks is what it is a number about.

The Food and Agriculture Organization of the United Nations published a report on this in 2003. Open chapter 3 of it at https://www.fao.org/4/y5022e/y5022e04.htm.

Almost everybody who has met this once remembers three steps: gross energy, then digestible energy, then metabolisable energy. Search the page for digestible energy.

The phrase is not in the chapter.

Search for gross energy instead.

> *Food that is ingested contains energy - the maximum amount being reflected in the heat that is measured after complete combustion to carbon dioxide (CO2) and water in a bomb calorimeter. This energy is referred to as ingested energy (IE) or gross energy (GE).*

That is your burning measurement, named.

> *The energy that remains after accounting for the important losses is known as "metabolizable energy" (ME) (see Figure 3.1).*

Now read one paragraph further. There is a third step, and it is not the one you went looking for. Some energy is spent on digesting the food and handling it afterwards, and some is lost to microbes. Take those off metabolizable energy and the report names what is left.

> *...the result is an expression of the energy content of food, which is referred to as net metabolizable energy (NME).*

So the report's cascade runs gross energy, then metabolizable energy, then net metabolizable energy. There is no digestible-energy step in it. The phrase is not in chapter 3, and it is not in chapter 4 either.

Search the same chapter for Atwater.

> *The energy values are 17 kJ/g (4.0 kcal/g) for protein, 37 kJ/g (9.0 kcal/g) for fat and 17 kJ/g (4.0 kcal/g) for carbohydrates.*

Open the National Food Security Act, 2013 at https://indiacode.gov.in/handle/123456789/496105 and go to Schedule II. Find the row for upper primary classes. The hot cooked meal is set at 700 in the energy column, and at 20 grams in the protein column.

Work out how much of that 700 the protein standard accounts for.

```
20 times 4.0 = 80
```

As a share of the meal, divide the part by the whole.

```
80 divided by 700 = 0.1142857143
```

Keep every figure for now. Multiply by a hundred to turn the proportion into a percentage.

```
0.1142857143 times 100 = 11.42857143
```

Round once, at the end: 11.4. About eleven per cent. So the Act fixes an energy figure and a protein figure, and the protein figure accounts for roughly a ninth of the energy. The other eight ninths have to come from fat and carbohydrate, and Schedule II sets no figure for either of them.

Now do the same sum in kilojoules and watch something strange happen. The factor for protein in kilojoules is 17.

```
20 times 17 = 340
```

And the 700 kilocalories, in kilojoules:

```
700 times 4.184 = 2,928.8
```

```
340 divided by 2,928.8 = 0.1160885004
```

```
0.1160885004 times 100 = 11.60885004
```

Round once, at the end: 11.6. Eleven point six per cent this time, against eleven point four the other way.

The report's kilojoule column and its kilocalorie column were each rounded on their own, and they are not exact conversions of one another. A footnote on the same page gives protein's precise value as 16.7 kilojoules a gram, and that one does convert properly.

```
16.7 divided by 4.184 = 3.991
```

The 17 is the rounded figure, and rounding it is what moved your answer in the third digit.

You do not have a measurement of any meal. Schedule II says what a meal must meet, not what any meal contained.

You do not have what reaches a child either, even if a meal met the standard exactly.

This book does not carry either number, and you should not supply one from memory.

Every time somebody hands you a food-energy figure, ask three questions in order. Was it burnt or was it calculated. Is it the gross figure or the metabolizable one. And is it a standard somebody set, or a measurement somebody took.

Figures quoted in this illustration are from [35, 36, 37].

WHERE THIS PICTURE BREAKS. Every figure in the calorimeter example is made up. Only the 4.184, the Atwater factors and the two Schedule II figures come from a source.

The energy equivalent belongs to one calorimeter on one day. Carry it to another machine and every answer it gives is wrong, and wrong silently, because the arithmetic still works.

The method needs the sample to burn completely. Nothing in the arithmetic will tell you it happened.

The Atwater factors are averages over foods. So the method cannot tell apart two foods with the same grams of protein, fat and carbohydrate.

The two printed columns pair up for protein and for carbohydrate. They do not pair up for fat. The footnote that gave protein's 16.7 gives fat's precise value as 37.4 kilojoules a gram, and the report prints 9.0 kilocalories beside it. Try the conversion both ways round.

```
37.4 divided by 4.184 = 8.9388
9.0 times 4.184 = 37.656
```

Read each column as its own set of numbers and do not work one out from the other.

The eleven per cent figure is a share of two numbers the Act sets, not a share of anything a child ate.

None of the arithmetic here says anything about a person. It gives you what is in a food and what an average person would get out of it. What one person absorbs, what they spend, and what any change in their body would be made of are three separate questions. This book answers none of them.

MUST KNOW POINTS FOR YOU.

- A bomb calorimeter figure is the food's total combustible energy. It is not the energy a person gets. So never quote a burning figure as what somebody received.
- The report's cascade has no digestible-energy step. It runs from gross or ingested energy to metabolizable energy. The losses named are faecal, gaseous, urinary and surface. A third step sits below, net metabolizable energy. So do not object to a three-step reading of this report. Object to the word digestible when it is credited to this cascade. Name the step that was made up.
- A temperature rise is not an energy until the machine that produced it has been calibrated. The calibration is repeated before each set of runs, not done once.
- The Atwater general factors are the numbers behind nearly every food-energy figure you will ever be handed. They are 17 kilojoules, or 4.0 kilocalories, for each gram of protein. For fat they are 37 or 9.0, and for carbohydrate 17 or 4.0. Knowing them means you can recompute any label yourself instead of trusting it.
- Those factors use one value for each of protein, fat and carbohydrate, whatever the food. It cannot show that one food is taken up better than another.
- The energy declared on an Indian packet is worked out from the grams, not measured by burning anything. So two packets showing the same figure have been through the same arithmetic, not the same measurement.
- When a journalist gives you a food-energy figure, the useful sentence is about what it describes. Give them that line before you argue about the number itself.
- Teach the calibration before the formula. A trainee who has seen a known burn turned into a constant will never again treat a thermometer reading as an energy. They will also carry the question to every instrument they meet afterwards: what was this measured against.

EXERCISE 1 (retrieval). Without looking back, write down the two steps that turn a bomb calorimeter reading into an energy, in order, and say what each step needs.

Then write down the cascade the Food and Agriculture Organization's report gives for what happens to a food's energy in a person, naming the losses.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A supplement company's sheet says this. The company and the sheet are made up for this exercise. Read it twice before you answer.

> *Independent laboratory testing by bomb calorimetry confirms that each serving delivers 480 kilocalories of energy to the body. The measurement was made by burning, so it is more accurate than the numbers worked out on paper for ordinary packets.*

Say what is wrong. Then say what you would ask for.

EXERCISE 3 (teaching). A health secretary has one page. They have been handed a proposal about a school meal scheme. It argues the scheme is meeting its target, because the menu adds up to the energy figure in the standard.

Write what you would put on the page. Answer first, no methods section.

PRACTICE. Twelve problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. One kilocalorie is 4.184 kilojoules.

Turn 2,092 kilojoules into kilocalories.

2. A calorimeter's energy equivalent is 9,500 joules for each degree.

In one run the thermometer rises by 3.4 degrees. Work out the energy released, first in joules and then in kilojoules. The figures in this problem are made up.

3. A sample whose energy is known to be 44,000 joules is burned in a calorimeter. The thermometer rises by 4.00 degrees.

Work out how many joules one degree is worth in that calorimeter. The figures in this problem are made up.

4. A sample holds 12 grams of protein, 8 grams of fat and 40 grams of carbohydrate.

Using 4.0 for each gram of protein, 9.0 for each gram of fat and 4.0 for each gram of carbohydrate, work out the total.

5. Food and Nutrition Paper 77 is a report on food energy. The Food and Agriculture Organization of the United Nations published it in 2003.

It gives the Atwater general factor for protein twice over. Once as 17 kilojoules for each gram, and once as 4.0 kilocalories for each gram. A footnote on the same page gives the precise figure as 16.7 kilojoules a gram.

Turn both of those kilojoule figures into kilocalories a gram. One kilocalorie is 4.184 kilojoules. Then say which of the two the report's 4.0 was rounded from.

6. A laboratory calibrates a bomb calorimeter by burning a sample whose energy is known to be 52,500 joules. The thermometer rises by 3.50 degrees Celsius.

It then burns a 2.00 gram dried pellet of a food, and the thermometer rises by 2.80 degrees Celsius.

Work out the energy the food released, in kilojoules, and then the energy for each gram of food. Write the unit on every line. Every figure in this problem is made up; no real calorimeter and no real food is being described.

7. Schedule II of the National Food Security Act, 2013 sets standards for a hot cooked meal. For lower primary classes it sets 450 in the energy column and 12 grams in the protein column. The energy column is headed Calories, with Kcal in brackets.

The Atwater general factor for protein is 4.0 kilocalories for each gram. Work out how many of those 450 kilocalories the protein standard accounts for. Then work out what share of the meal that is, as a percentage.

8. Two laboratories are given the same dried food and asked for its energy for each gram.

The first burns it. Its calorimeter is worth 8,000 joules for each degree, and a 1.25 gram sample makes the thermometer rise by 2.50 degrees.

The second does not burn anything. It measures the composition instead. In each gram it finds 0.10 grams of protein, 0.05 grams of fat and 0.70 grams of carbohydrate. The rest is water and minerals. It uses the Atwater general factors of 4.0, 9.0 and 4.0 kilocalories a gram.

Work out both answers in kilocalories for each gram. Say which is larger, by how much, and why you would have expected that before doing the arithmetic. All the figures are made up for this problem.

9. Here is a worked answer. Find the step that broke. The figures in this problem are made up.

```
the calorimeter is worth 10,000 joules for each degree
the thermometer read 21.4 degrees before the sample was lit
the thermometer read 24.9 degrees after the sample had burned
10,000 times 24.9 = 249,000
so the sample released 249,000 joules, which is 249 kilojoules
```

10. A packet's nutritional information panel reads: total carbohydrate 30 grams, of which sugars 12 grams; protein 6 grams; total fat 5 grams. The packet is made up for this problem.

Here is a worked answer for the energy in the packet. Find the step that broke.

```
carbohydrate: 30 times 4 = 120
sugars: 12 times 4 = 48
protein: 6 times 4 = 24
fat: 5 times 9 = 45
120 plus 48 plus 24 plus 45 = 237
so the packet holds 237 kilocalories
```

11. A packet in a shop carries this line. The packet and the figure are made up for this problem.

> *Contains 250 kilocalories.*

A colleague reads it and says that eating the packet adds 250 kilocalories to the person who eats it.

Decide what to compute, compute it, and say what your answer does not establish.

12. In a district meeting, somebody says this.

> *Schedule II of the National Food Security Act, 2013 sets the upper primary hot cooked meal at 700 calories. It also sets 20 grams of protein. So every upper primary child gets 700 kilocalories a day from school. We can take that straight off what they need from home.*

Decide what to compute, compute it, and say what your answer does not establish.

E3 · Conservation of energy, and a system with a boundary

DEFINITION. Energy is a quantity that can be counted.

A system is whatever has been enclosed for the purpose of an accounting. The boundary is the closed surface that encloses it, and the surroundings are everything outside that surface. The system is chosen by the person doing the accounting.

The law of conservation of energy states that total energy is constant in any process.

Applying that law across a boundary gives the energy balance. The energy inside the boundary at the end of an interval equals the energy inside at the start, plus every quantity of energy that crossed the boundary inwards, minus every quantity that crossed it outwards.

A system's energy changes only by energy crossing its boundary, because any other route would require the total to change. And the change in a system's energy is fixed only once the boundary is fixed: two boundaries drawn around one situation give two different changes, both correct, because they answer two different questions.

The balance constrains the numbers and nothing else. It does not say by which route energy crossed, what caused a crossing to change, or what a quantity of energy amounts to in any unit other than a unit of energy. [38, 39, 40, 41, 20, 42]

IN PLAIN TERMS. Energy is never made and never destroyed. It changes form, and it moves from one place to another, and the total stays where it was.

That sounds like a rule about the world. It is. But on its own it lets you compute nothing, because it talks about the total of everything, and nobody can count everything.

So here is the move that turns it into arithmetic. Draw a line round something.

The line is closed, like a circle on paper. Whatever you have drawn round is called the system. The line itself is called the boundary. Everything outside the line is called the surroundings.

The total for inside plus outside never changes. So if the amount inside goes up, the amount outside must have gone down by the same amount. And the only way that happens is if energy crossed the line.

Step one. Split everything in two. The line is closed, so every bit of energy in the world is either inside it or outside it. Nothing is both and nothing is neither.

```
the total = the amount inside, plus the amount outside
```

Step two. Use the rule. The total does not change over the interval you are looking at. So the total at the end is the same number as the total at the start.

```
the amount inside at the end, plus the amount outside at the end
= the amount inside at the start, plus the amount outside at the start
```

Step three. Take the starting amounts off both sides. This is the move from the section on rearranging an equation, done twice. Take away "the amount inside at the start", then take away "the amount outside at the start". The right-hand side becomes nothing at all. The left-hand side now has four pieces in it.

You may write those four pieces in any order you like. A run of adding and taking away comes out the same whatever order you do it in. Adding 3 and then taking away 5 lands you where taking away 5 and then adding 3 lands you. So put each end value next to its own start value.

```
the amount inside at the end, minus the amount inside at the start,
plus the amount outside at the end, minus the amount outside at the start = 0
```

Now read each pair on its own. An end value minus its own start value is a change. So the first pair is the change inside, and the second pair is the change outside.

```
the change inside, plus the change outside = 0
```

The two changes add to nothing. So whatever the inside gained, the outside lost, and by exactly the same amount.

Step four. Name where it went. Energy that left the outside and arrived inside crossed the line inwards. Energy that went the other way crossed it outwards. There is no third possibility, because step one used up the whole world in two pieces.

```
the energy inside at the end = the energy inside at the start, plus everything that
crossed in, minus everything that crossed out
```

Nothing was assumed there except the one rule at the top and the fact that your line is closed. It is the balance line from the section on stocks and flows, with energy as the stock.

Where you draw the line is your choice. Nothing in the situation tells you where it goes. A pan of water, a pan and its stove, a whole kitchen. Those are three different lines round one event. Each gives a different answer to "how much did the energy change?"

None of the three is the wrong answer. They are answers to three different questions, and the question is the line.

So a number for an energy change is not finished until somebody says what it was measured round. Ask "round what?" before you argue about the figure.

The warning is about what this line does not do. It tells you that the numbers have to add up. It does not tell you which way energy went. It does not tell you why a flow changed. And it does not tell you what a heap of joules is worth in kilograms of anything.

ILLUSTRATION. Open OpenStax College Physics 2e, section 7.6, Conservation of Energy, at https://openstax.org/books/college-physics-2e/pages/7-6-conservation-of-energy. Open your browser's find box. Type the word system and press enter.

> *Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.*

It tells you to decide what the system is. It never tells you what a system is.

A system is not a thing in the world. It is a line somebody drew. A physics book can tell you about things in the world. It cannot tell you where you decided to draw your line.

A small gas burner heats water in a steel pan for two minutes. The gas that burns in those two minutes releases 60,000 joules of chemical energy. Of that, 42,000 joules ends up in the water and 11,000 joules ends up in the steel of the pan. The other 7,000 joules ends up in the air of the kitchen. The kitchen door and window are shut for the two minutes.

Those four figures are made up for this example.

Line one, round the water only. What crossed in: 42,000 joules, from the flame and the hot pan. What crossed out: nothing. Put that through the balance line.

The energy inside went up by 42,000 joules.

Line two, round the water and the steel pan together. Now the 11,000 joules in the steel is inside your line instead of outside it. What crossed in is the 42,000 and the 11,000 together.

The energy inside went up by 53,000 joules.

Line three, round the burner, the unburnt gas, the pan and the water. This one is different, because the chemical energy of the gas now starts inside your line. Nothing crosses in. The 7,000 joules that reached the kitchen air crossed out.

```
0 minus 7000 = -7000
```

The energy inside went down by 7,000 joules.

Check it as changes. The gas gave up 60,000 joules. The water gained 42,000 and the steel 11,000.

```
42000 plus 11000 minus 60000 = -7000
```

Line four, round the whole kitchen. The door and window are shut. Nothing crossed in, nothing crossed out.

The energy inside did not change at all. The form changed. The total did not.

Put the four side by side.

where the line is drawn	change in the energy inside
the water	plus 42,000 joules
the water and the pan	plus 53,000 joules
the burner, the gas, the pan and the water	minus 7,000 joules
the whole kitchen	0 joules

Four numbers, one event, and every one of them is right.

One joule is small. OpenStax College Physics 2e, section 7.1, puts it like this.

> *One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.*

So 60,000 joules is about what you would spend lifting that apple 60,000 times. Two minutes of a small burner.

Draw the line, on paper, closed. Write on the outside of it every route by which energy can cross. The flame, the air, the light, the wire, the food, the water going down the drain. Then put a number on each route, and see whether the numbers add up to the change somebody claimed.

A balance that fails almost never fails because conservation failed. It fails because a route was left off the paper.

Figures quoted in this illustration are from [43].

WHERE THIS PICTURE BREAKS. No burner was measured. Only the size of the joule comes from a source, College Physics 2e section 7.1.

The shut kitchen is an idealisation and it has a time limit. Over two minutes, a closed room loses little enough through its walls that treating the crossing as zero is fair. Over an hour it is not fair, and over a day it is wrong.

So a balance that does not close has two possible causes. Either a route was left off the paper, or a figure on the paper is wrong. The balance cannot tell you which.

Conservation is exact and it is not the whole story about energy. The 7,000 joules spread through the kitchen air is every bit as real as the 42,000 in the water, and it will not cook anything. The balance counts joules. It says nothing about which joules you can still use.

The balance holds for a person exactly as it holds for a pan. What it will not do is turn joules into kilograms of a body.

MUST KNOW POINTS FOR YOU.

- An energy figure with no boundary attached is not a measurement yet. Ask "round what?" before you argue about the number.
- Energy is never lost, so refuse the word when somebody uses it to close an inquiry. Energy that has left your system is in the surroundings, by some route.
- A system gains or loses energy only by energy crossing its boundary. So every explanation of a change has to name a crossing.
- The balance tells you the numbers must add up. It never tells you why one of them moved. So do not use conservation to explain anything.
- Nothing turns joules into kilograms of a body. How much energy a kilogram of change holds depends on what the change is made of.
- A person is not a closed system, so "the energy has to go somewhere" is true and names nothing. Energy crosses a person's line inwards in food and drink. It crosses outwards as heat, as work done on things around them, and in what leaves the body unabsorbed. Two lines are both in use. One is at the mouth and skin, where unabsorbed food crosses in and back out. The other is at the gut wall, where it never crosses in. Say which one you mean.
- Teach the line before the equation. A trainee who draws a closed boundary and lists the crossings can set up a problem they have never seen.
- When a claim is defended on the ground that energy is conserved, say plainly what conservation does. It rules things out. On its own it establishes nothing.

EXERCISE 1 (retrieval). Without looking back, write down the balance line for energy across a boundary, in words.

Then say, in one sentence each, what it rules out and what it cannot tell you.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A note about a community kitchen contains this line.

> *We measured the gas going in and the heat going into the food. Only about two thirds of the energy reached the food, so a third of the energy was lost. The burners are faulty and should be replaced.*

The kitchen and the figures are made up for this exercise. Say what is wrong with the reasoning, and say what you would ask for.

EXERCISE 3 (teaching). A journalist has three minutes and no jargon. They have read this: “the body obeys the laws of physics, so weight is just energy in minus energy out”. They want to know whether it is true.

Say what you would tell them, and give them one quotable sentence.

PRACTICE. Eight problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section’s name.

1. A system holds 1,200 at the start of an interval. Over that interval 450 crosses in and 610 crosses out.

Work out the amount inside at the end, and the net crossing.

2. Each row below is one boundary over one interval. Two of the four numbers in each row are given. Work out the missing one.

inside at the start	crossed in	crossed out	inside at the end
900	300	100	
900	300		1050
900		250	1100
900	0	0	

3. A kerosene lamp burns in a room for an hour. The kerosene releases 900,000 joules of chemical energy in that hour. Of that, 840,000 joules ends up warming the air and the walls of the room, and the rest leaves through the window as light. The room is otherwise shut.

The lamp, the room and both figures are made up for this problem.

Work out how much energy leaves through the window. Then give the change in the energy inside each of these three boundaries, with its unit, and say for each one what crossed.

1. Round the kerosene in the lamp.
2. Round the lamp, the air and the walls together. Count the window as a hole in the boundary.
3. Round the room and everything outside it that the light reaches.

4. A solar water heater sits on a roof. Over one day, 5,000,000 joules of sunlight crosses into the tank of water. At the end of the day the energy in the water is 4,100,000 joules higher than it was at the start.

The heater and both figures are made up for this problem.

Work out how much energy crossed out of the tank over the day, and say what fraction of the sunlight that is. Then say what your answer does not tell you.

5. Here is a worked answer. Find the step that broke.

```
a burner releases 60,000 joules of chemical energy in two minutes
the water in the pan gains 42,000 joules in those two minutes
60,000 minus 42,000 = 18,000
so 18,000 joules was destroyed
energy cannot be destroyed, so one of the two measurements must be wrong
```

6. Here is a worked answer. Find the step that broke.

```
a room is shut for an hour and nothing goes in or out of the door
a fan inside the room runs for that hour on 300,000 joules of electricity from the
mains
energy is conserved, so the total energy inside the room cannot change
300,000 minus 300,000 = 0
so the room cannot have got any warmer
```

7. A wellness page says this.

> *Energy cannot be created or destroyed. So every kilojoule of energy in the food you eat has to end up somewhere in your body.*

Decide what to compute, compute it, and say what your answer does not establish.

8. A colleague says this in a meeting.

> *The label says this packet holds 1,200 kilojoules. So eating it puts 1,200 kilojoules into the person.*

The packet and the figure are made up for this problem. The figure on the label was calculated with the Atwater general factors.

Decide what to compute, compute it, and say what your answer does not establish.

E4 · Heat and temperature; how heat moves

DEFINITION. Temperature is a measure of how hot or cold something is. Heat is the spontaneous transfer of energy due to a temperature difference. Heat is measured in joules, the SI unit of energy. Temperature is measured in degrees Celsius or in kelvin.

The specific heat of a substance is the heat required to change the temperature of one kilogram of it by one degree Celsius. Its unit is joules per kilogram per degree Celsius.

The letter Q stands for the quantity of heat transferred. The letter m stands for the mass of the substance. The letter c stands for its specific heat.

```
Q = m times c times ΔT
```

The change in temperature is the final temperature minus the initial temperature.

Every process involving heat transfer takes place by only three methods. Conduction is heat transfer through stationary matter by physical contact. Convection is heat transfer by the macroscopic movement of a fluid. Heat transfer by radiation occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. [44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55]

IN PLAIN TERMS. **Temperature** is how hot a thing is. A thermometer reads it. It is a level, not an amount. It does not care how much of the thing there is. A spoonful of boiling water and a bucket of boiling water are at the same temperature.

Heat is energy moving, because one thing is hotter than another. It is an amount, and it is measured in joules, like every other energy in this book.

So the bucket and the spoonful are at the same temperature and hold very different amounts of energy.

Energy goes from the hotter thing to the colder thing, and it keeps going until the two are at the same temperature. Then it stops.

The letter Q stands for the quantity of heat. The letter m stands for the mass. The letter c stands for the number belonging to the material. The Greek capital letter delta, written Δ, is how mathematics writes “the change in”. So the pair ΔT stands for the change in temperature, and it is read “delta T”.

```
Q = m times c times ΔT
```

That number belonging to the material has a name: the specific heat. It is the joules you need for one kilogram of the stuff to go up by one degree.

material	joules for one kilogram, one degree
water	4186
glass	840
iron and steel	452

Water needs more than nine times as much energy as steel for the same rise. A bigger temperature rise does not mean more heat went in. It means the two things were different things.

One more word, because the section on the calorimeter used it without naming it. Heat capacity is the same idea for one particular object rather than for a material. It is the joules that object needs to go up by one degree. A calorimet-

er has one, and you have already used it. The energy equivalent from that section is the calorimeter's heat capacity. It is also why a calorimeter's degrees can be trusted: inside one instrument the object never changes, so the number never changes.

The change in temperature is the temperature at the end minus the temperature at the start. If the thing cooled, that subtraction gives a negative number, and so does Q. It means the energy went out rather than in.

Last, how heat actually gets from one place to another. There are exactly three ways, and that is not a list somebody stopped writing. It is all of them.

Conduction is touching. Energy moves through matter that is staying put, by contact. A steel tumbler of hot chai and your fingers.

Convection is carrying. A fluid — a liquid or a gas — moves, and the energy goes with it. Hot air rising off a tawa. Blood carrying warmth from the middle of you out to your skin.

Radiation is shining. Energy crosses as light, or as the invisible light below red that bodies give off, and it needs nothing in between. The warmth on your face from an open fire across the room.

Anything that keeps something warm or cool acts on one of them directly. So when somebody tells you their product keeps heat in, ask two questions. Which of the three does it act on? And what happens to the other two?

ILLUSTRATION. In the calorimeter, a bigger temperature rise meant more energy.

Go and break it, with two things you have in the kitchen.

Take a steel tawa that weighs one kilogram. Take one kilogram of water in a pan. Put them on two identical burners and give each exactly the same amount of energy.

Use 4,186 joules, because it makes the first sum come out at one.

Multiply the mass by the specific heat to get the joules it needs for one degree.

```
1 times 4186 = 4186
```

You are giving it exactly that, so the water goes up by one degree Celsius.

Now the steel, the same way.

```
1 times 452 = 452
```

So 4,186 joules will take the tawa a long way up.

Divide the energy by what it needs for one degree. That is 4,186 divided by 452, which comes to 9.26 degrees Celsius, to two decimal places. Check it backwards.

```
452 times 9.26 = 4185.52
```

That is within half a joule of 4,186.

One thermometer reads a rise of one degree and the other reads a rise of more than nine. The temperature rise did not measure the heat. It measured the heat divided by what the object needed per degree, and the two objects needed very different amounts.

Inside one calorimeter the object never changes. So the joules it needs per degree never change either, and that is the energy equivalent you were handed.

Within one instrument, twice the rise is twice the energy. Across two instruments it is worth nothing, because the two heat capacities differ and nobody has said by how much.

Two hundred grams of chai are poured at 95 degrees Celsius and left. An hour later they are at 30. The change in temperature is the end minus the start.

```
30 minus 95 = -65
```

Now the energy, taking 0.2 kilograms of a liquid that is mostly water.

```
0.2 times 4186 = 837.2
837.2 times -65 = -54418
```

Minus 54,418 joules. The minus sign says it left.

Pour chai into a steel tumbler and pick it up. Your fingers know within a second. The steel stayed where it was and the energy came through it into your hand. Now pour the same chai into a clay kulhar and pick that up. You can hold it. Nothing about the chai changed. The matter between the chai and your fingers did.

Hold your palm a foot above a hot tawa, not touching it. The air over the tawa is being heated, it rises, and it brings the energy up to your hand.

Stand three feet from an open tandoor and face it. Your face warms, and the air in between did not have to carry anything.

Now the part that makes the three names useful rather than decorative. Read what the same section says about controlling them.

> *We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight).*

Choosing the material acts on how easily heat gets through it. Stopping the air moving acts on convection. Choosing the colour acts on radiation.

Take a sheet of paper and write the three routes down the left side. Then put the claim against each one in turn and ask what it does about that route.

Try it on a white roof, which is the book's own example. Colour acts on radiation, so the roof absorbs less of the sunlight falling on it. That is the only route the colour touches. The other two are downstream of it. A cooler slab conducts less heat down into the ceiling, and a cooler ceiling warms the air under it less. So naming the route tells you where the control bites, not which routes end up different. Whether the white roof is enough depends on how much of the heat was arriving as sunlight on the roof in the first place.

Figures quoted in this illustration are from [43].

WHERE THIS PICTURE BREAKS. The heat equation holds only while the substance stays the same substance in the same phase. Boil the water and it stops. The energy then goes into turning liquid into steam, and the temperature sits still while that happens. Any calculation that crosses a boiling point or a melting point has left this section behind.

Specific heat is not truly a constant. Over the ranges used here the variation is small enough to ignore.

A person is not one substance and has no single specific heat you can look up. Nothing here gives you a figure for heating or cooling a person, and none of these numbers should be carried across to one.

All three routes run at once in every real situation. Naming the route a control acts on does not settle what the other two then do.

This section gives no rates. It says by which routes heat moves and never how fast.

Sweating is not one of the three, and the book does not treat it as one. Any claim that turns on evaporation is outside what you are holding.

"Only three methods" is a claim about routes across a boundary. It is not a promise that every warming or cooling you meet is explained by naming one. A person getting warmer may be doing it by running a chemical reaction inside their own boundary. That is not a transfer across the boundary at all.

MUST KNOW POINTS FOR YOU.

- A bigger temperature rise does not mean more heat went in. Give a kilogram of steel and a kilogram of water the same energy. The steel rises more than nine times as far. So a thermometer reading is not an energy.
- Heat is energy on the way, not a thing an object holds, so refuse the sentence "this object contains so much heat". Heat is the name of a transfer.
- When anyone turns a temperature reading into an energy, ask two things. What heat capacity did they use? Where did it come from? Inside one machine the heat capacity is fixed, so the reading can be trusted. Across two machines a degree is not a degree. Rises set side by side with no heat capacity named are not energies set side by side.
- Water needs more than nine times the energy that iron or steel needs, for the same rise in the same mass. The figures are 4186 joules per kilogram per degree Celsius at 15.0 °C, against 452.
- The heat equation holds only while the substance stays the same substance in the same phase. It breaks at every boiling point, every melting point and every chemical reaction. When a calculation reaches one of those points, stop rather than substitute.
- A temperature is not an amount of energy, so never read a patient's temperature as a statement about energy gained or lost. One is in degrees and the other in joules.
- Heat moves by conduction, convection and radiation, and by nothing else, so make every claim about keeping warm or cool name its route. Once the route is named, the question becomes how much was arriving that way, and that can be answered.
- Teach the two-objects demonstration before the formula. After that they will not read a rise as an energy again.

EXERCISE 1 (retrieval). Without looking back, write down the difference between heat and temperature, with the unit of each.

Then name the three ways heat moves, and give one example of each from a kitchen.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A sheet for a "heat-retaining" food container says this.

> *The double steel wall keeps food hot for six hours, so no energy escapes and the food keeps all its calories.*

The container and the sheet are made up for this exercise. Say what is wrong, and say what you would ask for.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They have used a calorimeter and they are confident that a bigger temperature rise means more energy.

Teach them why that is true inside one instrument and false between two, so that they leave able to challenge a temperature-based energy claim.

PRACTICE. Eight problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. Work out the value of m times c times ΔT when m is 2, c is 500 and ΔT is 30.

2. Two parts, both on the same relationship.

1. Q is 30,000, m is 2 and c is 500. Work out ΔT.
2. Q is 9,000, m is 3 and ΔT is 6. Work out c.

3. A steel degchi weighing 1.2 kilograms holds 2 kilograms of water. Both are heated from 28 degrees Celsius to 100 degrees Celsius.

Take the specific heat of water as 4186 joules per kilogram per degree Celsius. Take that of steel as 452.

Work out the energy that went into the water, the energy that went into the steel, and the total. Keep the unit on every answer. Then say which of the two took more, and why.

4. A glass tumbler weighing 0.18 kilograms and a steel tumbler weighing 0.18 kilograms are each filled with hot chai. Each tumbler itself rises by 55 degrees Celsius.

Take the specific heat of glass as 840 joules per kilogram per degree Celsius. Take that of iron and steel as 452.

Work out how much energy each tumbler absorbed, with its unit. Then say which tumbler takes more energy out of the chai, and what that means for the chai in each.

5. Here is a worked answer. Find the step that broke.

```
500 grams of water is heated from 30 degrees Celsius to 80 degrees Celsius
the specific heat of water is 4186 joules per kilogram per degree Celsius
80 minus 30 = 50
500 times 4186 times 50 = 104650000
so the water absorbed 104,650,000 joules
```

6. Here is a worked answer. Find the step that broke.

```
a 1 kilogram steel tawa and 1 kilogram of water start at 30.0 degrees Celsius
each is put on an identical burner for the same length of time
after that time the tawa reads 39.3 degrees Celsius and the water reads 31.0
the tawa's temperature rose by much more than the water's
so much more heat went into the tawa than into the water
```

7. A poster in a gym says this.

> *Drink four litres of chilled water a day, straight from the fridge at 5 degrees. Your body has to warm every litre up to 37 degrees, and that is energy spent without moving a muscle.*

The poster and both temperatures are made up for this problem. Take the four litres as four kilograms of water.

Decide what to compute, compute it, and say what your answer does not establish.

8. Someone says this in a meeting about food testing.

> *The bomb in our calorimeter has a heat capacity of 893 joules per degree Celsius. We ran two foods. One gave a rise of 4.2 degrees and the other 2.1 degrees. So the first food holds twice the energy of the second.*

Decide what to compute, compute it, and say what your answer does not establish.

E5 · The cell, membranes, proteins and receptors

DEFINITION. A cell is the smallest unit of a living thing, and cells are the basic building blocks of all organisms.

The nucleus houses the cell's DNA. Ribosomes build proteins. Mitochondria are responsible for making adenosine triphosphate (ATP), which is the cell's main energy-carrying molecule.

A phospholipid has a head area that is hydrophilic, meaning water-loving: it mixes with water, and a tail area that is hydrophobic, meaning water-hating: it does not. In water these molecules arrange themselves with their hydrophobic tails facing each other and their hydrophilic heads facing out.

The plasma membrane controls the passage of organic molecules, ions, water and oxygen into and out of the cell. Sugars, amino acids, fatty acids and proteins are among those organic molecules.

A protein's shape is critical to its function, and many different types of chemical bond maintain that shape. Changes in temperature, acidity or exposure to chemicals may change the shape permanently, which loses the function. Enzymes are proteins that act as catalysts in biochemical reactions.

A ligand is a molecule that binds another specific molecule, in some cases delivering a signal in the process. Receptors are protein molecules in the target cell or on its surface that bind ligand, and a specific ligand typically has a specific receptor that binds only that ligand. The cytoplasm is the cell's whole region between the plasma membrane and the nucleus. It is mostly water but semi-solid. Internal receptors sit in the cytoplasm and respond to hydrophobic ligand molecules that are able to travel across the plasma membrane. Cell-surface receptors span the plasma membrane and perform signal transduction, by which a signal outside the cell is converted into a signal inside it. Binding changes the shape of the receptor, and the changed shape is what does the work. [56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77]

IN PLAIN TERMS. Everything alive is made of cells.

A cell is a bag of watery jelly, called the cytoplasm, with a skin round it. The skin has a name: the plasma membrane.

The nucleus is the compartment that holds the cell's instructions, which are written in its DNA.

The ribosomes are the machines that build proteins.

The mitochondria make a small molecule called adenosine triphosphate, written ATP, which the cell uses to carry energy from one reaction to another.

The membrane is built out of one kind of molecule, and that molecule has two ends that behave differently. One end mixes with water. The other end will not mix with water and mixes with grease instead.

Put a lot of them in water and they sort themselves out. The water-hating ends turn inwards and huddle together, away from the water. The water-loving ends turn outwards, where the water is. What you get is a sheet two molecules thick.

So a membrane is wet on both faces and greasy in the middle.

A molecule that dissolves in grease can slip through the greasy middle on its own. A molecule that dissolves in water cannot, and has to be carried across. The carrying is done by proteins that sit in the membrane and take things through.

A protein is a chain. The links of the chain are called amino acids, and there is a small fixed set of kinds. The chain folds up into a shape, and the shape is decided by the order of the links.

The shape is what the protein does. Not the chain, the shape. Change the shape and you change what it does, and a shape that is pulled apart far enough stops working altogether. Heat does that. So does acid.

An enzyme is a protein that makes one particular reaction go faster. It works because of its shape: the thing it acts on fits into it. Every enzyme speeds its reaction up.

Some proteins are shaped so that one particular molecule fits into them. The molecule that fits is called the ligand. The protein it fits into is called the receptor.

When the ligand sits in the receptor, the receptor changes shape. The new shape does something the old shape could not do.

A water-hating ligand crosses the membrane on its own, so its receptor can sit inside the cell. A water-loving ligand cannot cross, so its receptor sits in the membrane itself, sticking out on both sides. The ligand lands on the outside end and never enters. The inside end changes shape and carries the message in.

ILLUSTRATION. Nearly everyone who has met the membrane once remembers the phrase selectively permeable. Go and find it.

Open OpenStax Biology 2e, section 5.1, Components and Structure, at https://openstax.org/books/biology-2e/pages/5-1-components-and-structure. Open your browser's find box. Type the words selectively permeable and press enter.

Nothing. The section that describes the membrane does not use the phrase anywhere.

Now search the same page for the word exclude. One sentence comes up.

> *Cells exclude some substances, take in others, and excrete still others, all in controlled quantities.*

It says the membrane picks.

Take a glass of water. Pour in a spoonful of cooking oil. Stir it hard, then stop and watch. The oil comes back together and sits apart from the water.

Now read the phospholipid paragraph on the page you already have open, with that glass in front of you.

> *...phospholipids arrange themselves with their hydrophobic tails facing each other and their hydrophilic heads facing out.*

Draw the result on paper. Two rows of molecules. Tails pointing at each other in the middle, heads facing out on both sides.

Open section 9.1, Signaling Molecules and Cellular Receptors, at https://openstax.org/books/biology-2e/pages/9-1-signaling-molecules-and-cellular-receptors. Search inside it for the word hydrophobic.

> *...respond to hydrophobic ligand molecules that are able to travel across the plasma membrane.*

Water-hating molecules travel across the membrane.

A signal arriving at a cell is a molecule. The word for it is the ligand, which means the thing that binds. The protein it binds to is the receptor.

Take any claim that a substance acts on the body and ask three questions in order. What is the molecule? What does it bind? What changes when it binds?

The third question is the one that does the work. A claim that cannot answer it has described something that was present, not something that acted.

WHERE THIS PICTURE BREAKS. The glass of oil and water gets you the greasy middle and nothing else. A membrane is not a puddle.

Water-hating in, water-loving out is a rule with real exceptions, and the book says so itself. The membrane controls the passage of water too, and water is the most water-loving thing there is. Take the rule as telling you what usually crosses unaided, not as a law about every molecule.

The key-in-a-lock picture for a ligand and a receptor breaks in two places. A lock does not change shape when the key turns, and a receptor does, and that change is the entire point. And one key can open more than one kind of lock here: the same signal can reach several kinds of cell.

Binding and acting are not the same event. Something can sit in a receptor and do nothing, and something can sit in it and stop the usual ligand getting in.

This whole section is about what can happen and never about how much. Any question that has a number in its answer is outside it.

MUST KNOW POINTS FOR YOU.

- A substance being present in a tissue is not the same as it acting there. So ask which cells carry the receptor before you accept any claim about where a medicine works.
- Ask three questions of every mechanism claim, in this order. What is the molecule, what does it bind, and what changes when it binds.
- Binding and acting are two claims, not one. A molecule can sit in a receptor and do nothing, or sit in it and keep the usual ligand out.
- A protein's shape is what it does, so anything that changes the shape changes the job. So before you accept that a protein can be given by mouth, ask what the stomach does to it.
- Refuse any claim that a mechanism settles whether an effect is big enough to matter. A mechanism tells you what can happen and never how much.
- When a product claims to work at the cellular level, ask what it binds and what changes when it does. The phrase names no molecule and no receptor, so it commits to nothing.
- Teach the two receptor places before you teach a single drug. A water-hating signal crosses the membrane and meets its receptor inside; everything else is handled by a receptor built through the wall.
- An enzyme makes one reaction go faster, and its shape is the reason. Ask which enzyme, and ask how the rate was measured.

EXERCISE 1 (retrieval). Without looking back, write down what happens from the moment a signal molecule reaches a cell to the moment something changes inside it. Do it twice: once for a water-hating signal and once for a water-loving one.

Then say, in one sentence, why the two go different ways.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A company sheet for a herbal capsule says this.

> *The active molecule has been shown to bind the same receptor as the prescription medicine. It therefore has the same action, without the side effects.*

Say what is wrong, and say what you would ask for. The capsule and the sheet are made up for this exercise.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They can recite that insulin acts on cells, and they cannot say what "acts on" means.

Teach them the membrane and the receptor so that they leave able to place a drug they have never met.

E6 · Metabolism as chemistry happening in a body

DEFINITION. Metabolism is the sum of all of the chemical reactions that are involved in catabolism and anabolism. Catabolic reactions break down large organic molecules into smaller ones, releasing the energy contained in their chemical bonds. Anabolic reactions use the energy produced by catabolic reactions to build larger molecules from smaller ones. Both sets of reactions run continuously in the same body.

A living cell cannot store significant amounts of free energy. Part of the energy released is transferred to adenosine triphosphate, written ATP, and the remainder is given off as heat, which tissues and body fluids absorb.

ATP carries energy between the reactions that release it and the reactions that consume it. ATP is therefore a carrier and not a store.

Removing its terminal phosphate group releases energy, leaving adenosine diphosphate (ADP), and the released phosphate group activates the molecule it binds.

Food molecules are absorbed only after being reduced in size. All carbohydrates are absorbed in the form of monosaccharides. Triglycerides are broken into free fatty acids, which cross the intestinal membrane and are recombined into triglyceride molecules once across.

Not everything eaten is absorbed. The human body produces no enzyme that can break down most fibrous polysaccharides, such as cellulose.

Glucose is the body's most readily available source of energy. Excess glucose is either stored as an energy reserve in the liver and skeletal muscles as the complex polymer glycogen, or converted into fat, that is triglyceride, in adipose cells.

A triglyceride is the molecular form that fat takes both in food and in the body. Glucose beyond what the body needs can be turned into fat, as well as stored as glycogen. Triglycerides are stored in adipose tissue until they are needed.

Where catabolic reactions release more energy than anabolic reactions use, the body stores the excess energy by building fat molecules for long-term storage. Where catabolic reactions release less energy than anabolic reactions use, the body uses stored energy to compensate. Nothing in this account fixes the size of either flow, the rate at which any of it happens, or the fraction of energy that survives any step. [78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 93, 94, 95, 96, 97, 98, 99, 100, 101]

IN PLAIN TERMS. Metabolism is a word people use as though it were a thing you could have more or less of. It is not a thing. It is a name for all the chemical reactions going on in a body at once, added together.

Some reactions take a big molecule and break it into smaller ones. Reactions that go this way are called catabolism.

Other reactions take small molecules and join them into bigger ones. Building costs energy. Reactions that go this way are called anabolism.

A cell cannot hold a useful amount of loose energy, which the textbooks call free energy. So the energy goes into a small molecule first, and you have already met that molecule: adenosine triphosphate, written ATP.

Think of ATP as a coin. Breaking food earns coins. Building things spends coins.

Here is what the coin is, physically. ATP has three phosphate groups on it, sitting in a row. Knock the last one off and energy comes out, and that loose phosphate goes and sticks onto whatever the cell wanted to set going. What is left is called ADP, which is the same molecule one phosphate short.

Put a phosphate back on and you have ATP again. Paying for that is what the energy from food is for.

Not all the energy in a food molecule ends up in coins. Some of it comes off as heat, and the tissue and fluid around it soak that heat up.

ATP is not where a body keeps its energy. It is what a body moves energy with. Anyone who tells you the body stores its energy as ATP has mistaken the coin for the bank.

Food has to get in. In, through the wall of the gut and into your blood.

So the gut takes the food apart, cutting big molecules down into pieces small enough to pass through the lining. That passing through has a name, and the name is absorption.

What you eat and what you absorb are two different amounts.

Some of what is on your plate never crosses. Your body makes no tool for breaking down most plant fibre, so fibre goes in at one end and out at the other.

A figure on a packet is a figure about the food. It is not a figure about you.

Break a starch — the stuff that rice, wheat and potato are mostly made of — and you get single sugar molecules, called monosaccharides. A chain of them, such as starch, is a polysaccharide. The commonest of them, and the fuel a body reaches for first, is glucose.

Glucose crosses the gut wall into the blood, and that blood runs first to the liver. A cell that takes glucose in breaks it down and makes coins with the energy.

The small store is glycogen. Glycogen is a great many glucose molecules strung together, kept in the liver and in the muscles.

The large store is fat, and the fat in food and the fat in a body are the same kind of molecule. It has a name: triglyceride.

Triglyceride gets into a body two ways. You eat it. Or the body builds it, out of sugar there is more of than there is a use for right now.

The store is a tissue, and its name is adipose tissue. It is made of cells whose job is to hold triglyceride, and they hold it until it is needed.

A triglyceride is pulled apart into its two kinds of piece, fatty acids and glycerol. The pieces are then broken down like any other fuel.

So read the two stores as what they are. Filling them and emptying them is the system working. That is what a store is for.

Catabolism lets energy go. Anabolism uses it. ATP carries it from one to the other.

And when catabolism lets go of more than anabolism is using, the extra is put away, in the small store and the large one. When it lets go of less, the body draws on what was put away before.

All of it is a direction. None of it is a size. You do not know how much, or how fast, or how much of it survives each step, and nothing in this section will tell you.

ILLUSTRATION. The obvious next question is how much. So go and get the figure.

Open OpenStax Anatomy and Physiology 2e, section 24.1, Overview of Metabolic Reactions, at https://openstax.org/books/anatomy-and-physiology-2e/pages/24-1-overview-of-metabolic-reactions. Open your browser's find box. Search the page for the word percent.

It gives you a figure for how much of the released energy goes into ATP, and another for how much leaves as heat.

Find out where those two numbers came from.

Look at the end of the sentence. There is no reference number. Look at the paragraph around it. No study is named. Look at the foot of the page. There is no reference list.

A number is a different kind of claim from a mechanism, and it needs a different kind of source. This course does not take that figure, and neither should you.

Section 24.7 of the same book, Nutrition and Diet, will hand you a figure for how many calories make a pound of body weight. It is stated the same way, with no study behind it.

What a kilogram of change in a body is made of decides the answer. Nobody quoting the figure has told you what it is made of.

Take a plate you have actually eaten from. Rice, a bowl of dal, a spoonful of oil in the tadka, and a raw onion sliced on the side. Follow each of the four, using only what this section has given you.

Rice is mostly starch, which is a long chain of sugar molecules. The gut cuts the chain into single sugars, and glucose is the one to watch. It crosses the gut wall into the blood. The blood carries it to the liver.

What the body has no immediate use for does not stay in the blood. Some goes into the small store, as glycogen in the liver and the muscles. Some goes into the large store, turned into triglyceride in adipose tissue.

Now the dal, and follow the protein in it. Protein is a chain too, made of amino acids. The gut cuts it into single amino acids and those cross the wall. On the far side they go one of two ways: into new proteins, or broken down further to make coins like anything else.

Now the oil. Oil is triglyceride, and a whole triglyceride molecule will not cross the gut wall. So it is broken into its pieces first. The pieces cross. On the far side they are put back together into triglyceride again. That triglyceride goes into the body, to be used or stored.

Now the onion, and follow the fibre in it. Your body makes no enzyme that can break most plant fibre down, so it is not absorbed as it stands. It travels on down the gut. While it is inside it is doing something useful: it helps move everything else along. Whether any energy reaches you from it further down, this section does not say.

So look at what happens if you price that plate out of a food table. The table gives a figure for each of the four, onion included. Your total is a number about the plate. It is not a number about the person.

When somebody hands you a figure for what a person ate, ask three questions in this order. Where was it measured — at the mouth, or anywhere else? Was it measured at all, or looked up in a table? And what happened to it between the plate and the blood?

WHERE THIS PICTURE BREAKS. A cell does not keep a purse. It makes coins and spends them almost at once.

This section gives no rule about which store is drawn on first, or how fast either fills or empties.

The dead end at section 24.1 was found on 23 September 2026 in the edition of the book at that address. The habit — find the number, then go and find where it came from — survives the edition.

Everything in this section is a direction and none of it is a size. Any question whose answer is a number is outside it.

MUST KNOW POINTS FOR YOU.

- What a person eats and what a person takes up are two different amounts. Some of what is eaten never crosses the gut wall at all, because the body makes no tool for most plant fibre. So a figure for intake is a figure about food.
- Adipose tissue holds triglyceride until it is needed. So describe fat storage as a store doing its work.
- ATP is what a body moves energy with. It is not where a body keeps it. The stores are glycogen and triglyceride. Correct a trainee who says energy is stored as ATP, because that error spoils every later sentence they write about stores.
- Somebody will hand you a figure for how many kilocalories make a kilogram of body weight. Book 0 does not give you one. Ask what the change is made of before you accept any sum built on it.

- Catabolism and anabolism are directions, not departments. Both run in the same cells at the same time, and a person is never in one or the other. The answer is always both.
- Metabolism is the name for every chemical reaction in a body added together. So a drink that boosts your metabolism has named nothing at all. Ask which reaction, in which direction, in which tissue, and how the change was measured.
- Direction is the whole of what you have here, and size is no part of it. Spare energy goes to glycogen and to fat. Refuse any sum that needs one of those numbers and does not say where it was measured.

EXERCISE 1 (retrieval). Without looking back, write down what happens to a mouthful of rice. Start at the moment it is chewed and stop when its energy is either used or put away. Then do the same for a spoonful of oil.

Then say in one sentence what is true of both, and in one sentence what is different.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A district nutrition note contains this paragraph.

> *We recorded what each woman in the sample ate over three days and looked every item up in the food composition tables. The mean intake was well above what these women need. Energy intake is therefore not the problem in this block.*

The block, the sample and the finding are made up for this exercise. Say what is wrong, and say what you would ask for.

EXERCISE 3 (teaching). A journalist has three minutes and no jargon. They have been sent an advertisement for a drink that says it boosts your metabolism. They want to know whether that is a real thing.

Say what you would tell them, and give them one quotable sentence.

E7 · Genes, DNA and inheritance in outline

DEFINITION. DNA is a molecule built from repeating units called nucleotides. Each nucleotide has three parts: a base containing nitrogen, a sugar with five carbons, and a phosphate group. The base is one of four — adenine, guanine, cytosine and thymine, written A, G, C and T.

A DNA molecule is two strands twisted around each other into a right-handed helix. A pairs with T, and C pairs with G. Bases that pair in this way are called complementary. Because the pairing is fixed, the order of the bases along one strand fixes the order along the other.

Genes are carried on chromosomes. An organism holding two matching copies of each chromosome, one inherited from each parent, is diploid. The gene that encodes a protein determines the order of amino acids in that protein.

Where one gene exists in more than one version, the versions are called alleles.

The traits an organism visibly expresses are its phenotype. Its underlying genetic make-up, counting both the alleles that are expressed and those that are not, is its genotype.

The account of alleles, genotype and phenotype above describes the case in which a single gene controls a single characteristic. A gene fixes the order of amino acids in a protein. It does not by itself fix when that protein is made. Whether the protein works once made depends on the conditions it sits in. [102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118]

IN PLAIN TERMS. Start with the molecule. DNA is long, and it is built from small units joined end to end. Each unit carries one part that matters here: a base. There are four bases and they have names. Adenine, guanine, cytosine and thymine. Nobody says those out loud. They are written A, G, C and T, and that is how you will meet them.

So a strand of DNA is a long line of letters. The order of those letters is the information.

DNA is not one strand. It is two, twisted around each other, which is why you have seen it drawn as a spiral staircase.

Each letter on one strand sits opposite one particular letter on the other. A always faces T. C always faces G. A pair that goes together this way is called complementary — a word meaning each one completes the other.

If you know one strand, you know the other. That is how a cell makes a second copy when it divides, and it is how a copy gets into an egg and a sperm.

What a gene does is this: it fixes the order of the amino acids in one protein. The protein is the thing that goes and does the work in the body.

Genes sit along structures called chromosomes, in a fixed order. The information a gene carries is written in the DNA. The cell turns the information in its DNA into the order of amino acids in a protein.

You get two copies of each chromosome, one from your mother and one from your father. An egg or a sperm carries only one copy of each chromosome, so when they join, the child has two. So you carry two copies of each gene, not one.

Where a gene comes in more than one version, the versions are called alleles.

What you can see about a person — the traits they show — is their phenotype. What they actually carry in their DNA, including the alleles that are not showing, is their genotype.

For some genes, one allele is dominant and the other recessive. Dominant means it shows whenever it is present. Recessive means it shows only when both copies are that version.

So a person with two dominant copies and a person with one of each look exactly alike. Same phenotype. Different genotype. You cannot get from what you see to what they carry.

The dominant-and-recessive picture is about one gene controlling one characteristic. It is describing a case.

So do not carry the picture over to a human trait and assume it holds.

A gene fixes the order of parts in a protein. It does not fix when that protein gets made.

A signal reaches a cell and crosses into it. Inside, it lands on a receptor, as the section on the cell showed you. What is new here is what some of these receptors then do: they set the making of a protein going. That is one route.

Then there is what happens after the protein exists. Whether it works depends on where it is. Heat, acidity and the chemicals around it can all change its shape. A protein that has lost its shape has lost its job.

So the chain from a letter in DNA to a thing that happens in a person has several steps in it. “It is in her genes” names the first step and says nothing about the rest.

ILLUSTRATION. Open OpenStax Biology 2e, section 14.2, DNA Structure and Sequencing, at https://openstax.org/books/biology-2e/pages/14-2-dna-structure-and-sequencing. Search inside the page for the word gene.

You will find it twice and neither one is a definition. The section that tells you what DNA is made of, and how the two strands pair, never says what a gene is.

Now do the same search on the genetics section. Open section 12.2, Characteristics and Traits, at https://openstax.org/books/biology-2e/pages/12-2-characteristics-and-traits. Genotype and phenotype are defined there.

When you want a definition, search the page for the word before you cite it. A section title tells you what a page is about. It does not tell you what the page says.

Take a gene that comes in two versions and call them A and a. Capital A is the dominant one, small a the recessive one. Everyone carries two copies, so there are three ways to be:

what the person carries	what shows
A and A	the A trait
A and a	the A trait
a and a	the a trait

Two different people, carrying different things, showing the same thing. Look at them and you cannot tell them apart.

A person is in front of you showing the A trait. What do they carry? You do not know. They are the first row or the second row and nothing you can see decides it.

Take a sentence you will hear in a clinic in Chennai or in Chandigarh, said kindly and meaning well.

> *It runs in her family, so it is genetic, so there is not much point.*

It runs in her family. That is an observation and it may well be right. Families share more than DNA. They share a kitchen, a neighbourhood, a food budget, the hours people work and the distance to somewhere safe to walk. A trait appearing in several members of one family is consistent with shared alleles. It is just as consistent with shared circumstances. The observation on its own does not separate the two.

So it is genetic. This is the step that does not follow, and nothing in this section supports it. This section gives you what a gene is and what an allele is. It gives you no way at all to move from a family pattern to a cause.

So there is not much point. This is the step that does harm, and it is wrong for a reason you now hold. A gene fixes the order of parts in a protein. It does not fix what happens next. A signal can reach a cell, cross into it, and set the mak-

ing of a protein going. And whether the protein works once made depends on the heat, the acidity and the chemicals around it. There are several steps between the letters and the person, and every one of them answers to something.

So the reply is not "no, it is not genetic". You do not know that either. The reply is that the sentence has three claims in it. Only the first has been observed. The third does not follow from either of the others.

WHERE THIS PICTURE BREAKS. Do not take the three-row table to a trait and assume it has three rows.

Not every gene works by one version hiding the other.

The A and a labels are letters on a page. They carry no quantity.

MUST KNOW POINTS FOR YOU.

- Carrying a gene variant is one claim. Having a fixed outcome is another, and the second does not follow from the first. A gene fixes the order of parts in a protein. It does not fix when that protein gets made.
- A trait showing up in several members of one family is an observation, not a cause. Families share alleles and they also share a kitchen, a work schedule, a food budget and a neighbourhood.
- Two people with different genotypes can show the same phenotype, so what you can see about a person does not fix what they carry.
- What this section gives you about dominant and recessive alleles is one case. So do not use it to say how many genes lie behind a trait in a person. And do not use it to say that a trait is or is not largely inherited. Both of those need a study that measured it.
- Call a variant a variant. An allele is one of the versions a gene comes in, and the neutral word is the accurate one. Calling a person's variant a defect or a fault adds a judgement the biology does not carry. A patient hears that word exactly as it sounds.
- Search the page for the word before you cite a source for a definition. A title tells you the topic and not the contents.
- When a journalist asks whether obesity is genetic, do not answer yes or no. Say that a gene sets the order of parts in a protein. Say that several steps sit between that and a person, and that each step answers to something. Then say what would settle the actual question, which is a study and not an opinion.
- One sentence hides three claims: "it runs in the family, so it is genetic, so there is not much point". Teach a trainee to write the three out on separate lines. Set out separately on the page, the second and third are visibly unsupported.

EXERCISE 1 (retrieval). Without looking back, write down the four bases by their letters. Say which pairs with which. Then say what that pairing lets a cell do.

Then say, in one sentence each, what a genotype is and what a phenotype is.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A district programme note contains this paragraph.

> *Three of the four adults in this household are living with obesity. Obesity is known to be strongly genetic. Counselling on diet in this household is therefore unlikely to be a good use of the worker's time, and the household should be deprioritised.*

Say what is wrong with it. Say which sentence you would remove, which you would change, and what you would ask for before writing anything in its place.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They have just told a patient that her weight is "in her genes".

Teach them why that sentence, said in a clinic, is doing something different from what they intended. They should leave with a replacement sentence they can actually say.

E8 · Organs and systems that matter for metabolism

DEFINITION. Absorbed material leaves the gut by two different routes. Water-soluble nutrients enter the capillary blood (the smallest blood vessels) in the villi, the tiny fingers lining the gut, and travel to the liver via the hepatic portal vein. Lipid-soluble nutrients diffuse through the plasma membrane, are packaged inside the cell, and enter the lacteals, the lymph vessels of the villi. Lymphatic vessels then carry them to the general circulation, the blood that goes round the whole body.

The liver is the largest gland in the body. Besides its role in digestion it has many roles in metabolism and regulation. Under the hepatic portal circulation, all blood from the alimentary canal (the gut tube) passes through the liver. Drugs and toxins are absorbed along with the nutrients. After processing what the blood brings, the liver releases the nutrients other cells need back into the blood. It stores excess glucose as glycogen and converts glycogen back into glucose.

The liver's digestive role is to produce bile and export it to the small intestine. The gallbladder stores, concentrates and releases that bile.

The pancreas has two separate functions. As an exocrine gland it produces pancreatic juice, containing digestive enzymes and bicarbonate ions, and delivers it to the small intestine through a duct. As an endocrine gland it secretes hormones directly into the surrounding fluid. Its pancreatic islets, formerly known as the islets of Langerhans, secrete glucagon, insulin, somatostatin and pancreatic polypeptide. The alpha cell produces glucagon and the beta cell produces insulin. Receptors located in the pancreas sense blood glucose levels, and the pancreatic cells secrete glucagon or insulin accordingly.

An endocrine organ secretes a chemical, the hormone, into the extracellular fluid, the fluid around its own cells. From there the blood vessels transport hormones throughout the body. A hormone binds to receptors on target cells, inducing a characteristic response. Adipose tissue is among the tissues known to produce hormones.

Elevated blood glucose stimulates the release of insulin. The primary function of insulin is to facilitate the uptake of glucose into body cells. Insulin also stimulates the liver to convert excess glucose into glycogen for storage, and promotes triglyceride and protein synthesis. Skeletal muscle cells and adipose cells are the primary targets of insulin.

Low blood glucose stimulates the release of glucagon. Glucagon stimulates the liver to convert its stores of glycogen back into glucose, which is released into the circulation. It stimulates the liver to take up amino acids from the blood and convert them into glucose. It also stimulates lipolysis, the breakdown of stored triglycerides into free fatty acids and glycerol.

Nothing stated here fixes how much of anything any of these organs handles. [119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158]

IN PLAIN TERMS. The section on the cell and the section on metabolism gave you a cell, and then the chemistry that goes on inside it. This one is about which parts of a body do which part of that work.

Six of them matter here. The gut, the liver, the pancreas, fat tissue, muscle and the brain.

The gut is a tube. Its proper name is the alimentary canal.

Until a molecule passes through the wall of the tube and into your blood, your body can do nothing with it.

So the gut has one job. Cut food down until the pieces are small enough to go through the wall, and then let them through. It is covered in tiny fingers, called villi, and the pieces cross into the vessels inside those fingers.

What dissolves in water and what dissolves in fat leave the gut by two different roads.

Whatever dissolves in water goes into the smallest blood vessels there, the capillaries. That blood runs straight to the liver, along a vein called the hepatic portal vein.

Whatever dissolves in fat goes into a different set of vessels, the lymph vessels. In the villi they are called lacteals. They empty into the blood further along, past the liver, into the general circulation, the blood that goes round the whole body. So fat reaches the rest of the body before the liver has looked at it, and sugar does not.

Now the liver.

Every drop of blood leaving the gut goes through the liver before it goes anywhere else. All of it. This route is called the hepatic portal circulation.

It sits in the doorway.

It also holds part of the small store. Sugar that nobody needs this minute is packed away there as glycogen, which you met in the section on metabolism.

And it makes bile, which it sends down into the gut, where it breaks large blobs of fat into tiny ones.

Next to the liver is a small bag called the gallbladder. It holds bile, concentrates it and lets it out.

Now the pancreas, and it does two completely separate jobs. Keep them apart in your head, or every sentence you ever read about the pancreas will confuse you.

The pancreas makes a juice full of enzymes and sends it down a tube into the gut, where the juice helps take food apart.

Scattered through the pancreas are small islands of another kind of cell, called islets. They put chemicals straight into the blood, and two of those chemicals run the rest of this section.

A chemical made in one part of the body that affects other parts some distance away is called a hormone.

A hormone starts in a gland, which is an organ whose job is to make it. The gland lets it out into the fluid around its own cells, called the extracellular fluid. From there the blood carries it on, and the blood goes everywhere.

A hormone does not travel to a destination. It goes everywhere, past everything.

It washes past every cell in the body. It does something only where there is a receptor shaped to hold it.

Two things follow, and both matter.

First, if you want to know where a hormone acts, never ask where it went. Ask which cells carry the receptor.

Second, the same hormone can do different things in different places. What happens next depends on the cell it landed in, not on the hormone. So “this hormone does that” is an unfinished sentence until it names a tissue.

Eat, and sugar crosses the gut wall into the blood.

The islets notice. Blood sugar, meaning the glucose in the blood, is high, and one kind of cell in them, called a beta cell, releases insulin.

Insulin’s main job is to move glucose out of the blood and into cells. It lands on a receptor. The cell responds by bringing glucose-carrying proteins to its own surface, where they can take glucose in.

Muscle cells and fat cells are the ones insulin acts on most.

Blood sugar falls.

The same islets notice that too. A different cell in them, called an alpha cell, releases a different hormone. Its name is glucagon.

Glucagon runs the other way, and mostly at the liver. It tells the liver to turn its glycogen back into glucose and push that glucose out into the blood. It tells the liver to build new glucose out of amino acids.

Sugar up, insulin, stores fill. Sugar down, glucagon, stores empty.

Neither half is the good half. A store that only ever filled would not be a store.

Glucose is the fuel every cell prefers. But some organs can use nothing else, and the brain is the one that matters. So blood sugar must not be allowed to fall too far. That is what glucagon is there for.

The brain does not need insulin to take glucose in. Neither do red blood cells, the liver, the kidneys or the lining of the gut. They take glucose out of the blood whether insulin is there or not.

Muscle and fat wait for insulin. The brain does not wait.

Adipose tissue, the large store from the section on metabolism, makes hormones of its own and puts them into the blood. That makes it an organ of the same kind as the pancreas.

Muscle is the other tissue insulin acts on most. It is a store too, and it holds glycogen, as the liver does.

All of it is what each part does. None of it is how much.

ILLUSTRATION. The rule is this. Insulin is what lets sugar into cells.

The brain runs on sugar. So the brain should be the first thing to fail, and it should fail for want of a way in.

Now go and check it.

Open OpenStax Anatomy and Physiology 2e, section 17.9, The Endocrine Pancreas, at https://openstax.org/books/anatomy-and-physiology-2e/pages/17-9-the-endocrine-pancreas. Open your browser's find box. Search inside the page for insulin receptors.

> *Red blood cells, as well as cells of the brain, liver, kidneys, and the lining of the small intestine, do not have insulin receptors on their cell membranes and do not require insulin for glucose uptake.*

The brain is on it. So is the liver.

Read to the end of the sentence: "do not require insulin for glucose uptake". That is the half to keep. The same page says insulin makes the liver store glycogen, so the liver is not deaf to insulin. What it does not need insulin for is taking glucose in.

The rule is right about some cells and silent about others, and nobody told you which were which.

Search the same page for primary targets.

> *...skeletal muscle cells and adipose cells are the primary targets of insulin.*

Muscle and fat wait for insulin. The brain does not wait at all.

Open section 24.2, Carbohydrate Metabolism, at https://openstax.org/books/anatomy-and-physiology-2e/pages/24-2-carbohydrate-metabolism, and search for only glucose.

> *Certain key organs, including the brain, can use only glucose as an energy source; therefore, it is essential that the body maintain a minimum blood glucose concentration.*

The brain can burn nothing but glucose. It takes glucose out of the blood whether or not insulin is there. So nothing protects the brain when blood sugar runs low except putting more sugar into the blood.

And that is why there are two hormones and not one. One to put sugar away when there is plenty. One to get it back out when there is not.

Here are the two, side by side.

hormone	released when blood sugar is	made by	what it does to the stores

insulin	high	beta cells	fills them
glucagon	low	alpha cells	empties them

Take a person who eats a meal at one o'clock and does not eat again until eight. Follow the seven hours.

One o'clock. Sugar goes into the blood, and the blood carrying it runs first to the liver.

Blood sugar rises. Beta cells release insulin, and the insulin goes everywhere in the blood.

Muscle cells and fat cells bring glucose carriers to their surface and take sugar in. The liver packs spare sugar away as glycogen.

Four o'clock. The brain has not stopped, and neither has anything else, so blood sugar starts to fall.

Alpha cells release glucagon.

The liver turns its glycogen back into glucose and pushes it out into the blood. It also starts building new glucose out of amino acids.

Eight o'clock. Blood sugar is still in a workable range and the brain has never gone short.

The store filling and the store emptying are both the mechanism working. Neither one is a failure, and neither one is a success.

Everything absorbed from the gut in water goes through the liver before the rest of the body sees it. All of it. So whenever somebody tells you that a substance taken by mouth does what that substance does when it is injected, you have one question ready.

What did the liver do to it first?

The answer is often that nobody checked.

WHERE THIS PICTURE BREAKS. Six organs is a working list and not the whole body.

The table gives no amount, no speed and no threshold at which anything happens.

Insulin and glucagon are not the only hormones the islands make, and they are not the only hormones acting on these stores.

The brain or the liver not needing insulin does not mean it is untouched by insulin. What the source establishes is narrower: it takes glucose without it.

The two roads out of the gut describe a route and nothing else.

The seven hours are a shape, not a measurement. The order of events is what the sources carry.

Any question whose answer is a number is outside this section.

What changes when part of the loop stops working is a later subject, and nothing here settles it.

MUST KNOW POINTS FOR YOU.

- "Insulin is what lets sugar into cells" is wrong for several of the cells that matter most. The brain, the liver, the kidneys, red blood cells and the lining of the gut do not need insulin to take glucose in. So make any claim about insulin name its tissue, and say whether it is about uptake or about something else.
- A hormone does not travel to a target. It goes everywhere in the blood and acts where a receptor holds it. Ask which cells carry the receptor, and what those cells do when it binds.

- So "this hormone causes that" is unfinished until it names a tissue. Ask for the tissue before you accept the claim.
- Everything taken up from the gut in water passes through the liver before the rest of the body sees it. All of it, every time.
- A store filling and a store emptying are both the mechanism working. Insulin fills, glucagon empties, and both run in the same person every day. So when weight comes back after it fell, say what the section has given you. A store fills under one signal and empties under another, and that loop runs the same way in every person. This section does not tell you which signals, or by how much. And it never licenses a sentence about the person's character.
- Adipose tissue makes hormones of its own and puts them into the blood. That makes it an organ with a job, not a bag that things end up in.
- The pancreas does two separate jobs and a trainee who merges them misreads every pancreas sentence after that. One job sends a juice full of enzymes down a tube into the gut. The other puts hormones straight into the blood.
- Nothing here carries an organ's share of energy use, a concentration in blood, or a rate. Refuse any argument that uses it that way.

EXERCISE 1 (retrieval). Without looking back, name the six organs or tissues this section covers. Write one line for each, saying what it does with energy.

Then say what a hormone is, and how it gets to the place where it acts.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A patient information leaflet contains this paragraph.

> *Fat cells are simply storage bags. Once they are full, the body has nowhere left to put the extra energy, and that is when the problems start. The answer is to stop filling them.*

The leaflet is made up for this exercise. Say what is wrong with it, sentence by sentence, and say what you would put in its place.

EXERCISE 3 (teaching). A first-year resident has ten minutes and a whiteboard. They have just told a patient that insulin is the key that unlocks the cell and lets sugar in.

Teach them insulin and glucagon so that they never say a sentence about insulin again without naming a tissue.

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Part F · Reading and reasoning

F1 · Reading a table

DEFINITION. A table presents values in a grid, so that every value is identified by the row and the column it sits in.

Reading a table means settling four things before reading any number. What does one row stand for? What does each column report about it? What units and denominators do the columns use? Where did the numbers come from? [1]

IN PLAIN TERMS. A number on its own tells you nothing.

You have to settle four things first. What does one row stand for — one person, one household, one whole state? That is called the unit of observation. What does each column report about that row? What units does a column use, and if it is a percentage, what is it a percentage of — its denominator? And where did the numbers come from, who counted them, and how?

Forty-five could be a headcount, a percentage, or a rank.

Illustration.	State	Women surveyed	Overweight or obese (%)
A	42,100	24.1	2019-21
B	18,750	39.6	2019-21

States A and B and all four figures in this table are made up for this section.

Ask first what one row stands for. Run your finger along the first row under the headings. It is one state, not one woman. So the figure 24.1 describes nobody in particular.

Ask next what each column reports. The second column counts people. The third column, sitting right beside it, looks like the same kind of thing and is not. It is a percentage — a fraction, always of something. Of what? Of the count in the column next to it.

'Overweight or obese' is one category, made by cutting a measurement at chosen values. It counts the overweight and the obese together, so it is larger than 'obese' alone. The heading does not say whose cut-offs were used, and that is a question to ask.

Look at the last column and read it slowly. It is not a single year but a span, 2019 to 2021, telling you the fieldwork ran across those years and not on one date.

Search the table for a line naming the source. There is none here.

Now put the question you were going to ask anyway: is state B worse than state A?

You cannot tell from this table alone. The two percentages differ, and that much sits on the page. But you have not been shown who was surveyed in each state, how those women were chosen, or how precise either figure is.

Say only what you can defend: the two reported figures differ.

WHERE THIS PICTURE BREAKS. Two people can build two honest tables from the same underlying numbers and leave two opposite impressions. Neither has lied. A table settles what was measured and reported. It does not settle whether the comparison you want to make is a fair one.

MUST KNOW POINTS FOR YOU.

- Before comparing two percentages across rows, check the count sitting beside each one. Twenty-four per cent of 42,100 women and twenty-four per cent of two hundred women are not the same evidence. The section on sampling says why: for a random sample, the typical error shrinks with the square root of the count. For a sample that was not random, no count rescues it.
- Ask the four questions in order, every time, before you look at any value.
- In an Indian national survey table, check the count in every row: states need not have been sampled equally.
- When a table arrives in a ministry deck or a press release, somebody chose its rows and columns before you saw it. Ask what was left out.
- If asked which of two percentages is higher, do not stop at naming the bigger number. Say whether the table supports a comparison at all, and name what is missing.
- This section gets you only to what a table says. It does not tell you whether the numbers are right, how precise they are, or whether a gap is real.

EXERCISE 1 (interpretation). In the table above, what exactly does the number 39.6 describe? Write one sentence, and make sure it contains the unit of observation, the denominator and the time span.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). Here is what a colleague writes. “State B has 15.5 percentage points more obesity than state A.” Name two things wrong with that sentence, read as a description of this table.

F2 · Reading a figure, and the ways a figure misleads

DEFINITION. Reading a figure means settling four things before its shape. What one mark stands for. What each axis measures, and in what unit. Where each axis starts, and how it is spaced. Where the numbers came from.

A length carries a value only when it is measured from zero. So the value axis of a bar chart must start at zero. The value axis is the one a bar's length is read against: the side axis when bars stand up, the bottom axis when they lie flat.

A figure can mislead while every number on it is correct. A bar chart's value axis may start above zero. Pictures or areas may stand in for lengths. Bins or time steps may be unequally spaced. The window of time shown may be chosen to suit a claim. Counts may be shown where rates are needed. And two vertical axes may be used, so that where the lines cross is set by the two scales chosen. [2, 3, 4, 5, 6, 7, 8, 9]

IN PLAIN TERMS. **What one mark is.** A mark is anything drawn that stands for one number: a bar, a point, a slice of a pie, a picture. Find what one mark stands for. It may be one group, one district or one year. This is the question you asked about one row of a table.

What each axis measures, and in what unit. Read the label along the bottom and the label up the side. If an axis is a rate or a percentage, ask what it is out of, as you did for a table's columns.

Where each axis starts, and how it is spaced. Read the lowest number printed on each axis. Then read along it, and check that equal steps on the page are equal steps in the numbers.

Where the numbers came from. Look under the figure for a source. With no source, you have nothing to check it against.

Only then look at the shape.

Next, see how a mark carries its number. There are three common ways, and each can mislead in its own way.

By position. A point on a line chart carries its value by where it sits. You read it by looking across to the numbers on the side.

By length. A bar carries its value by how long it is. Your eye compares bars by comparing their lengths. So a bar's length has to be its value, and that holds only when the bar starts at zero.

By area. A picture, a bubble or a slice of a pie carries its value by how much of the page it covers. The eye takes in area more readily than height. A pie chart is a circle standing for a whole. Each slice's share of the circle is its share of the total, so a slice shows a share, not an amount.

So a bar chart must start at zero, and a line chart need not. The section on graphs told you to redraw an alarming line from zero before you repeat it. Keep doing that. It is a check on the impression a slope gives, not a rule of honesty. A line on a printed non-zero axis is honest; a bar on one is not.

The two rules do not fight. A line chart asks you to read positions, and a printed axis starting at 400, stated in the caption, tells you where they are. A bar chart asks you to compare lengths, and a bar cut off at 400 has the wrong length. A line's steepness is still somebody's choice, as the section on graphs showed. So read the size of a change off the numbers, not off the slope.

Double the height and the width of a picture, and it covers four times the page. The number you multiply every length by is called the scale factor. The area goes up by the scale factor squared.

Here are six ways a figure misleads while every number on it is right.

A bar chart whose side axis does not start at zero. Read the lowest number on the axis. If it is not zero, subtract it from each value to see how long each bar was drawn.

Pictures in place of bars. Compare the numbers, never the pictures. A picture scaled both ways shows the scale factor squared where the numbers show only the scale factor. Two pie charts drawn the same size for two years hide any change in the total.

Unequal spacing. A bar can stand for a range of values grouped together, called a bin. Read the range printed under each bar, and check that each covers the same span of values. A label such as “over 80” does not say where its range ends, so ask.

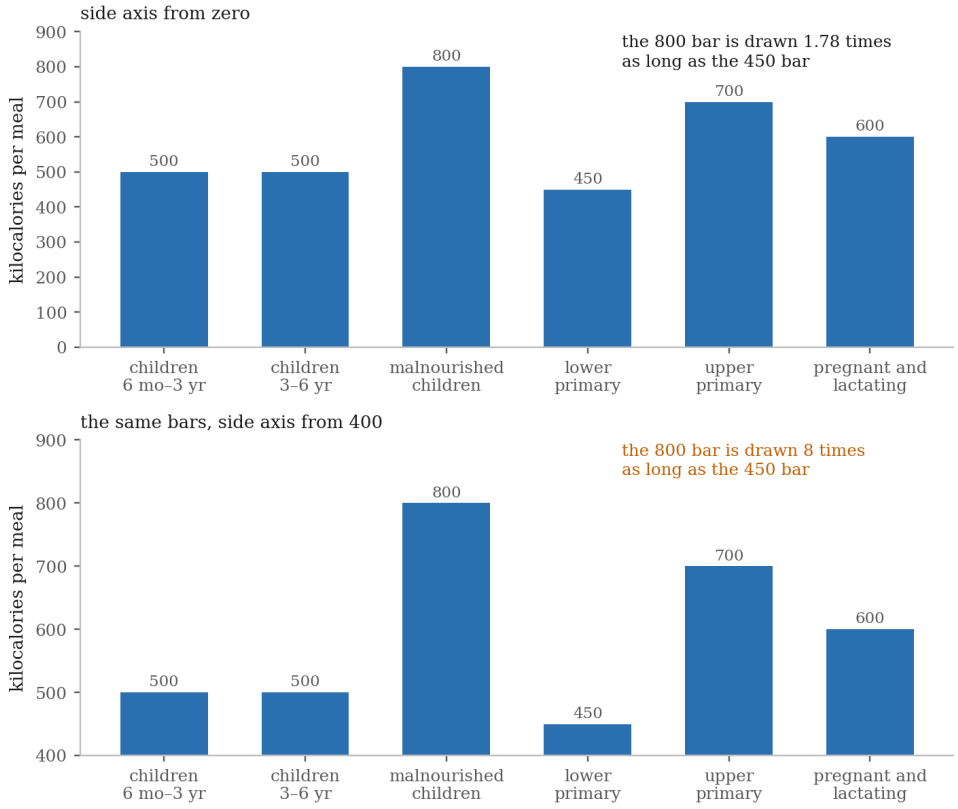
A bar covering a wider span catches more values. It may be drawn wider, or drawn the same width and taller. Either way it cannot be compared with the others as it stands. To compare it fairly, divide its count by how many of the ordinary spans it covers. That gives the count for each ordinary span, on average.

A time axis can also change its step part way across, say from years to months. Read the date under each point and work out the gap to the next.

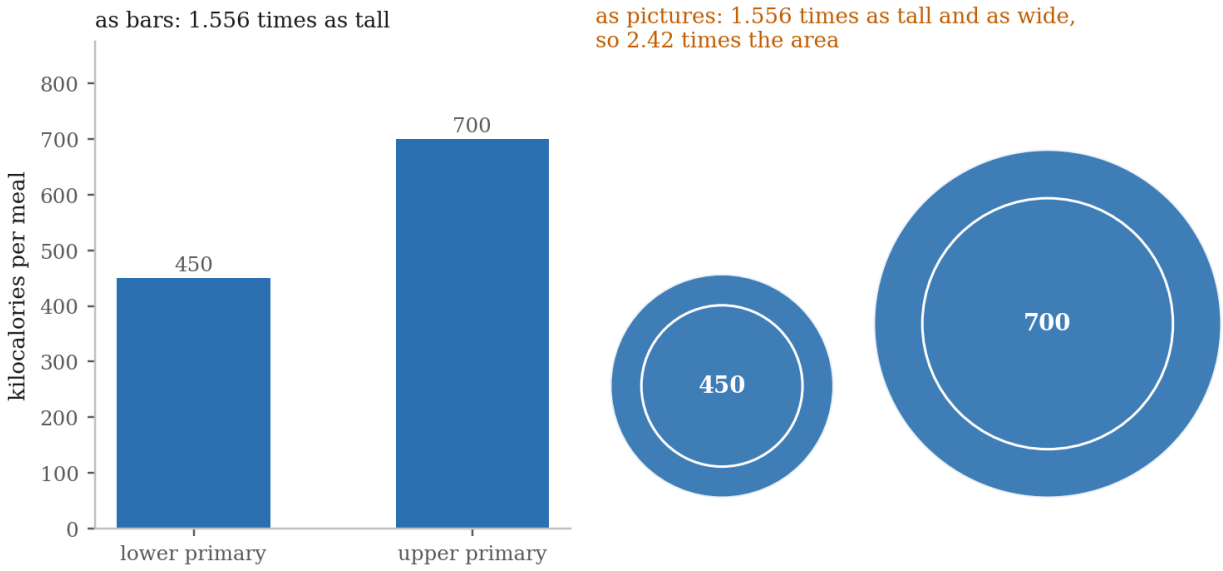
A chosen window. A chart over time starts and ends where somebody chose. That stretch is its window. A short window can show a fall inside a long rise. Ask for the whole series, and ask why the chart starts where it does.

Counts where rates are needed. A district with more people has more of almost everything. Divide each count by the population before you rank, as in the section on rates.

Two side axes on one figure. One line is read against the left axis and one against the right. Where the two lines cross depends on how the two axes were chosen. Read each line against its own axis, and never read the crossing as the moment the two were equal.



The six energy values of Schedule II drawn twice. In the upper panel every bar starts at zero, so each length is its value and the 800 bar is under twice the 450 bar. In the lower panel the side axis starts at 400, so the same 800 bar is drawn eight times as long as the 450 bar. Every printed number is correct in both panels.



Two meal energies, 450 and 700 kilocalories, drawn as bars and as pictures. The larger picture is 1.556 times as tall as the smaller and, to keep its shape, 1.556 times as wide, so it covers 2.42 times the page for 1.556 times the energy.

ILLUSTRATION. Schedule II of the National Food Security Act, 2013 sets the energy that each meal or ration under the Act is to give.

the group	the meal	kilocalories
children, 6 months to 3 years	take home ration	500
children, 3 to 6 years	morning snack and hot cooked meal	500
children, 6 months to 6 years, who are malnourished	take home ration	800
lower primary classes	hot cooked meal	450
upper primary classes	hot cooked meal	700
pregnant women and lactating mothers	take home ration	600

Picture these six as a bar chart on a slide. The side axis starts at 400 kilocalories.

Each bar is drawn up from 400, so its length is its value minus 400.

```
450 minus 400 = 50
800 minus 400 = 400
```

```
400 divided by 50 = 8
```

The long bar is drawn eight times as long as the short one.

```
800 divided by 450 = 1.7777…
```

About 1.78, which is under twice. Every number on the slide is correct, including the 400 at the bottom.

Redraw the chart with the side axis starting at zero. Each bar is now as long as its value. Or keep the axis from 400 and draw each value as a dot with no bar under it. A dot carries its value by position.

That is why the zero rule belongs to bars.

Take the hot cooked meals: 450 kilocalories for lower primary classes and 700 for upper primary. An infographic — a chart decorated with pictures — draws each meal as a plate. The bigger plate is made as tall as its value requires.

```
700 divided by 450 = 1.5555…
```

The bigger plate is 1.556 times as tall. A plate stretched upward alone would look wrong, so it is widened by the same amount.

Whatever its shape, a plate made 1.556 times as tall and 1.556 times as wide covers 1.556 times 1.556 as much of the page.

```
1.556 times 1.556 = 2.42
```

The bigger plate covers 2.42 times the page, for 1.556 times the energy.

Before the shape, ask yourself how each mark carries its number. A point carries it by position, so read the scale. A bar carries it by length, so find the zero. A picture carries it by area, so compare the numbers and ignore the pictures.

Figures quoted in this illustration are from [10].

WHERE THIS PICTURE BREAKS. The eight times belongs to an axis starting at 400. Start at 440 and the same two bars come apart far further.

```
800 minus 440 = 360
450 minus 440 = 10
360 divided by 10 = 36
```

The closer the start sits to the shortest bar, the larger the distortion.

Stretch a picture upward only, and its area grows in step with its value. It will look stretched, which is its own warning.

MUST KNOW POINTS FOR YOU.

- "Every chart must start at zero" is the error. A bar chart must, because a bar's length is its value. A line chart need not, because a point carries its value by position, read off the printed scale.
- Settle four things before you look at the shape. What is one mark? What does each axis measure, and in what unit? Where does each axis start, and how is it spaced? Where did the numbers come from?
- A picture enlarged by some factor in height and width covers that factor squared. When you are shown an infographic, compare the numbers and never the pictures.
- When a bar chart in a meeting starts above zero, ask for it redrawn from zero before anybody discusses the size of a gap.
- Ask for each count divided by its population before any district is named as the worst.
- On a figure with two side axes, where the lines cross is set by the two scales somebody chose.
- Teach a trainee to ask one question of every mark: is its number carried by position, by length or by area?

- These checks catch a figure that draws true numbers unfairly. They cannot catch a wrong number, and they cannot tell you whether a real gap matters.

EXERCISE 1 (interpretation). Look at the lower panel of the first figure: the six Schedule II values as bars, on a side axis starting at 400 kilocalories. A colleague says a line in the caption, stating that the axis starts at 400, would make it acceptable.

Say whether it would, and why. Then describe two honest ways to redraw it.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (critique). A budget document shows two pie charts of the same size, one for each of two years. Each is split into spending on grain, on cooking and on transport. The presenter says this.

> *Look how the grain slice has shrunk. We are spending less on grain.*

In the first year the scheme spent 100 crore rupees, 40 per cent of it on grain. In the second year it spent 150 crore rupees, 30 per cent of it on grain. The scheme and all four figures are made up for this problem.

Say what is wrong with what the presenter said, and what you would ask for.

EXERCISE 3 (teaching). A journalist has three minutes and no jargon. They have heard that every chart must start at zero. They want to know whether that is true before they file a story about a chart that does not.

Say what you would tell them, and give them one quotable sentence.

PRACTICE. Thirteen problems on this technique, easiest first. Work them on paper before turning to the answers, which are in the appendix at the back under this section's name.

1. A bar chart has a side axis that starts at 60. One bar stands for 80 and another for 100.

Work out how long each bar is drawn, in the units of the axis. Then work out how many times as long the second bar is drawn as the first, and how many times as large its value is.

2. A picture is enlarged so that its height and its width are both multiplied by the same number. Work out how many times its area grows when that number is 3, when it is 1.5, and when it is 0.5.

Then work backwards. A second picture was enlarged the same way and covers 16 times the area of the original. Work out what its height was multiplied by.

3. The side axis of a chart is printed with 200 at the bottom and 300 at the top, with a numbered line every 20.

Point A sits level with the third numbered line above the bottom. Point B sits halfway between the numbered lines for 220 and 240. Point C sits a quarter of the way from the numbered line for 280 to the numbered line for 300.

Read the value of each point. Then work out how far C is above A.

4. Schedule II of the National Food Security Act, 2013 sets 450 kilocalories for a hot cooked meal in lower primary classes. It sets 600 kilocalories for the take home ration for pregnant women and lactating mothers.

A slide draws the two as bars on a side axis that starts at 400 kilocalories. Work out how long each bar is drawn, with its unit. Work out how many times as long the longer bar looks, and how many times as large its value is. Then say what a chart redrawn from zero would show.

5. Schedule II of the National Food Security Act, 2013 sets 500 kilocalories for the take home ration for children from 6 months to 3 years. It sets 800 kilocalories for the take home ration for children from 6 months to 6 years who are malnourished.

An infographic draws each ration as a picture of a packet. The larger packet is as tall as its value requires, and it is widened by the same factor to keep its shape. Work out the scale factor, and how many times the page the larger packet covers. Then work out what the scale factor would have to be for the areas to match the values.

6. A printed bar chart shows two values from Schedule II of the National Food Security Act, 2013. One is the hot cooked meal for lower primary classes, 450 kilocalories. The other is the hot cooked meal for upper primary classes, 700 kilocalories. The side axis carries no numbers.

With a ruler you measure the bars at 10 millimetres and 60 millimetres. Work out how many kilocalories one millimetre stands for, and where the side axis starts.

7. Here is a worked answer. Find the step that broke.

```
a picture is enlarged 2 times in height and 2 times in width
its area grows 2 plus 2 = 4 times
a second picture is enlarged 3 times in height and 3 times in width
its area grows 3 plus 3 = 6 times
so the second picture covers 6 times the area of the original
```

8. Here is a worked answer. Find the step that broke.

```
a line chart shows six readings, one at each printed date
the dates along the bottom are 2000, 2005, 2010, 2011, 2012 and 2013
the six points are spaced evenly across the page
the readings are 20, 30, 40, 42, 44 and 46
the line climbs steeply over its first two steps and then flattens
so the quantity grew fast until 2010 and has slowed sharply since
```

9. A bar chart groups 60 readings into ranges. The readings and every number here are made up for this problem. Here is a worked answer about the chart. Find the step that broke.

```
the ranges printed under the bars are 0 to 20, 20 to 40, 40 to 60, 60 to 80 and over
80
the five bars are drawn the same width
the bars are 6, 10, 14, 12 and 18 readings tall
the largest reading in the data is 190
the last bar is the tallest
so readings over 80 are the most common kind
```

Then say what you would check on any chart like this.

10. Here is a worked answer. Find the step that broke.

```
a figure has two lines and two side axes
line A is read against the left axis, which runs from 0 to 100
line B is read against the right axis, which runs from 0 to 20
the two lines cross at year 4, 40 per cent of the way up the figure
so in year 4 the two quantities were equal
```

11. A colleague reviewing a district report says this.

> *This chart is dishonest. Its side axis starts at 400 kilocalories, not at zero.*

The chart is a line through six points, one a year, showing the average energy of a school meal in the district. The side axis runs from 400 to 500 kilocalories, and both numbers are printed. The caption says the axis starts at 400. The line runs from 440 in the first year to 470 in the sixth. The district, the report and every figure are made up for this problem.

Decide what to compute, compute it, and say what your answer does not establish.

12. A press line reads like this.

> *District P reported 1,800 cases of the illness last year and District Q only 900. The chart makes it plain: the burden in P is twice that in Q.*

The chart shows the two counts as bars from zero. District P has 12 lakh people and District Q has 3 lakh. The illness, both districts and all four figures are made up for this problem.

Decide what to compute, compute it, and say what your answer does not establish.

13. A note from a block office says this.

> *Meals served in our schools have fallen in every one of the last four years. The chart shows it clearly.*

The chart covers years 6 to 10. You ask for the whole series and get this. The block, the note and every figure are made up for this problem.

year	meals served a day
1	5000
2	5400
3	5900
4	6300
5	6800
6	7400
7	7300
8	7200
9	7150
10	7100

Decide what to compute, compute it, and say what your answer does not establish.

F3 · Checking a reference

DEFINITION. A reference is a pointer from a claim to the place that is supposed to support it.

Checking a reference means four things, and each one can fail on its own. Finding the source. Confirming it is the source named. Finding the exact passage pointed to. Reading that passage to see whether it supports the claim.

Finding a source is not the same as checking it. [11, 12, 13, 14]

IN PLAIN TERMS. Checking a reference means you go and look, not just glance at the name on it.

Four separate acts sit inside that looking, and each one can fail on its own. First, find the source. Second, check it is the source named, not something close to it. Third, find the exact page or section pointed to. Fourth, read that passage and ask whether it actually says what you were told.

Most people stop at the first act and call the job done. That is where the danger sits. A real source, spelled correctly and sitting on the right shelf, can still be attached to a claim it never makes. Nothing about the name looks wrong. The book is real, the page number is real, and the sentence you were promised is simply not on that page. You find this out one way only: by opening the source yourself and reading the words on it.

ILLUSTRATION. Here is the claim.

> *The GST Council is set up by Article 279A of the Constitution.*

The Council is the body that recommends goods and services tax rates to the Union and the States (Constitution of India, Article 279A(4)). That fact is not what you are testing. The claim is.

First, find it. Open India Code, the government's own website for Indian law. Search for "Constitution of India". Open it.

Second, check it is the right thing. The title reads The Constitution of India. It sits on a government site. Nothing looks wrong yet.

Third, find the exact place. The claim names Article 279A, so search inside the document for "279A". Nothing comes up.

Why not? Look at the list of amendments this copy names — later changes stitched into the law. The newest one is from 2006. Article 279A was written into the Constitution in 2016, ten years after that. This copy is old, and nothing on the page warns you. The second act should have caught it, by checking the newest amendment the copy names.

A second copy sits on the same website, marked as up to date, and that one does hold Article 279A. It also says when the Article came in, in the footnote under it: "Ins. by the Constitution (One Hundred and First Amendment) Act, 2016". "Ins." means inserted.

Fourth, read it. Clause (1) of Article 279A says the President "shall ... by order, constitute a Council". The same clause names it the Goods and Services Tax Council. So the claim holds, with one correction that only reading could give you. The Article does not set up the Council itself. It requires the President to, by order.

Figures quoted in this illustration are from [11].

WHERE THIS PICTURE BREAKS. A source can say something smaller than the claim built on it. A paper finds that two things move together; someone later cites it as proof that one causes the other.

For a law, the second act asks one more question: is this copy up to date? It is not a fifth act. The source named is the law as it now stands, and an outdated copy is not that source.

Above, the second act checked only the title and the site, so the old copy got through. The gap showed at the third act, because the Article did not yet exist in that copy. Skip the question when the article was there but has since been amended, and the old copy gets further. It passes locating, and you read wording that is no longer the law.

MUST KNOW POINTS FOR YOU.

- If a citation opens, that does not mean it is checked. Opening is only the first act, and the easiest one.
- Treat every reference as four separate acts: find it, confirm it, locate the passage, read it.
- Before you trust a copy, check the newest amendment it names. That check takes seconds.
- Expect the people reading your work to test your citations against you, not for you.
- A citation can be perfectly formatted and still say nothing like what you claimed.
- This check tells you only whether a source supports a claim. It does not tell you whether the source is good, or whether better evidence exists.

EXERCISE 1 (critique). A draft states: "Article 279A of the Constitution establishes the GST Council [Constitution of India, India Code]." The reference resolves and the document is genuine. Explain precisely what is still missing, and what you would do to settle it.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (build). Take any three references from a paper, report or guideline you have to hand and run all four acts on each. Record, for each, which act it failed at, or that it passed.

F4 · Argument: premise, inference, conclusion

DEFINITION. An argument is a set of statements offered as reasons, called premises, together with a further statement they are meant to establish, called the conclusion. The move from the first to the second is the inference.

An argument can fail in exactly two independent ways. A premise can be false. Or the inference can fail to carry, even when every premise is true. [15]

IN PLAIN TERMS. Someone hands you a claim and asks you to accept it. Along with the claim, they give you reasons — these are the premises. The claim itself is the conclusion.

Ask the two questions in order. First: if every premise were true, would the conclusion follow? Then, only then: are the premises actually true?

To show an inference fails, describe a case where every premise is true and the conclusion is false. If you can, the inference does not carry.

Two things that move together may both be moved by a third. And a pattern across countries need not hold for the people in them.

ILLUSTRATION. Here is a claim you will meet often once you work in obesity policy. Take it apart, on paper, in three lines.

- *Premise one:* obesity happens when a person takes in more energy than they use.
- *Premise two:* how much a person eats is something they choose.
- *Conclusion:* obesity is a matter of personal choice.

Test the inference first. Grant both premises. Now ask: does the conclusion follow?

It does not. Premise two says intake is chosen. The conclusion needs something far stronger — that intake is freely chosen. It would have to be untouched by price, by what sits on the shelf nearby, by advertising, by sleep, by the biology that drives appetite.

Somewhere between the premise and the conclusion, “is a choice” has quietly turned into “is only a choice”. That swap is the fault. It sits in the inference, and it stands whether or not the premises themselves are true.

Picture a person who eats more than they use, and who chooses what to eat. But they choose from what is cheap and on the nearest shelf. Both premises hold, and “a matter of personal choice” does not.

You reached “this argument does not work” without settling either premise. Run the inference test first, always.

WHERE THIS PICTURE BREAKS. Most arguments you meet in a hostile room are not laid out at all.

The premise doing the real work is usually the one left unspoken, left out because saying it aloud would expose it. Nobody in the claim above says out loud that eating sits untouched by price, or by what is on the shelf nearby. That premise stays hidden, and the argument leans on it anyway.

So when you cannot find the fault in an argument, the likely reason is simple. You have not yet written down the premise the speaker left out. Look there first.

MUST KNOW POINTS FOR YOU.

- True premises do not make a conclusion true. Whether the premises are true and whether the inference carries are separate questions, and you need both to hold before the argument does any work.
- Test the two failure modes in order: ask first whether the conclusion follows, then ask whether the premises are true.
- In Indian food policy debate, look for an unspoken premise that people already know what is in their food. Check for it in any argument that leaves the choice to the buyer.
- Grant every premise aloud, then show the conclusion still does not follow.
- Watch yourself when you are handed an argument you already agree with.
- This gives you the shape of an argument and the point where it breaks. It does not tell you whether a premise is true — that needs evidence — or whether a conclusion matters, which needs judgement.

EXERCISE 1 (critique). Take this claim: "Countries that eat more packaged food have more obesity, so packaged food causes obesity." Lay it out as premises and a conclusion. Then say whether the inference holds.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (teaching). Explain to a first-year resident, in ten minutes with a whiteboard, why "the inference is broken" is a stronger objection than "your facts are wrong". Use a food policy example of your own.

F5 · How government works, in the shape you need to read a regulation

Checked against the sources as they stood on 2026-09-19.

DEFINITION. In India the Constitution divides the power to make law from the power to apply it.

Parliament and the State Legislatures enact statutes, each called an Act. The Constitution is the founding law that every statute must fit.

The executive carries statutes out. The courts read statutes and decide whether the executive has acted within them.

The Central Government may make rules. A body that the statute creates may make regulations. [16, 17, 18, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]

IN PLAIN TERMS. Three different jobs, done by three different sets of people.

A legislature makes law — Parliament for the country, a State Legislature for its own state.

Parliament also amends the Constitution, but only by a special majority in each House. That means a majority of all its members, and two-thirds of those present and voting. Changes to some parts, among them the Seventh Schedule's lists, must also be ratified by at least half the State Legislatures. That is why the Constitution is the hardest law to change.

Below the legislature sits the executive — the day-to-day running of government. It is held by the President under Article 53. It is carried out beneath the President by ministries and departments.

A statute often leaves its detail blank on purpose, then hands the power to fill that blank onward. Neither a rule nor a regulation comes from a legislature. Yet both bind you, because a statute said they could.

Above all this sit the courts, reading statutes and testing whether the executive stayed inside its permission.

ILLUSTRATION. Pick up a packet of biscuits from your kitchen. Turn it over. Read the nutrition panel on the back.

Search for the Food Safety and Standards Act, 2006, on the India Code website and open it.

Now read section 23(1) of the Act. It says no one may sell a packaged food that is not labelled "in the manner as may be specified by regulations". Stop there. The Act hands you a duty, not the detail. The detail sits in a regulation, written by a body the statute set up, not by Parliament.

Section 4 sets up the body that writes it, the Food Safety and Standards Authority of India. Section 92 gives that body the power to make regulations.

Now suppose you think the rule on this packet is wrong. Do not go to a court expecting a new label. Write instead to the Authority that wrote the regulation, because that is where the power to change it sits.

Figures quoted in this illustration are from [28].

WHERE THIS PICTURE BREAKS. A programme may rest on nothing but an executive decision and a budget line. Look for the statute; if there is none, that is what it rests on. No section to open, no list of amendments to check. That makes such a programme quick to start and just as quick to stop.

MUST KNOW POINTS FOR YOU.

- A statute needs a legislature and a debate. A regulation under the Food Safety and Standards Act needs a draft published and the Central Government's approval. It also needs a notification in the Gazette (the government's official newspaper). It is then laid before Parliament.
- Four levels, hardest to easiest to change: the Constitution, a statute, a rule or a regulation (one level), and a notification. A notification here is an order issued under a power a statute gives, and published in the Gazette. Lawyers call any one of them an instrument.
- Before repeating a label rule to anyone, open the regulation itself. The Act tells you a label is compulsory; it does not tell you what goes on the label. For the energy figure, that is regulation 5(3)(b) of the Food Safety and Standards (Labelling and Display) Regulations, 2020. The section on energy units used it.
- Food is not one subject belonging to one government. Article 246 and the Seventh Schedule split subjects between Union, State and Concurrent lists, so both Parliament and the State Legislatures act in this space. Expect two authorities where you assumed one.
- When you want something changed, find the instrument first. Ask a minister to change a regulation and you get sympathy. Ask the body that wrote it, and you get a process with dates you can chase.
- Before relying on a programme, look for the statute behind it. If there is none, it rests on an executive decision and a budget line, and can be stopped as quickly as it was started.
- Asked who regulates food labelling in India, the weak answer is "the government". The strong answer names the body, the statute that created it, the section granting the power, and the kind of instrument it issues.
- This tells you who may act and through which instrument. It says nothing about whether the instrument is sensible, whether anyone enforces it, or whether it works.

EXERCISE 1 (retrieval). Write down, hardest to easiest to change, the four kinds of instrument named here. For each, say who is able to change it.

Record your confidence as a percentage before turning to the answer.

EXERCISE 2 (interpretation). A news report says "the government has banned the sale of energy drinks in school canteens". List three questions you must answer before you can tell whether this is a statute, a regulation, or an advisory with no legal force.

EXERCISE 3 (teaching). Explain to a journalist, in three minutes and without jargon. Why can a Health Minister not change what is printed on a food packet next week?

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An entry marked outstanding has not yet been checked against the source itself. Nothing in these sections rests on one: the reasoning is a derivation you can check with a calculator, and the entry records an anchor still to be supplied.

Appendix · Worked answers

A1 · Counting, place value, and what a calculator is actually doing

EXERCISE 1. Five lakh, Indian grouping: 5,00,000. International grouping: 500,000. Six digits either way.

Twenty-five thousand, in the Indian grouping: 25,000. In the international grouping: 25,000. They are identical, and that is worth noticing. Below one lakh the two systems group the digits the same way, so every number under a hundred thousand looks the same in both. The ladders only separate above that, which is exactly why the mistake feels safe right up to the point where it happens.

Now the division, line by line.

```
500,000 divided by 25,000
= 500 divided by 25    (take three zeros off each)
= 20
```

The ceiling is twenty times the proviso figure.

EXERCISE 2. The first half is right. Section 53 of the Food Safety and Standards Act, 2006 caps that penalty at ten lakh rupees, and ten lakh is one million.

"Ten crore" is wrong. One crore is ten million, so ten crore is a hundred million. That is a hundred times the real figure.

The step it happened at matters more than the size of it. No arithmetic was done here, so no arithmetic went wrong. The writer converted a number by matching names instead of by counting digits. Had they written 10,00,000 and 1,000,000 side by side, there would have been nothing left to get wrong.

1. Going from the left, one digit at a time.

9	ninety thousand
0	no thousands
8	eight hundred
0	no tens
7	seven

Five digits. The two zeros are doing work: they hold the other digits in their places. Take them out and you have 987, which is a different number entirely.

2.

```
15 minus 40 = -25
8 minus -5 = 13
-6 plus -9 = -15
-6 times -7 = 42
-36 divided by 4 = -9
```

Taking 40 from 15 runs 25 steps past zero. Taking away a negative is the same as adding, so 8 minus -5 is 8 plus 5. Adding two negatives goes further below zero. Two negatives multiplied give a positive. A negative divided by a positive is negative.

3. +————-+————-+————+ | two lakh | 2,00,000 | 200,000 | +————-+————-+————+ | twelve lakh | 12,00,000 | 1,200,000 | +————-+————-+————+ | three crore | 3,00,00,000 | 30,000,000 | +————-+————-+————+

Check each pair by counting digits rather than commas. Six, seven and eight digits, and the two columns agree every time.

4.

```
5 plus 3 times 6 = 5 plus 18 = 23
(5 plus 3) times 6 = 8 times 6 = 48
20 divided by 4 times 5 = 5 times 5 = 25
18 minus 6 divided by 3 = 18 minus 2 = 16
```

Now strictly left to right, ignoring the order.

```
5 plus 3 = 8
8 times 6 = 48
18 minus 6 = 12
12 divided by 3 = 4
```

The first and the fourth change. In each, a multiplying or dividing comes after an adding or taking away. Strict left to right then does the lower step first, which the agreed order does not. Mixing the two is not enough on its own. A line like 3 times 4 plus 5 comes out the same either way, because the multiplying already comes first. The second has brackets, so it comes out the same. The third has only multiplying and dividing, which run left to right anyway.

5. Count the digits first. Seven in each, so they are the same size of number and the commas are the only difference.

Now read the digits themselves, left to right.

```
90,90,000    = 9090000
9,900,000    = 9900000
```

The first two digits are 90 against 99, so the second number is larger. It is ninety-nine lakh against ninety lakh and ninety thousand.

How you decided: count the digits, then compare digit by digit from the left. Never compare comma patterns, because the two systems put them in different places.

6. The money first.

```
3,50,000 minus 4,20,000 = -70,000
```

Minus 70,000 rupees. The kitchen has spent 70,000 rupees more than it had. That is a deficit, and a budget can run one. So the sign is a real answer: the kitchen is 70,000 rupees short.

Now the rice.

```
40 plus 25 = 65
65 minus 70 = -5
```

Minus five sacks. A store cannot hold less than nothing, so this is not a fact about the store. It says the register is wrong somewhere. A delivery may not have been written down, or a sack written out twice. Somebody should count the sacks before anyone uses the figure.

7. +————+————+————+———-+ | one crore | 1,00,00,000 | 10,000,000 | 8 digits | +————+————+————+———-+ | ten crore | 10,00,00,000 | 100,000,000 | 9 digits | +————+————+————+———-+

One digit apart, so one is ten times the other.

Notice what the Indian commas do here and the international ones do not. The Indian grouping puts a break at the crore, so the word and the comma agree. The international grouping breaks at the million, which falls in a different place, so the same number looks unfamiliar.

8. The line is 50,000 plus 1,200 times 450. The agreed order multiplies first.

```
1,200 times 450 = 5,40,000
50,000 plus 5,40,000 = 5,90,000
```

Five lakh ninety thousand calories, which is 590,000 in the international grouping.

Now press equals after 50,000 plus 1,200, and carry on.

```
50,000 plus 1,200 = 51,200
51,200 times 450 = 2,30,40,000
```

That is two crore thirty lakh forty thousand calories. It has eight digits against six. Two extra digits say it is roughly a hundred times too big, and that is only a rough guide. Divide to get the real size of the error.

```
2,30,40,000 divided by 5,90,000
= 2,304 divided by 59    (take four zeros off each)
= about 39
```

It is about 39 times too big. The pressing did it. The calculator added the fruit to the count of meals, then multiplied calories by the result. Keep the label on each figure and that mistake shows itself: 50,000 calories and 1,200 meals cannot be added.

9.

```
450 times 1,00,000
= 450 times 100000
= 4,50,00,000    (Indian grouping)
= 45,000,000    (international grouping)
```

Eight digits either way, so nothing has slipped.

Named: four crore fifty lakh calories, or forty-five million calories.

Keep the label on it. These are calories, not rupees and not meals, and the number is meaningless the moment the word comes off it.

10. The calculator is not the step that broke. It divided exactly what it was handed, and 20 is the right answer to the sum that was typed.

The step that broke is the typing. Count the digits in each number against the Act.

```
ten lakh    10,00,000 = 1000000    7 digits    typed 8 digits
five lakh    5,00,000 = 500000    6 digits    typed 6 digits
```

One zero too many went into the first number. What they actually divided was one crore.

Done properly:

```
1000000 divided by 500000
= 10 divided by 5
= 2
```

Twice, not twenty times.

Why it looks reasonable: 20 is a clean whole number, and a clean answer feels like a confirmed one. Nothing on the display marks a mistyped figure, because from the calculator's side nothing went wrong. The only place this can be caught is before the equals sign, by counting the digits you typed against the digits in the source.

11. The calculator did what the agreed order says. It divided first, so it halved only Tuesday's figure.

```
1250 divided by 2 = 625
1150 plus 625 = 1775
```

The step that broke is the typing. The adding had to happen first, and nothing on the line said so. Brackets say so.

```
(1150 plus 1250) divided by 2 = 1200
```

The average is 1,200 meals a day. Pressing equals after the 1250 would also have worked, because then you choose the order yourself.

Why it looks reasonable: 1,775 has four digits, like both days, and the calculator gave it without complaint. But there is a quick check it fails. An average of two numbers must sit between them, and 1,775 is bigger than both.

12. The addition first.

```
10,00,000
+ 3,00,000
+ 5,00,000
= 18,00,000
```

That is right. Eighteen lakh.

Now the naming. A crore is 1,00,00,000, which is eight digits. 18,00,000 has seven.

```
1.8 crore = 1,80,00,000    eight digits
18 lakh    =    18,00,000    seven digits
```

So the report is out by a factor of ten. The total should be called eighteen lakh, or 1.8 million.

Why it looks reasonable: the digits 1 and 8 appear in both, in that order. Only the digit count separates them, which is exactly why you count digits.

13. The first line is right. The second is not.

```
150 minus 165 = -15
```

Taking 165 from 150 goes 15 steps past zero. The clerk worked out 165 minus 150 instead, and dropped the sign.

Now read what the right answer says. Minus fifteen sacks cannot exist. So the register is wrong somewhere: more sacks were written out than came in. The shelf needs counting, and the entries need checking.

Why the wrong answer looks reasonable: 15 sacks is an ordinary stock. Dropping the sign did more than get the number wrong. It hid the only evidence that the register did not add up.

14. What to check: the figure, by converting it into the grouping the Act uses.

Section 53 caps the penalty at ten lakh rupees.

```
ten lakh    = 10,00,000 = 1,000,000 = one million
ten million    = 1,00,00,000    = one crore
```

So the line is out by a factor of ten. It should say one million, or ten lakh.

What this does not establish. It does not tell you whether any such penalty has ever been imposed, or what was actually collected. It does not tell you who imposes it. And the figure comes from a copy of the Act that names no change to it after 2008. It is what this copy says, not what is in force today.

15. The two figures are changes, not amounts. Add them to get the change across both years.

```
300 plus -400 = -100
```

Across the two years, enrolment went down by 100. That part is right.

The mistake is the word “enrolment”. Enrolment is a count of children, and a count cannot go below zero. The change in it can. What the speaker has found is a fall of 100 from two years ago, not a school with minus 100 children.

What it does not establish. How many children are enrolled now. For that you need the count two years ago, and nobody gave it. Why the number fell. Whether a fall of 100 is large, which depends on whether the school had 200 children or 2,000.

16. Two claims are buried in that sentence. Check them separately.

First, 400 crore against 4 billion.

```
400 crore = 400 times 1,00,00,000
= 4,00,00,00,000
= 4,000,000,000
= 4 billion    correct
```

Second, the figure per person.

```
4,000,000,000 divided by 10,00,000
= 4,000,000,000 divided by 1,000,000
= 4,000 rupees per person    correct
```

Both hold. This is worth feeling: checking is not the same as finding a mistake, and a check that confirms is not wasted.

What it does not establish. Nothing here says whether 4,000 rupees a person is a lot or a little, what it buys, or whether the money was spent. Nor whether ten lakh is who the scheme reaches or who it was designed for. The arithmetic is sound and the sentence is still not yet a fact about the world.

17. First read the line the way the agreed order reads it, dividing first.

```
2,00,000 divided by 400 = 500
4,00,000 plus 500 = 4,00,500
```

Read that way, the line says 4,00,500 rupees a child. That cannot be what the writer meant. They meant the two costs added together, then shared among 400 children.

```
4,00,000 plus 2,00,000 = 6,00,000
(4,00,000 plus 2,00,000) divided by 400 = 1,500
```

So 1,500 rupees a child is right, and the line as written is wrong. Anyone who types it whole into a calculator that shows the whole line gets 4,00,500. They will either trust it or throw the note away. A basic desk calculator works each step as the next key is pressed, so it gives 1,500. That is the right figure from a line that is still wrong. The fix is brackets round the two costs.

What it does not establish. Whether the two sums are the whole cost. Whether 400 is the number of children served or the number enrolled. Whether 1,500 rupees buys what the scheme promises.

A2 · Fractions

EXERCISE 1. Start with 500 over 800.

```
500 over 800
= 50 over 80    (both ended in a zero, so divide both by ten)
= 5 over 8    (both are in the ten times table, so divide both by ten again)
```

Five eighths. Check it backwards: five times a hundred is five hundred, eight times a hundred is eight hundred.

What it is: the ordinary take-home ration standard for a child of six months to three years. It is expressed as a fraction of the standard set for a malnourished child of six months to six years. The two rows cover different age bands, so even this is a comparison across two groups.

What it is not: a fraction of what either child needs, or of what either child eats. Both figures are standards for a ration. The Act sets them; it does not measure anybody.

EXERCISE 2. Twelve over twenty does reduce to three fifths, so nothing has gone wrong in the sum.

The whole has been swapped. Three fifths is the lower primary standard as a fraction of the upper primary standard. The sentence turns it into a fraction of "their protein", which would have to mean a daily requirement for a child of that age. No such figure appears in those rows. One fraction has been quietly moved from one denominator to a different one.

A defensible rewrite. "The protein standard for a lower primary meal is 12 grams. That is three fifths of the 20 grams set for an upper primary meal."

Notice what the rewrite costs. It is longer, it names both wholes, and it makes no claim about what a child needs. That is the trade every time.

1.

```
6 over 8    = 3 over 4    (divide both by 2)
20 over 50 = 2 over 5    (divide both by 10)
36 over 48 = 18 over 24    (divide both by 2)
= 9 over 12    (divide both by 2)
= 3 over 4    (divide both by 3)
```

Check the last one backwards: three times twelve is thirty-six, four times twelve is forty-eight.

Notice that the first and the last came out the same. Two fractions that look nothing alike can be one number.

2. Forty is the smallest number that both five and eight divide into, so use forty.

```
3 over 5 = 24 over 40    (multiply top and bottom by 8)
5 over 8 = 25 over 40    (multiply top and bottom by 5)
```

Same denominator now, so the tops can be compared directly. Twenty-five is more than twenty-four, so five eighths is larger.

The gap is one fortieth, which is small. Guessing by eye would have been a coin toss.

3. Fifteen is the smallest number both five and three divide into.

```
2 over 5 = 6 over 15    (multiply top and bottom by 3)
1 over 3 = 5 over 15    (multiply top and bottom by 5)
6 over 15 + 5 over 15 = 11 over 15
```

Eleven fifteenths.

Why not add tops and bottoms: that would give 3 over 8, which is less than a half. Two fifths on its own is nearly a half, so adding something to it cannot leave you below a half. The method has to be wrong, and you can see it is wrong without knowing the right answer.

4. Protein first.

```
12 over 20
= 3 over 5    (divide both by 4)
```

Then calories.

```
450 over 500
= 45 over 50    (divide both by 10)
= 9 over 10    (divide both by 5)
```

Three fifths and nine tenths.

Two different columns, and two different pairs of things. The protein fraction compares two meals. The calorie fraction compares a meal with a ration. Neither can be quoted without naming the two things it sets against each other.

One thing to notice about the schedule itself. Four of its six rows give protein as a range — 12 to 15, 20 to 25, 18 to 20. Only the lower and upper primary rows give one figure. So those are the only two protein numbers you can divide without first picking an end of a range. Picking an end is a choice, and a choice has to be declared.

5.

```
240 over 400
= 24 over 40    (divide both by 10)
= 3 over 5    (divide both by 8)
```

Three fifths.

What the whole was: the 400 households in the block, not the people in them, and not the households in any other block. Say that out loud when you report the fraction. Three fifths of households and three fifths of people are different claims, and this table supports only one of them.

6. Check first that the two fractions can be added at all. Both are fractions of the same whole, the 320 households of this block, and no household is counted twice. Both of those have to hold before any arithmetic is allowed.

Eight is the smallest number that both eight and four divide into, so work in eighths.

```
3 over 8 = 3 over 8    (already there)
1 over 4 = 2 over 8    (multiply top and bottom by 2)
3 over 8 + 2 over 8 = 5 over 8
```

Five eighths. Now turn that back into households.

```
5 over 8 of 320
= 320 divided by 8, times 5
= 40 times 5
= 200 households
```

Check it the other way round: three eighths of 320 is 120, a quarter of 320 is 80, and 120 plus 80 is 200.

Keep the whole attached when you report it. Five eighths of the households in this block, in this month. Not of the people in them, and not of any other block.

7. The step that broke is the first one. The tops and the bottoms were both added.

```
1 over 4 + 1 over 4 = 2 over 4 = 1 over 2
```

Adding fractions adds the tops only. The denominator says what size the parts are, and that does not change when you take more of them. Two quarters is a half.

Why it looks reasonable: the answer came back as one quarter, which is a tidy fraction and one of the two you started with. A wrong method that returns a plausible-looking fraction is the hard case, because nothing about the shape of the answer warns you.

8. The arithmetic first, to confirm it.

```
450 over 700
= 45 over 70
= 9 over 14
```

Nine fourteenths is a little under two thirds, so that much holds.

What broke is the whole. Nine fourteenths is the lower primary meal as a fraction of the **upper primary meal**. The sentence turns it into a fraction of a child's daily energy, which is a different quantity and appears nowhere in the schedule.

Why it looks reasonable: both numbers came off the same table, so the fraction feels as though it belongs to that table. It does. The sentence built on it does not.

9. What to compute: whether two fifths and a half can be added at all.

They cannot, as they stand. They are fractions of two different wholes — centres in one case, schools in the other. Adding them adds parts cut from different things.

Watch what adding them properly would give, even if the wholes did match.

```
2 over 5 = 4 over 10
1 over 2 = 5 over 10
4 over 10 + 5 over 10 = 9 over 10
```

So nine tenths is where the number came from. But a fraction of all facilities needs counts, not fractions. With 50 centres and 200 schools it would be 120 out of 250, which is under a half.

What this does not establish. It does not tell you the real figure, because the note gives no counts. It does not tell you whether "a stock shortage" means the same thing in both places. Ask for the two counts, and ask for the definition.

10. What to compute: the size of the change, and whether the two fractions share a whole.

```
3 over 5 = 6 over 10
1 over 2 = 5 over 10
6 over 10 - 5 over 10 = 1 over 10
```

A tenth of the standard. That is the change, and it is worth stating that way rather than as "three fifths, up from a half", which sounds larger.

Now the whole. Both are fractions of "the standard", so they can be compared — but only if it is the same standard on both dates. The schedule sets different standards for lower primary, upper primary and take-home rations, so this needs saying.

What this does not establish. It says nothing about how many children that tenth reaches, or whether the standard itself changed between the two dates. A rising fraction of a falling standard is not an improvement, and this sentence cannot tell the two apart.

A3 · Decimals, rounding, and significant figures

EXERCISE 1. Monthly first.

```
5 kilograms per person per month, times 6 people
= 30 kilograms per household per month
```

That figure is exact. Both numbers that went into it are counts, not measurements.

Now daily. A month is not a fixed number of days, so work both ends.

```
30 divided by 31 = 0.9677…
30 divided by 28 = 1.0714…
```

So the household gets roughly one kilogram a day, somewhere between 0.97 and 1.07.

Where to stop: one figure. Round it to 1 kilogram a day. The spread across the ends is more than a tenth of a kilogram. A second decimal place would claim to know something the month length has already taken away from you.

EXERCISE 2. Ask two things, in this order.

First: what two numbers were divided to make it? An average per person is a total divided by a count of people. Say the total was in tonnes, rounded to the nearest tonne, and the count was a registered beneficiary count. Then the fourth decimal place is carrying a precision that neither input had.

Second: how exact is the count on the bottom? A beneficiary list is almost never exact to the last person, and the divisor moving by a few hundred would change the third decimal place.

What you expect: that 4.8732 is what the spreadsheet printed and nobody chose it. The defensible version is 4.9 kilograms, and the interesting sentence is not the average at all. It is the gap from 5, and whether that gap is real.

1. Take 0.408 one place at a time, starting just after the point.

4	four tenths
0	no hundredths

8	eight thousandths

Each place is worth a tenth of the place on its left. The zero is holding the 8 in the thousandths place. Take the zero out and you have 0.48, which is a different number.

Now the ordering. Line the numbers up by place rather than by how long they look.

0.7	7 tenths,	0 hundredths,	0 thousandths
0.65	6 tenths,	5 hundredths,	0 thousandths
0.089	0 tenths,	8 hundredths,	9 thousandths
0.7000	7 tenths,	0 hundredths,	0 thousandths

Smallest first: 0.089, then 0.65, then 0.7 and 0.7000, which are equal.

Two things to notice. First, 0.65 is smaller than 0.7 even though it has more digits, so compare the tenths first and stop as soon as they differ. Second, 0.7000 has the same value as 0.7. The extra zeros change nothing about its size. They do say the figure was carried to four places, and that is what the rest of this section is about.

2. Look at the third digit after the point each time. Four or less, the second digit stands. Five or more, it goes up by one.

0.3849	third digit 4	→	0.38
12.0761	third digit 6	→	12.08
0.0955	third digit 5	→	0.10
7.2950	third digit 5	→	7.30

The third one is worth a second look. Rounding 0.09 up carries into the next place and gives 0.10, not 0.1. Both are the same number, and 0.10 is the one that shows you stopped at two places on purpose.

3. Count from the first digit that is not zero.

450	2 or 3	the trailing zero is ambiguous
0.0072	2	leading zeros only hold the place
6.030	4	the trailing zero after a point is real
1,200	2, 3 or 4	ambiguous the same way as 450

Two of the four cannot be answered from the number alone, and that is the useful part of this problem. A trailing zero with no point after it may be a measured digit or a placeholder. Only whoever wrote it down knows, and if they are not available, say so rather than guess.

4.

```
17 divided by 7 = 2.428571429
```

To three significant figures, count from the 2.

```
2.42|8571429
the next digit is 8, which is five or more
so the 2 goes up to 3
= 2.43
```

Two point four three. The other seven digits the calculator offered were never yours to give away.

5.

```
5 divided by 4 = 1.25 kilograms a week
```

That division is exact: it stops after two places on its own, with nothing rounded away.

But the 4 is not exact. A month is not four weeks — it is closer to 4.3 — so work the honest end too.

```
5 divided by 4.3 = 1.16… kilograms a week
```

So the answer sits between about 1.16 and 1.25. Report it as roughly 1.2 kilograms a week, two significant figures, and say you assumed a month of about four and a third weeks.

6.

```
450 times 196 = 88,200 calories
```

Now the precision. Both inputs are exact counts: 450 is fixed by the schedule and 196 is a count of days, not a measurement. Nothing was rounded on the way in.

So 88,200 is exact, and all of it is significant. Write it as 88,200 calories.

This is the case worth noticing. The rule that an answer cannot beat its weakest input is about measurements. Where both inputs are exact, the answer is exact too, and rounding it away would throw something real overboard.

7.

```
1,850 divided by 387 = 4.780361757…
```

Now ask what each input was worth. The 1,850 is a weight, and a weight recorded to the nearest ten kilograms is good to three significant figures at best. The 387 is a count, and counts are exact if the register is right.

So the weakest input carries three significant figures, and the answer carries three.

```
4.78 kilograms per household
```

What fixed the precision: the weight, not the division and not the household count.

8. The arithmetic in every line is right. The step that broke is the second one: the rounding happened in the middle.

Carry the full figure through instead and compare.

```
2.4 times 3.6 = 8.64
8.64 divided by 4 = 2.16
to 2 significant figures = 2.2
```

Here the two routes agree, at 2.2. That is the trap. Rounding in the middle does not always change the answer, so it survives every time it happens to be harmless and then bites once.

Why it looks reasonable: the worked answer rounds to the stated precision at every stage, which looks like care. It is the opposite. Round once, at the end.

9. Nothing is wrong with the arithmetic. Both inputs are counts, so the division is exact and the digits are all really there.

What is wrong is what those digits claim. Four places after the point in a percentage means the figure is being pinned to one part in a million.

```
64.8135 per cent of 10,000 households = 6,481.35 households
```

You cannot have 0.35 of a household. The fourth decimal place is describing a fraction of a household that does not exist, so it carries no information about the world.

And the register itself is not exact in practice, whatever the arithmetic says. Report 64.8 per cent, or 65 per cent, and give the two counts underneath.

10. Check the stated figure first.

```
2,014.6 minus 1,987.3 = 27.3
27.3 divided by 2,014.6 = 0.013550…
times 100 = 1.36 per cent
```

The arithmetic holds. Now ask where five significant figures in a mean intake could have come from. Dietary intake is estimated, not weighed, and the estimate for one person is uncertain by a long way more than 27 calories.

So the decimal place on each mean is claiming a precision the method cannot deliver. The difference between the two means is smaller than the uncertainty in either.

What this does not establish. It does not establish that intake fell at all. The two figures could be two draws from the same underlying quantity. Before publishing, ask how intake was measured and how precise a single mean is. Those are later sections of this book. This section is enough to know the question has to be asked.

A4 · Percentages, and the percentage of a percentage

EXERCISE 1. Going up, from 12.

```
20 minus 12 = 8
8 divided by 12 = 0.6666…
times 100 = 66.7 per cent more
```

Coming down, from 20.

```
8 divided by 20 = 0.4
times 100 = 40 per cent less
```

Same 8 grams either way. The denominator changed, so the percentage changed.

Which to put in a note: neither on its own. Write the absolute figures and one percentage with its direction named. “20 grams against 12 grams, about 67 per cent more.” That sentence cannot be read the wrong way round. A bare “67 per cent” in a note will be quoted back at you as “40 per cent”. The person quoting it read from the other end, and you cannot prove which of you meant what.

EXERCISE 2. The spine of it. Fifty per cent is a comparison with the starting point, and the starting point is not in the sentence. Two in a hundred becoming three in a hundred is a rise of fifty per cent. So is twenty in a hundred becoming thirty, and those are not the same story at all.

Then give them the second half, because it is the half that makes you useful rather than merely cautious. One extra person in a hundred, across a population of several crore, is a large number of people. So the percentage alone can understate it as easily as overstate it.

What to hand them: “The share went from two in a hundred to three in a hundred. That is one more person in every hundred, which across this population is roughly this many people.”

Then give them the count, or say plainly that you cannot until you have the denominator. Say it in those words. A journalist can print a number with a caveat and cannot print a refusal.

1. Per cent means per hundred, so divide by a hundred each time.

7 per cent	0.07	7 over 100
40 per cent	0.40	40 over 100
125 per cent	1.25	125 over 100
0.5 per cent	0.005	0.5 over 100

The third one matters. A percentage can be more than a hundred: that just means more than the whole you started from. A proportion cannot, and the two are often confused.

2. Thirty-five per cent of 840. Turn the percentage into a decimal, then multiply.

```
35 per cent = 0.35
0.35 times 840 = 294
```

Now the other direction. Divide, then multiply by a hundred.

```
840 divided by 1,200 = 0.7
0.7 times 100 = 70 per cent
```

Two different questions. The first asks for a part; the second asks what share a part is. Read which one you have been given before you touch the calculator.

3. Going up, starting from 80.

```
92 minus 80 = 12
12 divided by 80 = 0.15
times 100 = 15 per cent more
```

Coming down, starting from 92.

```
12 divided by 92 = 0.13043…
times 100 = 13.0 per cent less
```

Same 12 either way. The number underneath changed, so the percentage changed with it. Fifteen up, thirteen down, and neither is wrong.

4. A percentage of a percentage multiplies. Turn both into decimals, then multiply.

```
0.40 times 0.30 = 0.12, so 12 per cent of the whole
0.05 times 0.60 = 0.03, so 3 per cent of the whole
```

Both answers are smaller than either percentage that went into them.

They have to be. You are taking a part of a part, and multiplying by anything under one makes a number smaller. Twelve is below thirty and below forty. Three is below five and below sixty.

So there is a check you can run before the arithmetic. The answer can never be larger than the smaller of the two percentages you started with.

5. Rural first.

```
75 per cent = 0.75
0.75 times 18,00,000 = 13,50,000
```

Then urban.

```
50 per cent = 0.50
0.50 times 6,00,000 = 3,00,000
```

Thirteen lakh fifty thousand and three lakh. Sixteen lakh fifty thousand between them.

Now read the words again. The Act says “up to”. These are ceilings, not counts and not targets. A figure worked out from a ceiling is the most that may be covered, and says nothing about how many are.

6. A percentage of a percentage multiplies.

```
0.20 times 0.45 = 0.09
= 9 per cent of all respondents
```

Now the count.

```
0.09 times 2,400 = 216 people
```

Nine per cent, which is 216 of the 2,400.

The count is the part to carry into a meeting. Nine per cent sounds like a rounding error and 216 people sounds like 216 people, and they are the same finding.

7. The subtraction is right. The step that broke is the last line, where a difference in percentage points is called a per cent.

Both statements exist and they are different quantities.

```
6 minus 4 = 2 percentage points
2 divided by 4 = 0.5, so a rise of 50 per cent
```

So the rise is two percentage points, or fifty per cent, and the worked answer has used the first number with the second name.

Why it looks reasonable: the words “per cent” sit in the original figures, so carrying them into the difference feels automatic. Percentage points subtract. Per cent divides.

8. The step that broke is the addition on the third line.

Two percentages can be added only when they are parts of the same whole. These are not. The 40 per cent is a percentage of the 30 per cent, not a percentage of everybody.

A percentage of a percentage multiplies.

```
0.40 times 0.30 = 0.12
= 12 per cent of all respondents
```

Twelve per cent, not seventy. The broken answer is nearly six times the right one.

There is a second way to see that 70 cannot be right, and it needs no arithmetic at all. The group being described sits inside the 30 per cent who buy a snack at all. So the answer cannot be bigger than 30, whatever the second percentage is.

Why it looks reasonable: both numbers are percentages, and both are quoted in one breath about one survey. Adding percentages is a thing you are sometimes allowed to do. Here the second percentage is a part of the first, so it is not.

9. Do the two steps on the actual number instead of on the percentages.

```
100 minus 20 per cent of 100
= 100 minus 20
= 80
80 plus 20 per cent of 80
= 80 plus 16
= 96
```

Ninety-six, not a hundred. The budget is four per cent below where it started.

The step that broke is treating the two percentages as though they could be added and subtracted. They cannot, because they are taken from different numbers: the first twenty per cent is of 100 and the second is of 80.

Why it looks reasonable: the two percentages are equal and opposite, which is exactly the shape of something that cancels. Percentages of different bases never cancel.

10. You cannot compute anything yet, and saying so precisely is the answer.

A sixty per cent rise is a rise measured against a starting value, and the line does not give it. Two readings fit the same sentence.

from 5 in 100 to 8 in 100	rise of 3 percentage points
from 20 in 100 to 32 in 100	rise of 12 percentage points

Both are a sixty per cent rise. The second is four times the first in people.

So what to ask for: the two underlying figures, and the two counts they came from.

What this does not establish, once you have them. It will not tell you whether the change is real or the two surveys simply differ. It will not tell you whether they measured the same thing both times, or what caused any of it. It will tell you how many more women, which is the number anybody acting on this needs.

11. Check that the two sentences describe one change.

```
12 minus 8 = 4 percentage points
4 divided by 8 = 0.5, so 50 per cent
```

They do. Both are true, and one sounds several times larger than the other.

Which to put in the brief: both, in one sentence, with the counts. Something of the shape “coverage rose from 8 to 12 per cent, a rise of half, which is this many more households”.

Why not choose. A brief carrying only the fifty per cent will be read as a transformation. A brief carrying only the four points will be read as nothing happening. Bringing both is what makes the sentence hard to quote against you in either direction.

What this does not establish. Neither figure says how many households the four points are, and neither says whether coverage is the right measure of whether the programme works.

A5 · Ratios, rates and proportions

EXERCISE 1. Priority households first. They are entitled per person, so count the people.

```
1,200 households times 5 people = 6,000 people
6,000 people times 5 kilograms per person per month = 30,000 kilograms per month
```

Antyodaya households are entitled per household, so count the households.

```
300 households times 35 kilograms per household per month = 10,500 kilograms per
month
```

The block as a whole.

```
30,000 plus 10,500 = 40,500 kilograms per month
```

Now put both on the same denominator, which is kilograms per household per month.

```
priority:    30,000 divided by 1,200 = 25
Antyodaya: 10,500 divided by 300    = 35
```

Antyodaya is higher on this denominator, at 35 against 25. Note what did the work: the average of five people a household. At an average of seven they would be equal, and above seven the priority households would come out ahead.

EXERCISE 2. The percentage is a rate of change in tonnes. It is being reported as a change in coverage, which is a proportion of people. Those are different quantities and nothing in the line connects them.

Tonnes can rise for at least three reasons that have nothing to do with more people being covered.

- More households on the list.
- The same households lifting a larger share of what they were already entitled to.
- A change in what the entitlement is.

What to ask for, in this order. How many people or households were entitled in each period, so the tonnes have a denominator. How much they were entitled to in total. Then the offtake can be read as a proportion of the entitlement rather than as a bare weight. Only then does the word coverage have anything underneath it.

And keep the 11 per cent. It is a perfectly good statement about tonnes. It has simply been given the wrong noun.

1.

```
18 to 24    = 3 to 4    (divide both by 6)
45 to 15    = 3 to 1    (divide both by 15)
200 to 500 = 2 to 5    (divide both by 100)
```

A ratio reduces the way a fraction does. It is a division written another way.

None of the three carries a unit. Whatever the 18 and the 24 were measured in has cancelled out, and that is what makes a ratio a bare number.

2. The ratio first. Lifting against not lifting means 90 against the rest.

```
250 minus 90 = 160 not lifting
90 to 160 = 9 to 16    (divide both by 10)
```

Now the proportion, which is out of all of them.

```
90 divided by 250 = 0.36
```

Nine to sixteen, and 0.36.

Notice the two denominators. The ratio is against the 160 who did not; the proportion is against all 250. Neither number is wrong and they answer different questions.

3. Per 100,000 first. That is a hundred times as many people, so a hundred times as many events.

```
15 times 100 = 1,500 per 100,000 per year
```

Per person, divide by a thousand.

```
15 divided by 1,000 = 0.015 per person per year
```

One rate, three ways of writing it. Nothing has been measured differently, and none of the three is more precise than the others. The denominator was chosen to keep the zeros off the page.

4.

```
5 kilograms per person per month, times 4 people
= 20 kilograms per household per month
20 times 12 months
= 240 kilograms per household per year
```

The denominators, said out loud. The first is per household per month. The second is per household per year.

Both had to change together when you multiplied by four: the per-person denominator became a per-household one, and that is the step worth watching. The number 20 means nothing until it carries "per household per month" with it.

5. Put both on the same denominator, which is kilograms per household per month.

```
priority household of n people = 5 times n
set that equal to 35
5 times n = 35
n = 35 divided by 5
n = 7
```

Seven people.

That number is worth carrying. Below seven, an Antyodaya household receives more; above seven, a priority household does. "They match at a household of seven" is a sentence that settles an argument, and it took one division.

6. First block.

```
48 divided by 3,200 = 0.015 per person per year
times 1,000 = 15 per 1,000 per year
```

Second block.

```
160 divided by 12,500 = 0.0128 per person per year
times 1,000 = 12.8 per 1,000 per year
```

The first block is higher, at 15 against 12.8, even though it recorded fewer events.

That reversal is the point of the problem. The raw counts say 48 against 160. The rates say the opposite, because the second block has nearly four times the people.

7. The first two lines hold. Seventy-two against 108 is the ratio, and it does reduce to 2 to 3.

The step that broke is the last one. A ratio has been divided out and the answer read as a proportion. A ratio compares one part with another part. A proportion is a part over the whole it came from, and the whole here is every household in the block.

```
72 plus 108 = 180 households in all
72 divided by 180 = 0.4
times 100 = 40 per cent lift their full entitlement
```

Forty per cent, not 67.

The 0.6666 is not a meaningless number. It says the lifting households are two thirds as many as the households that do not lift. That is true. It has been given the wrong denominator and then the wrong noun.

Why it looks reasonable: a ratio and a proportion are both one number divided by another, and both come out as a decimal. A decimal below one invites being read as a percentage. The check is to say the denominator out loud. Divided by 108 is per non-lifting household. Divided by 180 is per household.

8. The step that broke is the comparison, which sets two rates side by side without matching their denominators.

Put both on the same denominator.

```
block A: 15 per 1,000
= 15 times 100
= 1,500 per 100,000
block B: 40 per 100,000
```

Block A is nearly forty times block B, not the other way round.

Why it looks reasonable: both lines end in “people”, both numbers are small, and the larger number belongs to the block being called worse. Everything about the shape of the sentence is right except the only thing that mattered.

9. The first three lines hold. Two kilograms a week for four weeks is eight kilograms a month.

The step that broke is the last one. The denominator was changed from per person to per household with no arithmetic done at all.

```
8 kilograms per person per month, times 5 people
= 40 kilograms per household per month
```

Forty, not eight.

Why it looks reasonable: the household size appears in the line just above, so the reader feels it has been used. It has been mentioned. Mentioning a denominator is not the same as dividing or multiplying by it.

10. What to compute: both entitlements on one denominator, which is kilograms per household per month.

```
priority household of 5: 5 times 5 = 25 kilograms per household per month
Antyodaya household:    35 kilograms per household per month
35 minus 25 = 10
10 divided by 25 = 0.4, so 40 per cent more
```

The 40 per cent is right, for a household of five.

Now the part the line hides. It is a statement about the average household, and the average household is a calculation rather than an address. Run the same division at other sizes.

```
household of 3: 5 times 3 = 15, so 35 is well over twice as much
household of 7: 5 times 7 = 35, so the two are equal
household of 9: 5 times 9 = 45, so the priority household gets more
```

The gap is 40 per cent at one household size, nothing at all at another, and the other way round above seven.

What this does not establish. It says nothing about how household sizes are spread here. An average of five holds whether every household has five people or half have two and half have eight. It says nothing about how many households are in either group. And it says nothing about whether either figure feeds anybody, which is a different question from which is larger.

11. Nothing can be computed until one thing is known: what each rate is divided by.

Suppose both are per 1,000 people, and A has 2 lakh people and B has 8 lakh. Then a doubled rate is not a doubled count.

```
district A: 20 per 1,000, times 200 thousand people = 4,000 cases
district B: 10 per 1,000, times 800 thousand people = 8,000 cases
```

Twice the rate and half the cases. Resources follow people, so the conclusion is backwards.

Those population figures are made up, to show the move. The move is not. Ask for the two denominators before you agree or disagree with the sentence.

What this does not establish. Even with both counts in hand, nothing here says what resource a case needs, or whether the two districts counted cases the same way. A rate comparison decides which question to ask next, not what to do.

A6 · Powers, roots and scientific notation

EXERCISE 1. In scientific notation.

```
one lakh    = 1 times 10^5
thirty-five thousand = 3.5 times 10^4
ten lakh    = 1 times 10^6
one crore    = 1 times 10^7
one thousandth = 1 times 10^-3
```

Lakh in a crore, by subtracting exponents.

```
10^7 divided by 10^5 = 10^(7 minus 5) = 10^2 = 100
```

A hundred lakh in a crore.

Thousand in a million, the same way.

```
10^6 divided by 10^3 = 10^(6 minus 3) = 10^3 = 1,000
```

A thousand thousand in a million.

Notice that both answers came out of subtracting two small numbers, with no long division anywhere. That is what the notation is for.

EXERCISE 2. Step one is right. 2.4 times 10^5 is 240,000, and 2.4 lakh is 2,40,000. The same number.

Step two, the yearly figure, is right in its arithmetic.

```
2.4 times 10^5, times 365 days
= 876 times 10^5
= 8.76 times 10^7, which rounds to 8.8 times 10^7
```

Step three is wrong, and it is the naming step again rather than the arithmetic. 8.8 times 10^7 is 8,80,00,000. A crore is 10^7, so this is 8.8 crore. That part is right too.

So all three steps hold. The thing to challenge is not in the notation at all: it is the 365. Production every single day of the year, with no closure and no maintenance, is an assumption the note never states. Check the exponents, then check the assumption hiding in the multiplier.

EXERCISE 3. A square's area is one side multiplied by itself. So the side is the square root of the area.

```
side = 2,025^(1/2) = 45 metres
```

Check it by multiplying back. 45 times 45 is 2,025.

The second one does not come out whole.

```
side = 2,000^(1/2) = 44.72135…
```

The calculator will offer you eight digits. The area you were given has four. Write 44.72 metres, or say about 45 metres. Digits the question never gave you are not yours to pass on.

1.

```
10^2     = 100
10^5     = 1,00,000     (one lakh)
10^0     = 1
10^-2     = 0.01
9^(1/2)     = 3
144^(1/2) = 12
```

Two of these are worth a second look. Anything to the power zero is one, which surprises people and falls straight out of the rules. Divide 10^2 by 10^2 and the exponents subtract to give 10^0. Any number divided by itself is one, so 10^0 is one.

And 10^{-2} is small, not negative. It sits between zero and one.

Check the last two by multiplying back. Three times three is nine. Twelve times twelve is 144. A root is the only operation here you can check without trusting the calculator.

2. Multiplying adds the exponents. Dividing subtracts them.

```
10^4 times 10^3    = 10^(4 plus 3)    = 10^7
10^8 divided by 10^5 = 10^(8 minus 5)    = 10^3
10^3 times 10^-1    = 10^(3 minus 1)    = 10^2
```

The third one shows why a negative exponent is worth having. Multiplying by 10^{-1} is the same as dividing by ten, and the rule handles it without a special case.

3. Move the point until one digit is in front of it, and count how far it moved.

```
6,200    = 6.2 times 10^3
93,00,000    = 9.3 times 10^6
0.0047    = 4.7 times 10^-3
1,00,00,000 = 1 times 10^7
```

The third moved the other way, so its exponent is negative. The last is one crore, and a crore is exactly 10^7.

4.

```
one crore = 1,00,00,000    = 1 times 10^7
ten crore = 10,00,00,000 = 1 times 10^8
```

Now subtract the exponents rather than dividing the numbers.

```
10^8 divided by 10^7 = 10^(8 minus 7) = 10^1 = 10
```

Ten times larger. No long division, and no zeros counted.

5. Put the 450 into scientific notation first, so both numbers are in the same form.

```
450 = 4.5 times 10^2
```

Now multiply the digits and add the exponents, separately.

```
4.5 times 1.8 = 8.1
10^2 times 10^5 = 10^7
so 8.1 times 10^7 calories
```

Eight point one times ten to the seven. That is 8,10,00,000 calories, or a little over eight crore.

Keep the label. Calories, not meals and not rupees.

6. Write the area out first. That is the habit with any raised number.

```
8.0 times 10^5 = 8,00,000 square metres
```

A square's side is the square root of its area.

```
side = 8,00,000^(1/2) = 894.427…
```

The calculator offers more digits than the question gave you. The area was given as 8.0 times 10^5, which is two digits. So write 890 metres.

Now multiply back.

```
890 times 890 = 7,92,100
```

Seven lakh ninety-two thousand against eight lakh. Not the same, and that is correct. The rounding threw away the digits that would have closed the gap. A check like this tells you the root is the right size. It does not tell you it is exact.

Keep the two units apart. The area is in square metres and the side is in metres.

7. In full first, because that is the habit.

```
2.5E+04 = 2.5 times 10^4 = 25,000
3.0E-02 = 3.0 times 10^-2 = 0.03
```

Now multiply, digits and exponents separately.

```
2.5 times 3.0 = 7.5
10^4 times 10^-2 = 10^(4 minus 2) = 10^2
so 7.5 times 10^2 = 750
```

Seven hundred and fifty. Check it against the full numbers: 25,000 times 0.03 is 750.

8. The step that broke is the second line. Multiplying two powers **adds** the exponents; it does not multiply them.

```
10^6 times 10^3 = 10^(6 plus 3) = 10^9
```

Check it by counting zeros. Six zeros times three zeros gives nine zeros, and 10^{18} would be eighteen.

Why it looks reasonable: the operation between the two numbers is a multiplication, so multiplying the small raised numbers feels like the matching move. The exponents count how many copies there are, and putting two piles together adds the counts.

9. The third line took the two roots before the two areas were added. The square root of a sum is not the sum of the square roots.

Work out what sits under the root first.

```
900 plus 1,600 = 2,500 square metres
2,500^(1/2) = 50 metres
```

Fifty metres, not seventy.

Check both by multiplying back.

```
50 times 50 = 2,500    the area that was asked for
70 times 70 = 4,900    nearly twice it
```

Why it looks reasonable: every line that contains arithmetic is right. Both roots come out whole, which makes the working look clean and finished. And splitting a sum into its parts is something you are allowed to do with plenty of operations. A root is not one of them.

The habit that prevents it: finish everything under the root sign, then take one root of one number.

10. The third line inverts the comparison. A more negative exponent means a smaller number, not a larger one.

Write both out and the question answers itself.

```
5 times 10^-3 = 0.005
2 times 10^-6 = 0.000002
```

The first is the larger, by more than a thousand times.

Why it looks reasonable: the sentence contains a true statement. Minus three really is greater than minus six. The wrong conclusion is then drawn from it. A true clause sitting next to a wrong one is harder to spot than two wrong ones.

The habit that prevents it: write the number out in full before comparing anything.

11. Put both figures into the same form before comparing them.

```
1.2E+06 = 1.2 times 10^6 = 12,00,000 tonnes a year
3 lakh a quarter = 3,00,000 times 4 = 12,00,000 tonnes a year
```

The two agree. Annual production and four quarters of capacity are the same number, so "close to capacity" is supported: it is at capacity.

What this does not establish. It says nothing about whether capacity is the right measure of anything. It says nothing about whether production was steady across the four quarters, or whether the tonnes were produced or merely reported. Two numbers agreeing tells you the arithmetic is consistent. It does not tell you either number is true.

12. Do it in powers of ten, which is where the slip will show.

```
2 crore = 2 times 10^7 people
1 rupee each
2 times 10^7 times 1 = 2 times 10^7 rupees
= 2,00,00,000 = 2 crore rupees
```

Two crore rupees, not two lakh. The sentence is out by a factor of a hundred.

The slip is that a rupee a head for a crore of people costs a crore of rupees. The unit changes from people to rupees and the power of ten does not.

What this does not establish. Nothing here says whether two crore rupees is affordable, whether a rupee a head buys anything, or whether the two crore people figure is real. It establishes that the number in the room is a hundred times too small, which is enough to stop the meeting agreeing to it.

A7 · Logarithms — what they are for before how they work

EXERCISE 1. Equal distances up this axis are equal multiplications. A straight line means the quantity is multiplying by the same factor in each equal step along the bottom. Doubling every year, say, or rising tenfold every decade.

That is a very different claim from the one a straight line usually carries. On an ordinary scale a straight line means the same amount is being added each step.

On an ordinary scale, this same data would not be a straight line at all. It would hug the bottom of the page for most of its length and then turn sharply upward near the right-hand edge. Almost all of the visible movement would happen in the last part of the figure, and the early values would be indistinguishable from zero.

Which is worth saying out loud: the log axis did not change the data. It changed which part of the data you can see.

EXERCISE 2. By dividing.

```
10,00,000 divided by 25,000
= 1,000 divided by 25    (take three zeros off each)
= 40
```

By subtracting the logarithms.

```
6.0 minus 4.4 = 1.6
```

A difference of 1.6 on the logarithm scale means a factor of $10^{1.6}$. A calculator gives about 40, which agrees with the division. Note the shape of it: the division needed the zeros counted, and the subtraction did not.

Ten crore. A crore is 10^7, so ten crore is 10^8, and the logarithm is 8.

Check it against the ladder. Ten crore is a hundred times the largest penalty in the Act. A hundred is two powers of ten, so the logarithm should be 6 plus 2. It is.

1. The logarithm is the power of ten, so read the exponent off each number.

```
100    = 10^2    so the logarithm is 2
10,000 = 10^4    so the logarithm is 4
1    = 10^0    so the logarithm is 0
10^9    so the logarithm is 9
```

The third is the one people get wrong under pressure. The logarithm of one is zero, because ten to the power zero is one.

2. Raise ten to each one.

```
3    → 10^3 = 1,000
6    → 10^6 = 10,00,000    (ten lakh)
0    → 10^0 = 1
-2 → 10^-2 = 0.01
```

A negative logarithm belongs to a number between zero and one. It does not belong to a negative number, and no logarithm does.

3. Put 4,800 between the two powers of ten that bracket it.

```
1,000 = 10^3
4,800
10,000 = 10^4
```

So the logarithm lies between 3 and 4.

Which end it is nearer. The halfway point on a logarithm scale is not the halfway point in the number. Halfway between 10^3 and 10^4 is about 3,160, and 4,800 is above that. So the logarithm is above 3.5, nearer 4. A calculator gives 3.68.

4. The logarithms first. Both numbers are exact powers of ten, so read the exponent off each one.

```
100    = 10^2    so log10 100    = 2
10,000 = 10^4    so log10 10,000 = 4
```

Add them.

```
2 plus 4 = 6
```

Turn the total back into a number by raising ten to it.

```
10^6 = 10,00,000    (ten lakh)
```

Now the direct check.

```
100 times 10,000 = 10,00,000
```

The same number both ways. An addition did a multiplication, and that is the whole reason logarithms were invented. Underneath it is the exponent rule you already have: multiplying two powers of ten adds their exponents, and a logarithm is that exponent.

5. Write each as a power of ten first. Every one of the three is an exact power, which is why this problem has clean answers.

```
ten lakh    = 10,00,000    = 10^6    logarithm 6
one crore = 1,00,00,000    = 10^7    logarithm 7
ten crore = 10,00,00,000 = 10^8    logarithm 8
```

Six, seven, eight. Each step up the ladder is one on the logarithm scale and ten times in rupees, which is what a logarithm scale is for.

6. Read the steps, not the words. Subtract one mark from the next, then divide one by the next, and see which of the two comes out the same every time.

```
A    2 lakh minus 1 lakh = 1 lakh    3 lakh minus 2 lakh = 1 lakh
B    10 lakh divided by 1 lakh = 10    1 crore divided by 10 lakh = 10
C    250 minus 0 = 250    500 minus 250 = 250
D    1,000 divided by 100 = 10    10,000 divided by 1,000 = 10
```

A and C are ordinary. Equal distances up the page are equal amounts.

B and D are logarithmic. Equal distances are equal multiples, and one step is ten times.

C settles itself before you subtract anything. It has a mark at zero, and a logarithm scale has no position for zero, because no power of ten produces it.

B is the one that catches people. Lakh and crore are round-sounding words, so a reader skims the labels and takes the axis for an ordinary one. A is built from the same word and is ordinary. The word tells you nothing. The step tells you everything.

7. The two logarithms.

25,000	logarithm 4.4
3,00,000	logarithm 5.5

Subtract.

```
5.5 minus 4.4 = 1.1
```

A difference of 1.1 on the logarithm scale means a factor of $10^{1.1}$, which a calculator gives as about 12.6.

Check it by dividing directly.

```
3,00,000 divided by 25,000
= 300 divided by 25
= 12
```

Twelve against 12.6. The small gap is the rounding in the two logarithms, each given to one place. That is the price of working on the logarithm scale, and it is usually worth paying.

8. The two logarithms. A calculator gives the first; the second is an exact power of ten with a digit in front, so a calculator is easiest there too.

```
log10 5     = 0.7
2,00,000    = 2 times 10^5
log10 2,00,000 = 5.3
```

Add them, because adding logarithms multiplies the numbers.

```
0.7 plus 5.3 = 6.0
```

Raise ten to the total to get back to a number.

```
10^6 = 10,00,000 kilograms a month
```

Ten lakh kilograms, which is a thousand tonnes.

Now check it directly.

```
5 times 2,00,000 = 10,00,000
```

The same figure. Keep the label on it. Kilograms of grain in one month, not people and not rupees.

Notice what the logarithms bought here: nothing at all. Two numbers this easy are quicker multiplied. The move is worth having for the case where the numbers are not easy, and worth practising on the case where you can check it.

9. The logarithms, from a calculator.

300	logarithm 2.5
30,000	logarithm 4.5

The distance between them.

```
4.5 minus 2.5 = 2
```

Two whole steps on the axis, which is two powers of ten.

```
10^2 = 100
```

So one value is a hundred times the other. Check it directly: 30,000 divided by 300 is 100.

Notice what the axis showed you. Two marks apart always means a hundred times, wherever on the axis you measure it.

10. Adding logarithms corresponds to multiplying the numbers, not adding them.

Check what the working actually computed.

```
200 times 300 = 60,000
```

Sixty thousand, and $10^{4.8}$ is about 63,000, which is that figure with the rounding in the two logarithms. So the arithmetic performed a multiplication correctly and the answer was given the wrong name.

The sum itself is simply 500, and logarithms are no help with it at all.

Why it looks reasonable: the words “add” and “plus” appear in the working and in the question, so the operations look matched. Adding logarithms multiplies. There is no logarithm rule for adding two numbers.

11. Each mark is a multiplication by ten, so equal marks are equal multiples and not equal amounts.

Suppose the line started at 1.

2015	1	
2020	10	an increase of 9
2025	100	an increase of 90

The second rise is ten times the first in amount.

“Steady” is right only if it means multiplying by the same factor each period. That is exponential growth, and it is the opposite of what most readers hear in the word.

Why it looks reasonable: the line is straight and the gaps are equal. On an ordinary axis that is exactly what steady growth looks like. The axis changed and the eye did not.

12. What to work out: what happened to the zeros.

A logarithm scale has no position for zero, because no power of ten gives zero. So every observation with a value of zero has been dropped from the figure, and the caption’s first sentence is describing that without saying so.

```
logarithm of 1    = 0
logarithm of 0.1  = -1
logarithm of 0.01 = -2
logarithm of 0    = does not exist
```

Going down the axis makes values smaller for ever and never reaches zero.

What the figure cannot establish. It cannot tell you how many observations were zero. Those may be the most important ones: a district with no cases, a facility with no stock. A figure that drops them can show a rising line while the number of places reporting nothing also rises. Ask for the count of zeros before you quote the figure.

13. What to work out: what the argument is about. That decides the axis, and nothing else does.

If the question is **whether the thing is growing by a constant factor**, doubling every year say, the log axis answers it. Constant multiplication is a straight line there, and nothing else is.

If the question is **how many more people are affected**, the ordinary axis answers it, because that is the axis on which distance means amount.

from 100 to 1,000 on a log axis	one step
from 100 to 1,000 in people	900 more people

Same data. The log axis shows the shape of the growth and hides its size; the ordinary axis does the reverse.

What this does not settle. Neither chart tells you whether the rise is real or whether the counting changed. And whichever you choose, say in the briefing which axis it is. A reader who assumes the other one takes away the opposite of what you meant.

A8 · Orders of magnitude and the back-of-envelope sanity check

EXERCISE 1. Energy first, in powers of ten.

```
450 calories, to one digit: 5 times 10^2
2 crore meals = 2 times 10^7
```

```
5 times 10^2, times 2 times 10^7
= 10 times 10^9
= 10^10 calories
```

Now the comparison, and both numbers in it are guesses.

```
one person eats about 2,000 calories a day    = 2 times 10^3    ← a guess
days in a year, to one digit    = 4 times 10^2
so about 8 times 10^5, call it 10^6 calories a year
```

```
10^10 divided by 10^6 = 10^4
```

About ten thousand people's worth of annual eating.

Is 2 crore meals possible for a state? Work backwards to what it implies.

```
2 times 10^7 meals, over about 200 school days    ← a guess
= 2 times 10^7 divided by 2 times 10^2
= 10^5 children fed each day
```

A lakh of children a day. For a large state that is low rather than impossible, and for a small one it is high. So the verdict is: possible. The figure to ask for next is the number of children enrolled, because that is the assumption doing all the work.

The assumptions, gathered.

- Two hundred school days.
- Two thousand calories a day for one person.
- That every meal was a lower primary meal, when the schedule sets a larger one for upper primary.

The third is the one that would move the energy figure most.

EXERCISE 2. Cost per child, in powers of ten.

```
40 crore rupees = 4 times 10^8
12 crore children, to one digit = 1 times 10^8
```

```
4 times 10^8 divided by 1 times 10^8 = 4 rupees per child per year
```

Four rupees a child for a year. Compare that against something you know: the price of one small item of food. Four rupees is around the cost of a single item, once.

So the conclusion is that one of the two figures is wrong, or the scheme does something much smaller than "reach" suggests. The likeliest reading is that 40 crore is an administrative line and not the cost of whatever the children receive.

What the check establishes: that the two numbers, taken together, cannot mean what the sentence implies. That is a strong finding and it took one division.

What it cannot establish: which of the two numbers is wrong, or what the right figure is. It also cannot establish that the scheme is bad. A four rupee scheme that hands out one piece of information might be exactly right. The check tells you to ask, not what the answer is.

1. Keep the first digit, look at the next one to decide whether it goes up, and count the places.

4,820	→ 5 times 10^3
61	→ 6 times 10^1
938,000	→ 9 times 10^5
0.0037	→ 4 times 10^{-3}

The first one rounded up from 4.8 to 5. At this level of roughness that is the point: you are not trying to keep the 8.

2.

```
412 times 7,900
→ 4 times 10^2, times 8 times 10^3
= 32 times 10^5
= 3.2 times 10^6, call it 3 times 10^6
```

```
61,000 divided by 290
→ 6 times 10^4, divided by 3 times 10^2
= 2 times 10^2
```

Above or below, for the first. The 412 was rounded down to 400 and the 7,900 up to 8,000. One each way, so the two pull in opposite directions and the estimate should be close. The true answer is 3,254,800, and the estimate of 3 million is a little under.

3. Divide one by the other and see whether the answer is under about ten.

```
9 times 10^7, divided by 4 times 10^6
= (9 divided by 4) times 10^(7 minus 6)
= 2.25 times 10^1
= about 22
```

Twenty-two is more than ten, so no: they are not of the same order of magnitude. One is about an order of magnitude and a bit above the other.

Notice that the exponents alone, 6 against 7, would have told you they were one step apart. The digits in front moved that to a bit more than one step. So divide, rather than reading the exponents off.

4. Two inputs are needed and neither was given, so both are guesses and both get marked.

```
children fed in a district    1,00,000 = 10^5    ← a guess
school days in a year    200 = 2 times 10^2 ← a guess
```

Multiply.

```
10^5 times 2 times 10^2
= 2 times 10^7 meals
```

Two crore meals, so 10^7 to the nearest power of ten.

Both guesses are the answer's real content. Suppose a district turns out to feed three lakh children. The answer moves to 6 times 10^7, which is still 10^7 to the nearest power of ten. That stability is what the method buys: the answer survived a guess being wrong threefold.

5. One input is given, one is a guess.

```
5 kilograms per person per month
people covered in a district    5,00,000 = 5 times 10^5    ← a guess
```

Work it through.

```
5 kilograms times 5 times 10^5 people = 25 times 10^5 kilograms a month
= 2.5 times 10^6 kilograms a month
times 12 months = 30 times 10^6
= 3 times 10^7 kilograms a year
```

Now the unit. A tonne is a thousand kilograms.

```
3 times 10^7 divided by 10^3 = 3 times 10^4 tonnes a year
```

About thirty thousand tonnes.

Which input it depends on most: the population guess, because it is the only one that can be wrong. The 5 kilograms and the 12 months are both exact.

6. The digits were multiplied correctly and then the power of ten was mishandled in the last step.

```
60 times 10^4 = 6 times 10^5
```

Sixty is 6 times 10^1, and that extra ten has to go into the exponent, not disappear.

```
3 times 10^4, times 2 times 10^1
= 6 times 10^5 kilograms a month
```

Six lakh kilograms, or 600 tonnes, and the worked answer said sixty thousand.

Why it looks reasonable: the digit 6 and the exponent 4 both sit in the line above. So the last line looks like a tidy-up rather than a step. It is a step. Putting one digit in front changes the exponent every time the digits reach ten.

7. The third line crosses from the Indian ladder to the international one and drops a power of ten on the way. A lakh is 10^5, so thirty lakh is thirty of them.

```
30 lakh = 30,00,000 = 3 times 10^6
```

Three million is right. Three times 10^5 is not. That is three lakh, ten times too small.

Carry the corrected figure through.

```
3 times 10^6, times 2 = 6 times 10^6 kilograms a month
6 times 10^6 divided by 10^3 = 6 times 10^3 tonnes a month
```

Six thousand tonnes, and the worked estimate said six hundred.

The comparison would have caught it without any of this. Six hundred tonnes shared among thirty lakh people is 0.2 kilograms each, and the line above says two. An estimate that contradicts its own input is finished before you look at the exponents at all.

Why it looks reasonable: the line holds a true statement. Thirty lakh really is three million. The 3 was then taken from "million". The 10^5 was left behind from "lakh". Two halves of a correct sentence got fitted to the wrong number.

8. Check the arithmetic first, since it is offered.

> 10^6 times 0.02 = 2 times 10^4
> 2 times 10^4 times 3 = 6 times 10^4

That is right. Sixty thousand visits.

What is wrong is that every input is a guess and nothing was compared against anything. The method has two halves and only the first was done. An estimate that is never checked against a quantity you actually know is three guesses multiplied together and presented as a finding.

What the second half would look like. Sixty thousand visits a year is about 160 a day across the district. How many facilities and how many staff does that imply, and is that possible? That comparison is where an impossible answer shows itself.

Why it looks reasonable: the guesses are all marked, which is the honest part, and it makes the working look complete. Marking a guess does not test it.

9. Work out the factor the conclusion rests on.

> 1.8 times 10^6, divided by 1.2 times 10^6 = 1.5

One and a half. That sits well inside a factor of two, and this method is blind to a factor of two. Every input was rounded to one digit, and the answer is claimed only to the nearest power of ten.

> 1.2 times 10^6, to the nearest power of ten = 10^6
> 1.8 times 10^6, to the nearest power of ten = 10^6

Both districts come out at 10^6 visits. The estimate says the two are the same size and says nothing else. A staffing sentence needs a real count of households in each district.

What the method could have supported. Had district B come out at 10^7, a tenfold gap would have survived the rounding and the conclusion would have stood.

Why it looks reasonable: the arithmetic is faultless, both guesses are marked, and the 1.5 came out of a division rather than being asserted. A number carries the precision of what went into it, and one-digit inputs cannot carry one and a half.

10. Compute the cost of one glass, since that is the only quantity you have any feel for.

> 50 lakh children = 5 times 10^6
> 200 days = 2 times 10^2
> glasses = 5 times 10^6, times 2 times 10^2 = 10^9 glasses
> 100 crore rupees = 100 times 10^7 = 10^9 rupees
> 10^9 rupees divided by 10^9 glasses = 1 rupee a glass

One rupee a glass of milk. Compare that against a price you know: a glass of milk costs several times that at any shop in India.

So something in the sentence is not as stated. Perhaps one of the three figures is wrong. Perhaps the milk is not bought at retail. Perhaps "provide" is doing work the note never explains.

What this does not establish. It does not say which figure is wrong, and it does not say the programme is a bad one. It establishes that the three numbers cannot all be right as stated, which is enough to ask before repeating any of them.

11. Work backwards to something you can judge. Per person is usually the way.

```
40,000 crore = 4 times 10^4 times 10^7 = 4 times 10^11 rupees
population of India, roughly 1.4 times 10^9    ← a guess
4 times 10^11 divided by 1.4 times 10^9
= about 2.9 times 10^2
= roughly 300 rupees per person per year
```

Three hundred rupees a person a year, from every person in the country, on sugary drinks tax alone. Now compare against something you know. That is a meaningful share of what many households spend on all drinks in a year, and it assumes nobody avoids the tax.

What to tell the journalist, in one sentence.

> *That works out at about 300 rupees a year from every person in India. That is high enough that I would want the assumed consumption and tax rate before repeating it.*

What this does not establish. It does not show the figure is wrong. A narrow tax at a high rate on heavy consumers could reach it. It establishes which two numbers to ask for, and it took one division and one comparison.

B1 · What a unit is, and the SI base and derived units

EXERCISE 1. The seven base units are the second, the metre, the kilogram, the ampere, the kelvin, the mole and the candela.

Four of them do nearly all the work in this course. The second measures time, the metre measures length, the kilogram measures mass, and the mole measures an amount of a substance. The ampere is for electric current, the kelvin for temperature and the candela for the brightness of a light source. You will meet those three rarely.

A derived unit is one built out of base units by multiplying and dividing them.

Two you can build without being told.

```
speed    = metres divided by seconds = m/s
area     = metres times metres       = m^2
```

If you also wrote kg/m^3 for density, or m^3 for volume, those are right too. There is no fixed list to have learnt. There is a way of building them.

EXERCISE 2. What is missing is the unit. The header names the thing measured and not the quantity, so none of the four numbers is a measurement yet.

Ask for two things, in this order. First, the unit each column is recorded in. Second, who enters it and on what form. A column with no unit in its header is usually one where the unit lives in somebody's habit.

Now the 4.2. The tempting move is to work out what it must be from its size. It sits beside numbers in the hundreds. So it looks like a different unit, or a typing slip.

Do not do that. You can say that the 4.2 is evidence the column is not uniform, which is a reason to stop and ask. You cannot say what it is. Guessing the unit from the size of the number is the habit this section exists to remove. A guess written into a report becomes a figure somebody else quotes.

EXERCISE 3. Ten minutes, for a resident who expects a mechanism.

Start by agreeing with them. Memorising it is a bad plan, and forgetting it is what happens to everybody who tries.

Then show the two halves of the answer.

First, where the statement lives. Open the SI's own page at bipm.org and search for "J =". The line is there, on the SI's own page. A unit is decided by a body. So the body's own page is where you settle it. That takes twenty seconds and you never have to trust your memory again.

Second, how to read it once you have it. Write on the board.

```
J = kg m^2 s^-2
```

Say it as a sentence: a kilogram, times a square metre, divided by a second squared. Ask them to say it back. Then ask what kg m s^{-2} would be a sentence about instead. Let them notice that it names a different quantity. It is not a worse way of writing the same one.

Close on the move you actually want them to keep. When they meet a unit they do not recognise, write it out in base units and read it as a sentence. That works for every unit they will ever meet, and there is nothing in it to forget.

B2 · Converting units as multiplication by one

EXERCISE 1. The height first. One metre is 100 centimetres, so the factor is 1 m over 100 cm, and the cm cancels.

```
96 cm times 1 m over 100 cm
= 0.96 m
```

A metre is bigger than a centimetre, so the number had to get smaller. It did.

Now the sack, in two steps. First the kilograms in the sack.

```
0.4 kg per packet, times 25 packets
= 10 kg
```

The packets cancelled there, which is worth noticing: a unit does not have to be a unit of measurement to cancel.

Then kilograms into grams, with the factor the other way up from the height one.

```
10 kg times 1,000 g over 1 kg
= 10,000 g
```

Ten thousand grams. A gram is smaller than a kilogram, so the number had to get bigger.

EXERCISE 2. The arithmetic that produced the 10 is a division by 1,000 somewhere, after a correct multiplication.

```
250 times 40 = 10,000
```

Ten thousand millilitres. The number is right; the unit on it is not.

A millilitre is a thousandth of a litre, so ten thousand millilitres is ten litres.

```
10,000 mL times 1 L over 1,000 mL
= 10 L
```

So the carton holds 10 litres, and the sentence dropped the conversion while keeping its result. That is the most dangerous shape of this error, because the 10 survives and only its unit is wrong.

What to change in how they work. Write the unit on every line. Read the unit off the last line, rather than deciding it from what the sentence is about. The unit is an output of the working, not a label added at the end.

1.

```
3 km times 1,000 m over 1 km    = 3,000 m
4,500 g times 1 kg over 1,000 g    = 4.5 kg
2.5 m times 100 cm over 1 m    = 250 cm
```

Check each one for direction. Metres are smaller than kilometres, so 3 became 3,000. Kilograms are bigger than grams, so 4,500 became 4.5. Centimetres are smaller than metres, so 2.5 became 250.

2. One kilogram is 1,000 grams, so the two orientations are these.

```
1,000 g over 1 kg
1 kg over 1,000 g
```

Both are equal to one, because the top and the bottom of each are the same mass written two ways.

The one that turns grams into kilograms is 1 kg over 1,000 g. The grams you start with sit on the top, the grams in the factor sit on the bottom, and they cancel. Kilograms are left.

The other one does not cancel anything. Grams on the top, multiplied by grams on the top, give grams squared per kilogram, and there is no such quantity in the question.

3. Two hours, through minutes.

```
2 h times 60 min over 1 h times 60 s over 1 min
= 2 times 60 times 60 s
= 7,200 s
```

The hours cancel against the hours, then the minutes cancel against the minutes. Seconds are left.

Now the mass, through grams.

```
0.75 kg times 1,000 g over 1 kg times 1,000 mg over 1 g
= 0.75 times 1,000 times 1,000 mg
= 750,000 mg
```

Both answers got bigger, which is right: seconds are smaller than hours and milligrams are smaller than kilograms.

4. Grams first.

```
35 kg per household per month, times 1,000 g over 1 kg
= 35,000 g per household per month
```

Now the year. Twelve months make a year, so the factor is 12 months over 1 year, and the months cancel.

```
35 kg per household per month, times 12 months over 1 year
= 420 kg per household per year
```

Say both denominators out loud. The first is per household per month. The second is per household per year. Only one half of the denominator changed in the second conversion, and the per-household half was carried through untouched.

Now the last line. The proviso does not hand 35 kilograms to every Antyodaya household by itself. The figure reads "thirty-five kilograms of foodgrains per household per month at the prices specified in Schedule I". And the entitlement runs "to such extent as may be specified by the Central Government for each State in the said scheme". So you can convert the 35. You cannot say, from this clause alone, that a named household in a named State is due 35 kilograms. You cannot say it comes free either.

What the extent is, in that State, is set under the scheme. Look it up before you quote a figure for anybody.

The arithmetic was never the risk here. Quoting a conditional entitlement as a flat quantity is.

5. One milligram is 1,000 micrograms. The symbol for micro is the Greek letter mu, written μ, so a microgram is μg.

Going down to the smaller unit, put mg on the bottom.

```
0.25 mg times 1,000 μg over 1 mg
= 250 μg
```

Going back up to the bigger unit, turn the same factor over.

```
4,000 μg times 1 mg over 1,000 μg
= 4 mg
```

Two conversions, one agreement, two orientations. The direction check holds both times: the number grew going to the smaller unit and shrank going to the bigger one.

6. The first two lines hold. The agreement is right: one gram is 1,000 milligrams.

The step that broke is the third. The factor is upside down. It puts grams on the top, where there are already grams, so nothing cancels.

Turn it over and the grams cancel.

```
0.5 g times 1,000 mg over 1 g
= 500 mg
```

Five hundred milligrams, not 0.0005.

Two checks would each have caught it on its own. The direction check: a milligram is smaller than a gram, so the number had to get bigger. And the unit check: the working as written leaves grams squared per milligram, which is not a unit anybody prescribes in.

Why it looks reasonable: the number 0.0005 is small, doses are often small, and the line contains the correct agreement one line above. Every ingredient of a right answer is present, in the wrong arrangement. That is the shape this error always has, and it is why the unit that falls out matters more than the number.

7. The step that broke is the third one, and what broke it is the square.

The factor between metres and kilometres was applied once. The quantity is not a length. It is a length times a length, so the factor has to be applied twice.

```
1 km² = 1 km times 1 km
= 1,000 m times 1,000 m
= 1,000,000 m²
```

Now convert with that.

```
12,000 m² times 1 km² over 1,000,000 m²
= 0.012 km²
```

Twelve thousandths of a square kilometre, not twelve.

The answer as given is out by a factor of a thousand, which is the factor that was left out.

Why it looks reasonable: the arithmetic is right, the agreement quoted is right, and the words "square kilometres" appear in the answer. Nothing in the line says the square was ignored. The guard is to write the unit with its power on every line. Squared metres on one side and plain kilometres on the other is a mismatch you can see. The same mismatch written out in words is not.

8. What to compute: both strengths in one unit, so that they can be compared at all. Take micrograms.

```
0.5 mg times 1,000 µg over 1 mg
= 500 µg
```

The two tablets are the same strength. Five hundred micrograms is 0.5 milligrams, written another way.

The sentence has read a change of unit as a change of amount. That is what a conversion factor is for, and it is why multiplying by one is worth a whole section: the amount did not move.

What this does not establish. It says nothing about whether the two tablets behave the same way in a person, because strength is only the amount of drug in them. It says nothing about how many tablets are taken or how often. And it says nothing about why the label changed units, which is the question actually worth asking in that meeting.

B3 · Dimensional analysis as an error check

EXERCISE 1. A unit check tests the shape of a formula. It cannot see a plain number multiplying it, because a plain number carries no unit and changes none.

So the check can reject, and it can never confirm. What your colleague has established is that the formula is not wrong in one particular way.

An example. The area of a circle is worked out from its radius, which is a length.

```
radius times radius
= m times m
= m²
```

Anything of the form "a number times the radius squared" comes out in square metres. So all of these pass the unit check.

```
3.14 times r²
2 times 3.14 times r²
6 times r²
```

Only the first is the area of a circle. The other two are square metres of something that does not exist, and no unit check will ever tell you which one you are holding.

What to say back. Ask where the formula came from, and check the number in front. The unit check is the first gate, not the last one.

EXERCISE 2. Ten minutes, for somebody who expects a mechanism rather than a rule.

Open by moving the goalposts. Tell them they are not going to remember formulas, and that you do not either. The useful skill is being able to throw the wrong ones out.

Write a wrong one on the board without saying it is wrong.

```
time for the journey = distance times speed
```

Ask them for the units of each piece. They will give you kilometres and kilometres per hour. Write those in.

```
km times km per hour
= km² per hour
```

Ask what kind of quantity that is. Let them get to "an area divided by a time" on their own, and then ask whether it can be a journey time. That is the moment the idea lands, and it lands because they did it rather than watched it.

Now turn it round. Ask what you would have to do to those two units to be left with hours. They will find the division. Write the four steps under it.

```
1. throw the numbers away
2. write the unit of every quantity
3. cancel what appears on the top and the bottom
4. compare the two sides
```

Close with the limit, because a resident who over-trusts this will be caught out by it. Tell them the check throws out wrong formulas and never certifies a right one, and give them the circle example if there is time.

1.

```
m/s times s    = m
m times m times m    = m³
kg divided by m³, times m³ = kg
```

In the first, the seconds on the top cancel the seconds on the bottom, and metres are left.

In the second, three metres multiplied together give metres to the power three.

In the third, the m^3 on the bottom cancels the m^3 on the top, and kilograms are left. That one is worth remembering as a shape: a density multiplied by a volume gives a mass.

2.

```
3 m and 4 m    as they stand    3 plus 4 = 7, so 7 m
3 m and 4 s    never
3 kg and 4 g    after converting
3 m² and 4 m    never
```

The second pair is a length and a time. There is no conversion between them, because they are not the same kind of quantity.

The third pair is two masses in different units, so convert one and add.

```
3 kg times 1,000 g over 1 kg = 3,000 g
3,000 plus 4 = 3,004
```

Three thousand and four grams.

The fourth pair is an area and a length. Squared metres and metres are not the same unit, whatever they look like on the page. That is the pair people actually get wrong, because both of them say “metres” out loud.

3.

```
a mass divided by a volume    = kg divided by m³    = kg/m³
a length divided by a time    = m divided by s    = m/s
that, divided by a time again    = m/s divided by s    = m/s²
people divided by an area    = people divided by m²
```

The third one is worth a second look. Dividing by seconds twice gives seconds squared on the bottom, which you write as m/s^2 or as $m\ s^{-2}$.

The fourth is a reminder that a count can sit in a unit. People per square metre is a perfectly good unit, and you will more often meet it as people per square kilometre.

4.

```
kg m² s⁻² divided by kg = m² s⁻²
kg m² s⁻² divided by s    = kg m² s⁻³
m² divided by m    = m
```

The first: the kilograms cancel outright.

The second: there was already a second squared on the bottom, and dividing by another second makes it a second cubed. The exponent went from minus two to minus three, which is the rule for dividing powers you already have.

The third: two metres on the top, one on the bottom, one left over.

5. Put kg m^2 s^{-2} in place of J every time, then cancel.

```
J divided by kg    = kg m² s⁻² divided by kg    = m² s⁻²
J times s² over m²    = kg m² s⁻² times s² over m²    = kg
J divided by s    = kg m² s⁻² divided by s    = kg m² s⁻³
```

The middle one is the one to notice. Multiplying by s^2 cancelled the s^{-2}, and dividing by m^2 cancelled the m^2, so a kilogram was left standing on its own.

That is what it means to say the joule is built out of base units. You can take it apart again and get them back.

6. Write the units and cancel.

```
mg per mL, times mL
= mg times mL over mL
= mg
```

The millilitres cancel and milligrams are left. So the total amount comes out in milligrams, which is a unit an amount of a drug is measured in. The formula is the right shape.

Now try it the other way round, so you can see the check working.

```
mg per mL, divided by mL
= mg per mL squared
```

Milligrams per square millilitre is not a quantity anybody uses. Had the protocol divided, the units alone would have told you to send it back.

7. You are given kilograms per household per year and asked for kilograms per person per month. So two things have to come off the answer: the year has to become months, and the household has to become people.

Write the units first and decide the arrangement from them.

```
kilograms per household per year
divided by months per year
divided by persons per household
= kilograms per person per month
```

The years cancel against the years and the households cancel against the households. Kilograms, one person and one month are what is left standing.

Now the numbers. A year holds twelve months, and the note's household holds six people.

```
360 divided by 12 = 30
```

Thirty kilograms per household per month.

```
30 divided by 6 = 5
```

Five kilograms per person per month.

That is the rate section 3(1) of the Act states for a person in a priority household. The 360 in the note is not in the Act. It is somebody's arithmetic on the monthly rate, and working back down is how you check whose arithmetic it was.

8. Units first.

```
minutes divided by kilometres
= min/km
```

Minutes per kilometre is a real quantity. It is a pace, which is how long each kilometre takes. It is not a speed, because a speed is a distance divided by a time.

Turn it over and you get kilometres divided by minutes.

```
kilometres divided by minutes
= km/min
```

That is a speed, and it is not yet the speed you were asked for. Kilometres an hour is what a road speed is quoted in, so the minutes have to become hours. Use the conversion factor from the section before this one, with the minutes where they cancel.

```
km/min, times 60 min over 1 h
= km/h
```

Now the numbers. The time is in minutes and the speed you want is per hour, so convert first.

```
30 min times 1 h over 60 min = 0.5 h
```

Then divide.

```
18 divided by 0.5 = 36
```

Thirty-six kilometres an hour.

Both quantities are useful and they answer different questions. The unit is what tells you which of the two you are holding.

9. A density is a mass divided by a volume, so its unit has to be kilograms per cubic metre.

Now the note's arrangement.

```
m³ divided by kg
= m³/kg
```

Cubic metres per kilogram is upside down. It is a volume in every kilogram, which is a real quantity and is not what was asked for.

Turn it up the right way.

```
kg divided by m³
= kg/m³
```

Now the numbers.

```
2,400 divided by 1.2 = 2,000
```

Two thousand kilograms per cubic metre.

Run the size check on that answer as well. A cubic metre is a large volume, so a density in kilograms per cubic metre is a large number. Had the note's arrangement been used, the answer would have been 0.0005, and a number that small for a heavy block is its own warning.

10. The step that broke is the second one, and the units say so before the arithmetic does.

```
30 m times 20 m
= 600 m²
```

Metres times metres give square metres. That is an area, and fencing is a length. The formula is the wrong shape for what was asked.

Fencing runs round the edge, so add the four sides.

```
30 plus 20 = 50
50 times 2 = 100
```

A hundred metres of fencing.

Why it looks reasonable: multiplying the two sides of a rectangle is a move everybody knows, and 600 is a plausible-looking quantity of fencing. The word "metres" even appears in the answer. It appears because somebody typed it, not because it fell out of the working, and that is the difference this section is about.

11. The step that broke is the third. A quantity given for each month was multiplied by a number of days.

Write the units of that multiplication out.

```
kilograms per person per month
times days
= kilogram days per person per month
```

Kilogram days per person per month is not a quantity of grain. Nothing cancelled, because nothing matched: there is a month on the bottom and a day on the top.

What goes on the top has to be months, and a year holds twelve of them.

```
kilograms per person per month, times 12 months
= kilograms per person per year
5 times 12 = 60
```

Sixty kilograms per person per year. That figure is your arithmetic and not the Act's. What the Act states is five kilograms per person per month. So whenever you quote the sixty, say that you multiplied by twelve months to get it.

Why it looks reasonable: the 365 is a real and familiar number, and the arithmetic is right. The answer is even labelled per year, which is the label that was wanted. Every part of it is correct except the one that mattered, which is what the 365 was counting.

12. The step that broke is the fifth. Containing the same letters is not carrying the same units.

Write both out with their powers side by side.

what it is	its unit
the proposal	kg m s^{-1}
the joule	kg m^2 s^{-2}

The metre is to the power one in the proposal and to the power two in the joule. The second is to the power minus one in the proposal and minus two in the joule. Two of the three powers differ, so these are different quantities that happen to be built from the same three base units.

So the proposal is rejected. A mass times a speed is not an energy.

Notice what you cannot say. You cannot say what energy is a mass times, because the check does not produce right formulas. It only throws out wrong ones, and here it has thrown one out.

Why it looks reasonable: the last two lines have the form of a check. The units are written down and compared, so the working looks like the work. Reading the letters and ignoring the powers is the commonest way this check is done badly. It is worse than not doing it at all, because it ends with the word "fine".

13. What to compute: litres per person per day. The unit tells you the arrangement, because litres per person per day means litres divided by people.

```
litres per day, divided by people
= litres per person per day
```

Write both figures out in full first.

```
4.5 crore litres a day = 4.5 times 10^7 = 45,000,000 litres a day
30 lakh people    = 3 times 10^6    = 3,000,000 people
```

Now divide.

```
45,000,000 divided by 3,000,000 = 15
```

Fifteen litres per person per day, not 135. The line is out by a factor of nine.

What this does not establish. It does not tell you what a person needs. Both 15 and 135 are only numbers until somebody names a standard and says who set it. It does not tell you that the water reaches anybody, since a plant treating water and a household receiving it are two different claims. And both figures came from the note itself, so the arithmetic being wrong is the only thing here you have checked.

14. What to compute: the mass of chlorine for one fill. Start from the units, because the brochure has divided and you should check whether dividing can give a mass.

```
milligrams per litre, divided by litres
= mg per litre squared
```

Milligrams per square litre is not a mass. So the arrangement is wrong, whatever the number comes out as.

Multiplying is what leaves a mass standing.

```
milligrams per litre, times litres
= milligrams
```

Now the numbers.

```
2 times 5,000 = 10,000
```

Ten thousand milligrams. Convert that to grams, since the brochure answered in grams.

```
10,000 divided by 1,000 = 10
```

Ten grams a fill, against the 0.0004 grams claimed. The brochure is out by a factor of twenty-five thousand, and the error is one inverted arrangement.

What this does not establish. It says nothing about whether 2 milligrams per litre is the right dose for that tank, which is a question for whoever sets the standard. It says nothing about whether the unit delivers what it claims to deliver. And it says nothing about what the chlorine is mixed with. Ten grams of a product is ten grams of chlorine only if the product is pure.

B4 · Energy units: joule, kilojoule, calorie, kilocalorie

EXERCISE 1. The word that fails is measurement. The value is a definition. A committee set the thermochemical calorie at 4.184 joules exactly, and a different committee set the International Table calorie at 4.1868 joules.

You can tell from the number itself. A measured quantity comes with a spread and gets sharper as instruments improve. This one is exact and always was, because it was chosen.

The replacement sentence names the decision and leaves the door open.

> *The thermochemical calorie is defined as 4.184 joules exactly. A second definition, the International Table calorie, is 4.1868 joules, so a paper should say which one it used.*

One more thing follows. Nothing an experiment finds can change either figure. Only a standards body can, which is why this section carries an as-of date and the arithmetic sections do not.

EXERCISE 2. Answer first, in their words.

> *The number on the packet is in kilocalories, even when the packet just says calories. One kilocalorie is 4.184 kilojoules, and that factor is fixed by agreement rather than measured.*

Then the part they asked about without knowing it. There are two calories in use and they differ by about seven parts in ten thousand. For a reader looking at a packet that difference is invisible. It only shows up when two published figures are compared to four digits.

Say the confidence out loud. The conversion is exact and not in doubt. What is worth doubting is the energy figure itself, and that is a different question from the unit.

What not to say: that one of the two calories is the right one. Neither is. They are two decisions, and a paper should tell you which it used.

1. Kilo means a thousand, so every line here is a multiplication or a division by 1,000.

```
1,000 divided by 1,000 = 1
2.5 times 1,000 = 2,500
3 times 1,000 = 3,000
7,000 divided by 1,000 = 7
```

Going to the bigger unit divides. Going to the smaller unit multiplies. Nothing about the amount of energy changed on any line.

2. Multiply each by 4.184.

```
1 times 4.184 = 4.184
10 times 4.184 = 41.84
250 times 4.184 = 1,046
1,500 times 4.184 = 6,276
```

Every answer is bigger than the number you started with. A joule is smaller than a calorie, so it takes more of them to hold the same energy.

3. Divide each by 4.184, because you are going back the way you came.

```
8.368 divided by 4.184 = 2
2,092 divided by 4.184 = 500
4,184 divided by 4.184 = 1,000
```

Check the direction on the last one. Four thousand joules is a thousand calories, so the calorie figure is the smaller of the two. If your answer came out larger, you multiplied where you should have divided.

4. Kilocalories to kilojoules uses the same factor as calories to joules.

```
450 times 4.184 = 1,882.8
```

So 1,882.8 kilojoules.

Now the digits. The Act gives 450, which carries two digits you can trust. Writing 1,882.8 claims five. Write 1,900 kilojoules, or say about 1,900 kilojoules a meal.

Keep the label with it. Kilojoules of food energy per hot cooked meal, not per child per day.

5. Once each way.

```
700 times 4.184 = 2,928.8
700 times 4.1868 = 2,930.76
```

The gap between them.

```
2,930.76 minus 2,928.8 = 1.96
```

Just under two kilojoules. As a share of the figure, divide the gap by either answer.

```
1.96 divided by 2,928.8 = 0.00067
```

About seven parts in ten thousand, which is under a tenth of one per cent.

Both answers are right. Which one you report depends on which calorie you say you used, and the Act's two digits cannot tell the two apart.

6. Divide, because kilojoules are the smaller unit and the count must come down.

```
2,000 divided by 4.184 = 478.0
```

So 478 kilocalories, or about 480 with the digits the figure deserves.

Now the same sum with the other calorie.

```
2,000 divided by 4.1868 = 477.7
```

478.0 one way and 477.7 the other. At two digits both are 480 kilocalories, so the choice of calorie could not have changed the answer you say out loud. Note which one you used anyway, so somebody checking you to four digits knows.

7. Into kilocalories first.

```
8,400 divided by 4.184 = 2,007.6
```

Round it the way a four-digit figure allows, and you would report about 2,000 kilocalories.

Now back again, starting from the rounded figure.

```
2,000 times 4.184 = 8,368
```

Eight thousand three hundred and sixty-eight against the 8,400 you began with. Not the same number.

The conversion did not lose anything. The rounding did. Convert once, from the figure you were given, and never chain a conversion onto a rounded answer.

8. Take each part of the food in turn, grams times its factor.

```
60 times 4 = 240
8 times 4 = 32
22 times 9 = 198
```

Now add the three together.

```
240 plus 32 plus 198 = 470
```

So 470 kcal for every 100 g. That is the figure the label should carry, because the regulation says the declared energy is worked out this way rather than measured.

Now into kilojoules, with the same 4.184 that turns kilocalories into kilojoules.

```
470 times 4.184 = 1,966.48
```

Round it before you write it. The grams on the label carry two digits each, so the last digits of 1,966.48 are not yours to keep. Report about 2,000 kilojoules for every 100 g.

Two things follow from this problem. Where a label declares only carbohydrate, protein and fat, you can reproduce its energy figure from the grams beside it, with no equipment. Where the food carries dietary fibre, polyols or organic acid, the panel need not print those grams. Your total then falls short of the declared figure, and nothing is wrong. And the regulation asks for no kilojoule figure, so the number you have just produced is one the label will never show you.

9. The third line divides where it should multiply.

A kilojoule is smaller than a kilocalorie, so the same energy needs more of them. The number has to go up, and this one went down.

```
450 times 4.184 = 1,882.8
```

One thousand eight hundred and eighty-three kilojoules, not a hundred and eight.

Why it looks reasonable: the working names the right factor and uses it once, so it has the shape of a finished conversion. Nothing in the digits shouts. Only the direction is wrong, and direction is the one thing the number cannot tell you.

The habit that prevents it: before you touch the calculator, say which unit is the smaller one and whether your answer should grow or shrink.

10. The arithmetic on line three is right and the unit on line four is wrong. The 250 was in kilocalories and the factor was applied as though it were in calories.

Take it in two steps.

```
250 times 1,000 = 2,50,000
2,50,000 times 4.184 = 10,46,000
```

Ten lakh forty-six thousand joules, which is 1,046 kilojoules. The digits 1,046 were right all along. The prefix was dropped.

Why it looks reasonable: the answer carries a plausible number and a real unit, and the multiplication checks out if you redo it. A prefix error never shows up in the digits. It shows up only as a factor of a thousand, and you have to be looking for it.

The habit that prevents it: write the prefix into every line of the working, not just the first.

11. What to compute: put the two figures in the same unit before deciding anything.

```
600 times 4.184 = 2,510.4
```

Two thousand five hundred and ten kilojoules, against the 2,510 in the other paper. The two papers agree exactly at the digits they printed. Nobody is wrong.

What this does not establish. The agreement is arithmetic, not evidence. Both papers may be quoting one original measurement, and a conversion between two units cannot tell you whether that measurement was any good. It does not tell you the two papers measured the same food, or prepared it the same way, or counted the same parts of it.

Two numbers agreeing after a conversion tells you the arithmetic is consistent. It says nothing about either number being true.

12. What to compute: whether the unit is the obstacle. Take a figure in one form and put it in the other, then reverse it.

```
500 times 4.184 = 2,092
2,092 divided by 4.184 = 500
```

The number came back unchanged. The conversion factor is exact, so moving between kilocalories and kilojoules loses nothing at all. No data need collecting again for this reason, and the proposal in the room would spend money on a rename.

What this does not establish. It does not show that the two datasets can be compared. Two sets of energy figures can differ because different foods were sampled. They can differ because the energy was worked out by different methods. No conversion touches either of those.

Say both halves out loud in the meeting. The unit is not the problem. Whether the two sets measured the same thing still is, and that is the question worth the money.

B5 · Concentration units: mg/dL, mmol/L, and converting between them

EXERCISE 1. The word that fails is “any”. The 18 is not a property of the two units. It is glucose’s own molar mass divided by ten.

```
180.16 divided by 10 = 18.016
```

So the rule as written is a glucose rule wearing a general coat. Apply it to a different substance and you get a number with the right unit on it and the wrong value inside it.

You can tell from the shape of the claim. Grams and kilograms measure the same kind of quantity, so one fixed factor crosses between them. mg/dL and mmol/L do not. One is a mass and the other is a count. Crossing between those has to involve the substance. So any rule that never mentions a substance has lost something on the way.

The replacement sentence names the substance and names the step that fetches the number.

> *To convert a value from mg/dL to mmol/L, multiply by 10. Then divide by the molar mass of that substance in grams per mole. For D-glucose the molar mass is 180.16 g/mol, so the divisor is 18.016. For any other substance, look its molar mass up first.*

One thing follows from that. A protocol that lists conversions must list them substance by substance. A single line covering all of them is not a shortcut, it is a defect.

EXERCISE 2. Answer first, in their words.

> *Both papers are right. They are describing the same blood in two different ways, the way the same distance can be a mile or a kilometre.*

Then the mechanism, briefly. One paper is weighing the sugar in a fixed amount of blood. The other is counting the sugar molecules in a fixed amount of blood. Weighing and counting give different numbers for the same blood. The sum that links them needs the weight of one molecule of that particular sugar.

Show them the sum if they want it.

```
92 times 10 = 920
920 divided by 180.16 = 5.1066
```

Round it to the digits the first paper gave you, and 92 becomes 5.1. That is the second paper's number.

Say the confidence out loud. This part is arithmetic and it is not in doubt at all. What is worth doubting is the blood sugar figure itself, and that is a different question from the unit.

What not to say: that one unit is the correct one. Neither is. What matters is that a paper says which unit it used, and which substance it measured.

1. Going from a decilitre to a litre multiplies by ten, because a litre holds ten of them. Going the other way divides by ten.

```
5 times 10 = 50
0.7 times 10 = 7
120 divided by 10 = 12
96 divided by 10 = 9.6
```

Nothing about the amount changed on any line. Only the size of the container the figure is quoted against changed.

2.

```
1,000 divided by 180.16 = 5.551
920 divided by 180.16 = 5.107
2,500 divided by 180.16 = 13.88
```

Every answer is much smaller than the number you started with, because you divided by a number near two hundred. If an answer came out bigger, you multiplied where you should have divided.

3. Two steps each, in that order.

```
72 times 10 = 720
720 divided by 180.16 = 3.996
110 times 10 = 1,100
1,100 divided by 180.16 = 6.106
45 times 10 = 450
450 divided by 180.16 = 2.498
```

Notice that multiplying by ten and then dividing by about a hundred and eighty leaves you with roughly a twentieth of what you started with. That rough check is worth keeping in your head for later.

4.

```
4 times 180.16 = 720.64
720.64 divided by 10 = 72.064
6.5 times 180.16 = 1,171.04
1,171.04 divided by 10 = 117.104
9 times 180.16 = 1,621.44
1,621.44 divided by 10 = 162.144
```

This is the previous problem run backwards. Multiplying undoes dividing and dividing undoes multiplying, so nothing is lost either way.

5. Milligrams per decilitre into milligrams per litre first.

```
92 times 10 = 920
```

So 920 mg/L. Now divide by the molar mass, because one millimole of glucose weighs 180.16 milligrams.

```
920 divided by 180.16 = 5.1066
```

Now the digits, and the reasoning is the one the illustration used. The laboratory gave 92 to the nearest whole milligram, so every digit it printed is one you can stand behind, and it printed two. Writing 5.1066 claims five. Write 5.1 mmol/L.

Keep the substance with it. That is 5.1 millimoles of glucose per litre, not 5.1 millimoles of anything else.

6. You are going the other way now, so the two steps reverse. Multiply by the molar mass first, which turns millimoles per litre into milligrams per litre.

```
7.2 times 180.16 = 1,297.152
```

Then divide by ten, which turns milligrams per litre into milligrams per decilitre.

```
1,297.152 divided by 10 = 129.7152
```

Now the digits. The paper gave 7.2, which carries two. Write 130 mg/dL.

Check the direction before you move on. A mg/dL figure for glucose is always much bigger than the same value in mmol/L. Yours went from 7.2 to 130, which is the right way.

7. The two steps can be done as one. Multiplying by ten and then dividing by the molar mass is the same as dividing by the molar mass over ten.

```
180.16 divided by 10 = 18.016
```

Now the same value both ways.

```
180 divided by 18.016 = 9.9911
180 divided by 18 = 10
```

The gap between them.

```
10 minus 9.9911 = 0.0089
```

As a share of the answer, divide the gap by the exact figure.

```
0.0089 divided by 9.9911 = 0.00089
```

About nine parts in ten thousand, which is under a tenth of one per cent. At the two digits a laboratory glucose value deserves, the shortcut and the exact divisor give the same answer.

So the shortcut is fine, and it is fine for one reason only: it is built out of glucose's molar mass. It carries no information about any other substance.

8. The glucose first, because that one has everything it needs.

```
110 times 10 = 1,100
1,100 divided by 180.16 = 6.1057
```

Three digits are what 110 deserves, so write 6.11 mmol/L of glucose.

Now the cholesterol, and this is the part of the answer that matters.

You know the substance. You know its formula, atom by atom. You know the method, and you have just used it. You still cannot do this sum, because the method needs cholesterol's molar mass in grams per mole and you do not have it.

So the answer is not a number. It is this: the cholesterol conversion is pending, and what is needed is cholesterol's molar mass, which is on PubChem compound 5997. Read it off the page, then run the same two lines you ran for the glucose.

What you must not do is divide 200 by 18.016. That divisor is glucose's molar mass divided by ten. It would give you a number, and the number would carry the unit mmol/L. Nobody reading your form could tell that it is wrong.

9. The second line broke, and everything after it inherits the break.

The 18 is not a property of the units. It is glucose's molar mass divided by ten.

```
180.16 divided by 10 = 18.016
```

Cholesterol is a different substance, so its molar mass is a different number, so its divisor is a different number. Nothing in the working says which substance the 18 came from. That is how it travelled from a glucose sum to a cholesterol one unnoticed.

What is the right answer? There isn't one here, and saying so is the point. The sum needs cholesterol's molar mass. Go to PubChem compound 5997, read the molecular weight off the page, and then finish.

Why it looks reasonable: both figures are in mg/dL, both answers are asked for in mmol/L, and the arithmetic on line three is correct. The working has the shape of a finished conversion. Only the provenance of the 18 is wrong, and provenance is the one thing a number cannot carry on its face.

10. The third line broke. It divided a per-decilitre figure and reported a per-litre answer.

The 90 is milligrams in a decilitre. The answer is wanted in millimoles in a litre. A litre holds ten decilitres, so the decilitre figure has to be taken up to a litre before anything else happens.

```
90 times 10 = 900
900 divided by 180.16 = 4.9956
```

At the two digits 90 deserves, that is 5.0 mmol/L, not 0.50.

Why it looks reasonable: the molar mass named on line two is the right one, and the division on line three is arithmetically correct. The answer even carries the right unit. The missing step is a factor of ten hidden inside a prefix. Ten is a small enough factor that the answer still looks like a laboratory number.

The habit that prevents it: write the unit on every line, not just the first and the last. The line that reads "900 mg per litre" cannot be confused with the line that reads "90 mg per decilitre".

11. The last line broke, and it broke on digits rather than on arithmetic.

The laboratory reported 100 milligrams per decilitre, to the nearest whole milligram. Three digits are real. The conversion factor renames the amount and cannot make it better known, so the answer has three real digits too.

```
1,000 divided by 180.16 = 5.5506
```

Write 5.55 mmol/L.

Why it looks reasonable: nothing in the working is wrong. Each line follows from the one before it. The molar mass is the right one. The step from decilitres to litres is there. A calculator produced every digit of the final figure. This is the failure the earlier section on significant figures exists for. It survives a unit check, a direction check and a size check.

What the long figure actually says, to a reader who takes it at face value, is that somebody measured that blood to ten digits. Nobody did.

12. What to compute: whether the unit is the obstacle. Take a figure, move it into the other unit, then move it back, and see whether it comes home unchanged. PubChem gives the molecular weight of D-glucose as 180.16 g/mol.

Out first.

```
5 times 180.16 = 900.8
900.8 divided by 10 = 90.08
```

And back.

```
90.08 times 10 = 900.8
900.8 divided by 180.16 = 5
```

The figure came home unchanged. The molar mass is one fixed number, so moving a glucose value between mg/dL and mmol/L loses nothing. Nobody needs sampling again for this reason, and the proposal in the message would spend money on a rename.

What this does not establish. It does not show the two sets can be pooled. Two sets of glucose figures can differ because different assays were used. They can differ because one set was taken fasting and the other was not. They can differ because the two groups of people are not alike. No conversion touches any of those.

And it worked only because both columns hold glucose. Put a different substance in one of them and the same two lines give a wrong answer with a right-looking unit on it.

13. What to compute: put the two figures in one unit, which means turning 4.5 mmol/L into mg/dL. Write the move out with the molar mass left standing as the letter M. M is the number you are about to discover you do not have.

```
4.5 mmol in a litre, times M grams in a mole
gives 4.5 times M milligrams in a litre
then divided by ten
gives 0.45 times M milligrams in a decilitre
```

Now try to put a number on M. You hold the molar mass of D-glucose and you do not hold cholesterol's. So the computation stops here, and stopping is the answer.

Notice what stopping costs you here, because it is more than it cost in the earlier problems. You cannot even eyeball this one. Without M you have no idea whether 0.45 times M lands near 210 or nowhere near it. Near enough is not a claim you can agree with or dispute. Anyone in the room who says the two figures match is telling you they hold a molar mass they have not named.

What you say in the room is short. You cannot check that claim yet. You need cholesterol's molar mass in grams per mole, it is on PubChem compound 5997, and you will have the comparison in five minutes.

What you must not do is reach for 18.016. That divisor is glucose's molar mass divided by ten. Using it here would give you a number that agrees or disagrees with the claim for no reason at all.

What your answer does not establish, even once you have the molar mass. Two figures matching after a conversion is arithmetic, not evidence. It would not tell you the two groups are alike in anything else. It would not tell you the trial

and the audit measured cholesterol the same way. It would not tell you they sampled people of the same ages. The unit was never the only question, and it is the easiest one in the room to answer.

B6 · Body-size units: kg, m, cm and kg/m^2

EXERCISE 1. Height into metres first.

```
155 divided by 100 = 1.55
```

Square it.

```
1.55 times 1.55 = 2.4025
```

Divide the mass by that.

```
64 divided by 2.4025 = 26.6
```

26.6 kilograms per metre squared.

What it does not tell you: anything about this person's health. The number is a mass divided by a squared length. Turning it into a judgement needs a cut-off, and a cut-off is a decision taken by some body, on some date, for some population. Book 0 gives you the arithmetic and the unit. The subject books take up whose line it is and whether it holds here.

EXERCISE 2. Start with the unit, not the formula. Write kg on the board, then m times m underneath it, and say that dividing one quantity by another divides their units too. That is where kg/m^2 comes from. It was built, not chosen from a list.

Then square it on purpose. Draw a shape, then the same shape with every length doubled. Mass follows volume and volume carries three lengths, so mass goes up eight times. The denominator carries two lengths, so it goes up four times. Eight divided by four is two: the index doubles for the same shape at a different size. That is the honest statement of what the square does and does not do.

The error to watch for is the centimetre. Show it on the board once.

```
100 times 100 = 10,000
```

Tell them the value comes out ten thousand times too small. It looks nothing like a small slip, so it usually survives a read-through of a spreadsheet.

Then stop. Do not go on to what any value means. That is a separate decision, taken by a named body on a named date. A resident who learns a cut-off as though it fell out of the arithmetic has learned something to unlearn later.

1. Square first, divide second.

```
1.8 times 1.8 = 3.24
72 divided by 3.24 = 22.2
```

```
1.5 times 1.5 = 2.25
81 divided by 2.25 = 36
```

```
1.5 times 1.5 = 2.25
45 divided by 2.25 = 20
```

The last two share a denominator, so the answers are in the same ratio as the two masses. That is worth noticing: with the height fixed, the index moves in step with the mass.

2. One metre is 100 cm, so divide each by 100.

```
168 divided by 100 = 1.68
155 divided by 100 = 1.55
180 divided by 100 = 1.8
96 divided by 100 = 0.96
```

The decimal point moved two places left every time, which is what dividing by 100 does.

3. One metre is 100 cm, so the number in centimetres is 100 times the number in metres. The denominator is squared, so the factor is squared with it.

```
100 times 100 = 10,000
```

The denominator comes out 10,000 times too big, so the answer comes out 10,000 times too small.

Say the direction out loud. A bigger denominator makes a smaller answer, every time.

4. The height is already in metres, so square it.

```
1.52 times 1.52 = 2.3104
```

Divide the mass by that.

```
58 divided by 2.3104 = 25.1
```

25.1 kilograms per metre squared.

Keep both parts of the answer. The number without the unit could be anything, and the unit is how anybody checks you used metres.

5. Read the height in each row before you compute anything. The first is in centimetres and the second is in metres, so the two rows do not take the same working.

The first row needs the conversion.

```
158 divided by 100 = 1.58
1.58 times 1.58 = 2.4964
61.5 divided by 2.4964 = 24.6
```

24.6 kilograms per metre squared.

The second row is already in metres, so square it and divide.

```
1.72 times 1.72 = 2.9584
73.4 divided by 2.9584 = 24.8
```

24.8 kilograms per metre squared.

How the row told you: by the size of the number. A height in metres is a small number with a decimal point near the front. The same height in centimetres is a three-digit whole number. A row holding 158 is not a row holding metres, whatever the column heading says.

The two answers come out close together. The two rows did not. A register that holds both units at once hands you one right answer and one that is out by ten thousand. Find out which unit each row is in before you compute any of them.

6. The conversion on line two is right. The division on line three is by the height, not by the square of the height.

Square it first.

```
1.7 times 1.7 = 2.89
65 divided by 2.89 = 22.5
```

22.5 kilograms per metre squared, not 38.2.

Look at the unit the broken working actually produced. Kilograms divided by metres is kg/m, and the last line claims kg/m^2. The label was written from memory rather than read off the division.

Why it looks reasonable: the centimetre conversion was done, which is the step everybody warns about, so the working feels careful. The answer is a two-digit number of the ordinary size, so nothing about it looks alarming. Only the unit gives it away.

7. Two things broke, and the second one hides the first.

The height was never converted. Line two squares 158 centimetres, so line three is kilograms per centimetre squared, whatever the last line calls it.

Then the repair used the wrong factor. There are 100 centimetres in a metre, but the denominator is squared, so the factor is 100 times 100.

```
100 times 100 = 10,000
```

Do it from the start instead of patching it.

```
158 divided by 100 = 1.58
1.58 times 1.58 = 2.4964
62 divided by 2.4964 = 24.8
```

24.8 kilograms per metre squared. The patched answer, 0.248, is a hundred times too small.

Why it looks reasonable: the working noticed its own error and did something about it, which reads as care. A correction applied in the right direction feels finished even when the size of it is wrong. And 0.248 still looks like a number from a calculation rather than an absurdity, where 0.00248 did not.

The habit that prevents it: convert the unit before the arithmetic, never after it.

8. What to compute: the factor that a centimetre height actually puts into the index.

The height enters the denominator squared, so the factor is squared.

```
100 times 100 = 10,000
```

The values are 10,000 times too small, not 100 times. Multiplying the column by 100 leaves every value a hundred times too small. The repaired figures will look far more plausible than the ones they started with. That is the dangerous part of the proposal.

What this does not establish. It does not establish that every row was in centimetres. A column filled in by different people can hold both units at once. One multiplication applied to all of it repairs some rows and ruins the rest. Before multiplying anything, look at the heights themselves and see which unit each row is in.

It also says nothing about whether the heights and masses are any good. Fixing a unit fixes a unit. It does not make a badly taken measurement into a good one.

C1 · Variables and algebra: what a letter stands for

EXERCISE 1. In words: T is five times a, plus two times b.

First case, with a as 6 and b as 4.

```
5 times 6 = 30
2 times 4 = 8
30 plus 8 = 38
```

T is 38.

Second case, with a as 6 and b as 9.

```
5 times 6 = 30
2 times 9 = 18
30 plus 18 = 48
```

T is 48.

What you repeated was the arithmetic. What you did not repeat was the rule. You wrote it once, and it held for both cases, which is the whole reason for putting letters in it.

EXERCISE 2. Ten minutes, and do not start with the notation.

Start with a number they already read that way. Write a price on the board and ask for the cost of three of them. They multiply without being asked. Then write the same thing with the price replaced by a letter.

```
3x
```

Ask what they would do if you told them x was 40. They will say 120. Let them say it before you name the rule.

```
3 times 40 = 120
```

Now name it. Two things side by side are multiplied. That is a convention, and it was chosen for a reason. A multiplication sign between every pair of symbols makes a long line unreadable.

Then show them what the convention costs, because this is the part they will meet in the wild. Write a formula with three letters in it and ask them to read it aloud.

```
E = 4c + 4p + 9f
```

Make them say “four times c” rather than “four c”. The moment they say times, the line stops being a string of symbols and starts being an instruction.

Close with the error to watch for. A letter is a number, not a unit. Tell them that f in that line is a count of grams. It is not the symbol for gram. Read it as a unit and the multiplication gets dropped, which is how a wrong formula reaches a spreadsheet.

1. Put 4 in place of x every time.

```
3 times 4 = 12
```

```
4 plus 9 = 13
```

```
2 times 4 = 8
8 plus 5 = 13
```

```
4 times 4 = 16
```

So the four answers are 12, 13, 13 and 16.

The second and third came out the same, which is worth a second look. Two different expressions can agree at one value of x and part company at every other one. Try x as 5 and they do.

2.

```
5n
n + 7
2n minus 3
n²
3(n + 1)
```

The first one is where the convention does its work. You write 5n and you read it as five times n.

The last one needs the brackets. Without them, 3n + 1 means three times the number, and then one added. With them, the whole of n + 1 is tripled. Try n as 4 and the two differ.

```
3 times 4 = 12
12 plus 1 = 13
```

```
4 plus 1 = 5
3 times 5 = 15
```

Thirteen against fifteen, from the same words said in a different order.

3. Multiply every term inside by the number outside.

```
4(x + 2) = 4x + 8
5(10 minus y) = 50 minus 5y
2(3n + 1) = 6n + 2
```

Now check each with the letter as 3. Work the bracketed side first, then the expanded side.

```
3 plus 2 = 5
4 times 5 = 20
4 times 3 = 12
12 plus 8 = 20
```

```
10 minus 3 = 7
5 times 7 = 35
5 times 3 = 15
50 minus 15 = 35
```

```
3 times 3 = 9
9 plus 1 = 10
2 times 10 = 20
6 times 3 = 18
18 plus 2 = 20
```

All three agree. The one to watch is the third. The 2 multiplies the 3n and the 1, and people who expand in a hurry write 6n + 1.

4. Put 5 in place of a and 2 in place of b.

```
5 times 2 = 10
```

```
3 times 5 = 15
2 times 2 = 4
15 plus 4 = 19
```

```
5 times 5 = 25
25 minus 2 = 23
```

```
5 plus 2 = 7
4 times 7 = 28
```

So the four answers are 10, 19, 23 and 28.

The first one is the one to watch. Reading ab as a multiplication is the whole move. Nothing on the page says to multiply except that the two letters are touching.

5. The rule is four kilocalories for every gram of carbohydrate. Four for every gram of protein. Nine for every gram of fat.

```
E = 4c + 4p + 9f
```

Substitute the three figures.

```
4 times 50 = 200
4 times 15 = 60
9 times 10 = 90
200 plus 60 plus 90 = 350
```

350 kilocalories per 100 grams.

What it does not tell you: anything about what the food does in a person. The number is what the rule says a label must declare. It is worked out from grams, by multiplying. It is not a measurement of the food. And it says nothing about anybody's health.

6. The step that broke is the substitution, and it broke in the same way three times.

4c means 4 times c. Putting 55 in place of c gives 4 times 55. It does not give a 4 with 55 written after it.

Do it properly.

```
4 times 55 = 220
4 times 9 = 36
9 times 18 = 162
220 plus 36 plus 162 = 418
```

418 kilocalories, not 1,422.

Why it looks reasonable: every digit of every number given is there in the answer. The addition at the end is correct. And 1,422 is a number a label could carry. Nothing about the working looks careless. Only one thing was missed: the convention that two symbols side by side are multiplied. It is invisible on the page, because it is written with nothing at all.

7. What to compute: the energy each of the three contributes, rather than the grams of each. That is what the claim is actually about.

```
4 times 40 = 160
4 times 5 = 20
9 times 30 = 270
160 plus 20 plus 270 = 450
```

450 kilocalories in total, of which 270 come from the fat.

```
270 divided by 450 = 0.6
0.6 times 100 = 60
```

Sixty per cent of the energy comes from the fat, which has fewer grams than the carbohydrate. So the claim does not follow. The letters in the rule each carry their own multiplier, and the multiplier on fat is more than twice the one on carbohydrate. More grams does not mean more energy.

What this does not establish. It says nothing about what the food does in a person, which is not a question arithmetic on a panel can reach. It says nothing about any other packet, since a different set of grams can go the other way. And both the grams and the energy came from the same panel. So what you checked is whether the panel agrees with itself, not whether it is right.

C2 · Rearranging an equation

EXERCISE 1. Start from the formula and get the term holding n by itself.

Take F off both sides.

```
T minus F = F + rn minus F
T minus F = rn
```

Now the side holding n is a single term, so divide both sides by r.

```
the whole of T minus F, divided by r = n
```

That is n on its own, which is what was asked for.

Now the numbers. Subtract first.

```
950 minus 200 = 750
```

Then divide.

```
750 divided by 25 = 30
```

Thirty items.

Check it by putting 30 back into the formula you started from.

```
25 times 30 = 750
200 plus 750 = 950
```

950, which is the total you were given.

EXERCISE 2. The division by r was applied to T only. The right side of the formula is a sum of two terms, F and rn, so dividing that side by r divides both of them.

What they have written gives F its full size where it should have been shrunk by r. The larger r is, the further out the answer goes.

Do it in the other order and the problem disappears.

```
T minus F = rn
the whole of T minus F, divided by r = n
```

Take the F off first, while the side is still a sum. Then divide, when the side is a single term and there is nothing for the division to miss.

Try it on numbers to make the size of the error visible. Take T as 950, F as 200 and r as 25. Their line gives this.

```
950 divided by 25 = 38
38 minus 200 = -162
```

Minus 162 items. The correct rearrangement gives 30.

What to change in how they work. Two habits, and the second is the one that matters. Move the added terms before the multiplied ones, so that every division happens to a single term. And put the answer back into the original formula before sending it. A negative count of items would have stopped this line at their own desk.

1. Take 7 off both sides.

```
x + 7 minus 7 = 12 minus 7
x = 5
```

Add 4 to both sides.

```
x minus 4 + 4 = 9 + 4
x = 13
```

Divide both sides by 3.

```
3x divided by 3 = 21 divided by 3
x = 7
```

Check all three by putting the answers back.

```
5 plus 7 = 12
13 minus 4 = 9
3 times 7 = 21
```

2. Undo what was done last, first. In both lines the multiplication was done first and the adding or subtracting second, so the adding or subtracting comes off first.

```
2x + 3 minus 3 = 11 minus 3
2x = 8
2x divided by 2 = 8 divided by 2
x = 4
```

```
5x minus 4 + 4 = 26 + 4
5x = 30
5x divided by 5 = 30 divided by 5
x = 6
```

Check both.

```
2 times 4 = 8
8 plus 3 = 11
```

```
5 times 6 = 30
30 minus 4 = 26
```

Doing it the other way round is not wrong, but it is harder. Divide first and you have to divide the 3 by 2 as well. The side is a sum, and a division applies to all of it.

3. The first one has x on both sides. Gather the x terms on one side by taking 3x off both.

```
5x minus 2 minus 3x = 3x + 8 minus 3x
2x minus 2 = 8
```

Now it is the shape you have already done. Add 2 to both sides, then divide by 2.

```
2x = 10
x = 5
```

Check by working out each side of the original with 5 in place of x.

```
5 times 5 = 25
25 minus 2 = 23
```

```
3 times 5 = 15
15 plus 8 = 23
```

Both sides come to 23, so the two sides really are the same number when x is 5. That is what solving an equation means.

The second one is one step. The x was divided by 4, so multiply both sides by 4.

```
x = 24
```

```
24 divided by 4 = 6
```

4. In the first, W is multiplied by L, so divide both sides by L.

```
A divided by L = W
```

In the second, x was multiplied by m and then k was added. Undo the adding first.

```
y minus k = m x
```

Now the side holding x is a single term, so divide both sides by m.

```
the whole of y minus k, divided by m = x
```

The third works the same way. Take u off both sides, then divide by a.

```
v minus u = a t
the whole of v minus u, divided by a = t
```

In all three, the subtraction came before the division. That order is what keeps the division working on a single term instead of on a sum.

And in all three you should ask whether the letter you divided by could be zero. A width of zero, a slope of zero or an acceleration of zero each make the division meaningless, and the page does not warn you.

5. Write the rule down first. Let the letters stand for the three quantities: I for the index, m for the mass in kilograms, h for the height in metres.

```
I = m divided by h²
```

The mass was divided by the square of the height, so multiply both sides by the square of the height.

```
I times h² = m
```

The mass is the index times the square of the height. Now the numbers, one line at a time.

Square the height first.

```
1.60 times 1.60 = 2.56
```

Then multiply.

```
27.0 times 2.56 = 69.12
```

69.1 kilograms.

Check it by going forwards again.

```
69.12 divided by 2.56 = 27.0
```

What it does not tell you: anything about this person's health. The index is a mass divided by a squared length. What any value of it means is a separate decision. Some body took that decision, on some date, for some population. This arithmetic does not reach it.

6. The rule is four kilocalories for every gram of carbohydrate, four for every gram of protein and nine for every gram of fat.

```
E = 4c + 4p + 9f
```

Get the fat term on its own by taking the other two terms off both sides.

```
E minus 4c minus 4p = 9f
```

Now divide both sides by 9.

```
the whole of E minus 4c minus 4p, divided by 9 = f
```

Put the numbers in.

```
4 times 45 = 180
4 times 9 = 36
180 plus 36 = 216
396 minus 216 = 180
180 divided by 9 = 20
```

Twenty grams of fat.

Check by going forwards.

```
9 times 20 = 180
180 plus 216 = 396
```

396 kilocalories, which is the figure on the panel.

7. Write the rule and put in what you know.

```
E = 4c + 4p + 9f
```

The energy is 450 kilocalories, the protein is 12 grams and the carbohydrate is 78 grams.

Get the fat term on its own.

```
E minus 4c minus 4p = 9f
```

Now the numbers.

```
4 times 78 = 312
4 times 12 = 48
312 plus 48 = 360
450 minus 360 = 90
90 divided by 9 = 10
```

Ten grams of fat.

Check by going forwards.

```
9 times 10 = 90
360 plus 90 = 450
```

What this does not establish. The Act sets an energy figure and a protein figure. It sets no figure for fat and none for carbohydrate. So the 10 grams follows from the recipe somebody chose. It is not a standard anybody laid down. Quote it as your own arithmetic on a made-up recipe, and quote the 450 and the 12 as the Act's.

8. Name the three quantities. Let d stand for the energy in each gram and E for the energy of the whole packet. Let w stand for the weight of the packet in grams.

```
d = E divided by w
```

This time the letter you want is underneath. Multiply both sides by the weight to bring it up. A packet does not weigh nothing, so you know that multiplication is by something other than zero.

```
d times w = E
```

Now the side holding the weight is a single term, and the weight is multiplied by d. So divide both sides by d. The register gives d as 4.5, which is not zero, so that division is a move you are allowed.

```
w = E divided by d
```

The weight is the energy of the packet divided by the energy in each gram. Now the numbers.

```
900 divided by 4.5 = 200
```

Two hundred grams.

Check by going forwards.

```
900 divided by 200 = 4.5
```

What this needed that the other direction does not: a second step. A letter that already sits above the division line comes free in one multiplication. This one started underneath, so you had to lift it out from under the division first, and only then divide it loose.

Why the direction changes what you have to do: the rule was written to give the energy in each gram. So the energy stands on top and the weight underneath. Recovering the energy runs the rule the way it was written. Recovering the weight runs it backwards, and every letter you have to cross on the way costs a step.

9. The step that broke is the third one, and the fourth line carries it out faithfully.

The height enters the rule squared. So undoing the division puts back the square of the height, not the height.

Square the height first.

```
1.80 times 1.80 = 3.24
```

Then multiply.

```
25.0 times 3.24 = 81
```

Eighty-one kilograms, not 45.

The unit check catches it too. The index is in kilograms per metre squared. Multiply that by metres and you are left with kilograms per metre, which is not a mass.

Why it looks reasonable: 45 is the right order of magnitude for the quantity asked for, so nothing about the answer looks alarming. The arithmetic in the line is correct. And multiplying to undo a dividing is the right idea, applied to the wrong quantity. Every part of the move is right except what it was applied to, which is the shape this error nearly always has.

10. Every line of the rearranging is right. The step that broke is the last one, and it is a reading error rather than an arithmetic one.

The line before it says that the carbohydrate and the protein together come to 80 grams. It does not say the carbohydrate comes to 80 grams. Two letters are still on that side.

Take the protein off.

```
80 minus 15 = 65
```

Sixty-five grams of carbohydrate.

Check it by going forwards through the whole rule.

```
4 times 65 = 260
4 times 15 = 60
9 times 20 = 180
260 plus 60 plus 180 = 500
```

500 kilocalories, which is the figure the panel gives.

Why it looks reasonable: the rearranging was done correctly and shown line by line, so the working reads as careful. The 80 arrived from a division that was genuinely valid. And the figure that came out is the same size as the other figures on the panel, so nothing about it looks out of place. The error is in what the final line was taken to be about, and a rearranged equation always needs reading before it is used.

11. What to compute: the mass the row gives back. Then how much that mass moves, given that both figures in the row were rounded before they were written down.

The mass is the index times the square of the height.

```
1.65 times 1.65 = 2.7225
22.0 times 2.7225 = 59.895
```

About 59.9 kilograms. So the colleague is right that the mass can be recovered.

Now the part the note does not mention. Both figures in that row were rounded before anybody wrote them down, and both roundings move the answer.

Start with the index. An index written as 22.0 was rounded to one decimal place, so the real value lies somewhere between 21.95 and 22.05. Work the mass out at each end, with the height left where it is.

```
21.95 times 2.7225 = 59.76
22.05 times 2.7225 = 60.03
```

A quarter of a kilogram, and that is not the whole of it. A height written as 1.65 was rounded in the same way, to two decimal places. So the real height lies somewhere between 1.645 and 1.655. The height enters the rule squared, so square both of those ends.

```
1.645 times 1.645 = 2.706025
1.655 times 1.655 = 2.739025
```

Now put the two roundings together. The smallest mass the row can stand for is the smallest index times the smallest square. The largest is the largest index times the largest square.

```
21.95 times 2.706025 = 59.40
22.05 times 2.739025 = 60.40
```

The recovered mass is somewhere between about 59.4 and 60.4 kilograms.

```
60.40 minus 59.40 = 1.00
```

About a kilogram of uncertainty, not a quarter, and most of it came from the height rather than from the index. Leave either rounding out of the arithmetic and you will quote a tighter answer than you have.

What this does not establish. It does not establish that the recovery is good enough. Whether a kilogram matters depends on what the register is for, and nothing in the arithmetic decides that.

It says nothing about whether the height or the index was measured well in the first place.

And it says nothing about the rows where the index was worked out from a height in centimetres. No amount of rearranging will find those. A wrong index rearranges into a wrong mass just as smoothly as a right one.

12. What to compute: what a change in one letter does to the total. That is a question about the rule, not about any one meal.

```
E = 4c + 4p + 9f
```

The fat sits in the line multiplied by 9. So a change in the fat of five grams changes the energy by nine times five.

```
9 times 5 = 45
```

Forty-five kilocalories, not five. The speaker has read the change in grams straight across as a change in kilocalories, which is the multiplication being dropped again.

Now the part that matters more. Taking five grams out of a recipe usually puts something else in, and whatever goes in carries its own factor. If five grams of carbohydrate replace the five grams of fat, work both terms.

```
4 times 5 = 20
45 minus 20 = 25
```

The meal loses twenty-five kilocalories, not forty-five and not five. Which answer is right depends on a fact about the recipe that nobody in the meeting has stated.

What this does not establish. It says nothing about whether the meal still meets any standard set for it, which is a separate question with its own figures. It says nothing about what anybody eats, since a meal served is not a meal eaten. And it says nothing about what the change does to a child, which is not a question the label rule was built to answer.

C3 · Two equations at once

EXERCISE 1. What the line gives you is a relation between two quantities. Call the grain cost g and the fuel cost k, both in rupees a meal.

```
g + k = 18
```

What you can say: the grain cost and the fuel cost add to 18 rupees a meal. So neither of them can be more than 18, and if one goes up the other must come down by the same amount.

What you cannot say: what the grain costs. Any pair adding to 18 fits this line, and the line carries nothing that prefers one pair to another.

What would settle it is a second statement about the same two quantities, true at the same time. It could take either of these forms, among others.

The first form names one of them outright.

```
k = 5
```

Then the grain cost follows at once.

```
18 minus 5 = 13
```

The second form relates them again, differently.

```
g = 2k
```

Then substitute and solve.

```
2k + k = 18
3k = 18
18 divided by 3 = 6
```

The fuel is 6 rupees and the grain is 12.

```
18 minus 6 = 12
```

What matters in the answer you give the officer is the order of the two sentences. First: I cannot answer that from this line. Second: here is what I would need. Giving a number instead would be inventing one.

EXERCISE 2. Three minutes, no symbols, and one picture.

Start with the shape of what they have. Two numbers went up. The thing they want to write about is a third number, built out of both. Knowing that two numbers rose does not tell you what happened to something made by dividing one by the other.

Give them a case they can hold. Say a scheme spent more money and served more meals, and that the meals went up faster than the money. Then the cost of each meal went down while both totals went up. Nothing in the two rising figures rules that out.

Then say what would settle it. The spend in each year and the meal count in each year, all four numbers. With those, the cost of a meal in each year is one division, and the comparison is a fact rather than a guess.

The quotable sentence: “Two totals going up tells you nothing about the cost of a meal. The four numbers that would settle it take a morning to get.”

What not to say: do not say the claim is false. You do not know that either. You know it does not follow. That is a different statement, and a stronger one. It survives whatever the four numbers turn out to be.

1. Test each pair in both statements.

The first pair.

```
4 plus 3 = 7
4 minus 3 = 1
```

Both hold, so this pair is a solution.

The second pair.

```
5 plus 2 = 7
5 minus 2 = 3
```

The first holds and the second does not, because 3 is not 1. So this pair is not a solution.

The third pair.

```
6 plus 1 = 7
6 minus 1 = 5
```

Again the first holds and the second does not. Not a solution.

Notice that all three pairs passed the first test. That is exactly why a pair is checked in both statements and never in one.

2. Four pairs, and any four will do.

x	y
0	12
3	9
5	7
10	2

Check one of them.

```
5 plus 7 = 12
```

How many are there: as many as you like. Pick any number for x at all and subtract it from 12 to get y, and the pair works. There is no end to them.

What that means: this line cannot tell you what x is. It can only tell you what y is once somebody else has told you x. One statement, two unknowns, no answer.

3. The first line already has y standing alone, so put 2x in place of y in the second line.

```
x + 2x = 18
3x = 18
```

Divide both sides by 3.

```
18 divided by 3 = 6
```

So x is 6. Now go back to the first line for y.

```
2 times 6 = 12
```

So y is 12.

Check the pair in both statements.

```
6 plus 12 = 18
```

And 12 is twice 6, so the first statement holds too.

4. Add the two statements together, left side to left side and right side to right side. The y on the first line and the minus y on the second cancel each other.

```
2x = 15 plus 3
15 plus 3 = 18
18 divided by 2 = 9
```

So x is 9. Put that into the first statement to get y.

```
15 minus 9 = 6
```

So y is 6.

Check the pair in both statements.

```
9 plus 6 = 15
9 minus 6 = 3
```

Both hold.

Why elimination was the easier route here: the y carried the same number in both lines. One line had a plus and the other a minus. So adding removed it in a single step. Had neither letter matched, you would have had to make a match first by multiplying a whole line.

5. Write the label rule and take the protein term out of it.

```
E = 4c + 4p + 9f
4 times 10 = 40
440 minus 40 = 400
```

So the first statement is this.

```
4c + 9f = 400
```

The note gives the second statement.

```
c + f = 75
```

Rearrange the second to put c on its own, then substitute it into the first.

```
c = 75 minus f
4 times the whole of 75 minus f, plus 9f = 400
```

Work the bracket out. Four times 75 is 300, and four times minus f is minus 4f.

```
4 times 75 = 300
300 minus 4f + 9f = 400
300 + 5f = 400
```

Take 300 off both sides, then divide by 5.

```
400 minus 300 = 100
100 divided by 5 = 20
```

Twenty grams of fat. Now the carbohydrate.

```
75 minus 20 = 55
```

Fifty-five grams of carbohydrate.

Check the pair in both statements.

```
4 times 55 = 220
9 times 20 = 180
220 plus 180 = 400
```

```
55 plus 20 = 75
```

Both hold.

6. Name the two unknowns. Let L stand for the number of lower primary meals and U for the number of upper primary meals.

The first statement is the count of meals.

```
L + U = 50
```

The second is the total kilocalories, with each meal carrying its own figure from Schedule II.

```
450L + 700U = 27,500
```

Rearrange the first statement and substitute into the second.

```
L = 50 minus U
450 times the whole of 50 minus U, plus 700U = 27,500
```

Work the bracket out.

```
450 times 50 = 22,500
22,500 minus 450U + 700U = 27,500
```

The two U terms collect into one.

```
700 minus 450 = 250
22,500 + 250U = 27,500
```

Take 22,500 off both sides, then divide by 250.

```
27,500 minus 22,500 = 5,000
5,000 divided by 250 = 20
```

Twenty upper primary meals. Now the lower primary count.

```
50 minus 20 = 30
```

Thirty lower primary meals.

Check the pair in both statements.

```
30 plus 20 = 50
```

```
450 times 30 = 13,500
700 times 20 = 14,000
13,500 plus 14,000 = 27,500
```

Both hold. The two figures from Schedule II come from the Act. The split of 30 and 20 is your own arithmetic on a made-up plan. Say so whenever you quote it.

7. Let L stand for the number of lower primary meals and U for the number of upper primary meals. Each kind carries its own pair of figures from Schedule II, so each total gives you one statement.

```
450L + 700U = 27,500
12L + 20U = 760
```

To eliminate U, make the U terms match. The first line carries 700U and the second carries 20U, so multiply the whole of the second line by 35.

```
20 times 35 = 700
12 times 35 = 420
760 times 35 = 26,600
```

So the second statement, multiplied through, says this.

```
420L + 700U = 26,600
```

Now subtract that from the first statement, left from left and right from right. The U terms cancel.

```
450 minus 420 = 30
27,500 minus 26,600 = 900
30L = 900
```

Divide both sides by 30.

```
900 divided by 30 = 30
```

Thirty lower primary meals. Now go back for U, using the protein statement.

```
12 times 30 = 360
760 minus 360 = 400
400 divided by 20 = 20
```

Twenty upper primary meals.

Check the pair in both statements.

```
450 times 30 = 13,500
700 times 20 = 14,000
13,500 plus 14,000 = 27,500
```

```
12 times 30 = 360
20 times 20 = 400
360 plus 400 = 760
```

Both hold. Notice what you were given and what you were not: no meal count at all, and two totals of different kinds. Two totals were enough, because energy and protein are different statements about the same two unknowns.

8. The step that broke is the sixth line. The rearranged first statement was put back into the first statement.

That can only ever give something true and useless. The rearranged line says the same thing as the line it came from, so putting one into the other says it twice. The 20 equals 20 that came out is not a finding. It is the working going round in a circle.

The whole point of substituting is to carry what you learned from one statement into the other one. Put the rearranged first line into the second.

```
20 minus y minus y = 6
20 minus 2y = 6
```

Take 20 off both sides, then divide by minus 2. Easier still, add 2y to both sides and take 6 off both sides.

```
20 minus 6 = 14
2y = 14
14 divided by 2 = 7
```

So y is 7. Now go back for x.

```
20 minus 7 = 13
```

So x is 13.

Check in both statements.

```
13 plus 7 = 20
13 minus 7 = 6
```

Why it looks reasonable: every line of the working is a true statement, and the last line is the truest of all. Nothing was miscalculated. A reader watching for arithmetic errors finds none. The error is in which statement the substitution went into, and the page does not label them.

9. Two things broke, and the second one is the one that matters.

The first is the last line. The working reached a statement that is true for every pair, and then a pair was named anyway. Nothing in the working produced 25 and 25. The numbers were chosen because the two unknowns looked symmetrical.

The second is upstream, and it is why the working could not have ended any other way. The second statement is the first statement doubled. Multiply the first by 2 and you get the second exactly, so the second says nothing the first did not already say.

```
50 times 2 = 100
```

So this is one statement written twice, not two statements. Two unknowns and one statement means many answers, and 25 with 25 is only one of them.

```
30 plus 20 = 50
10 plus 40 = 50
```

Both of those pairs satisfy the note just as well as 25 and 25 does.

What a real second statement would look like: one that says something the first cannot. A total of kilocalories for the day does that, because a lower primary meal and an upper primary meal carry different figures.

Why it looks reasonable: the working counted two lines and two unknowns and drew the usual conclusion, and counting lines is what most people do. The substitution was carried out correctly. And 25 with 25 is a pair that does satisfy the statement, so checking it back against the note would not have caught this either. The only thing that catches it is asking whether the second line says anything new.

10. What to compute: whether a pair of counts exists in which both totals rise and one of the two counts falls. If one does, the claim does not follow, whatever this kitchen actually did.

Start with last year's two totals.

```
30 plus 20 = 50
```

```
450 times 30 = 13,500
700 times 20 = 14,000
13,500 plus 14,000 = 27,500
```

Fifty meals and 27,500 kilocalories.

Now try this year as 26 lower primary meals and 25 upper primary meals.

```
26 plus 25 = 51
```

```
450 times 26 = 11,700
700 times 25 = 17,500
11,700 plus 17,500 = 29,200
```

Fifty-one meals and 29,200 kilocalories. Both totals went up. The lower primary count went down, from 30 to 26.

So the claim does not follow. Two totals are two statements about two unknowns, and knowing that both totals rose is not knowing either unknown. The upper primary meal carries a larger figure, so shifting meals towards it lifts the kilocalorie total even while one count falls.

What this does not establish. It does not establish that the lower primary count actually fell, and nothing here says the note's conclusion is false. It says the two totals do not reach it. It also does not establish what the kitchen did at all, because the counts used here were invented to test the reasoning. And it says nothing about whether any child ate a meal, since a meal counted and a meal eaten are two different things.

What to ask for instead: both counts for both years. Two counts and two totals for one year would also do it, because then you have two statements and two unknowns and the pair solves.

C4 · Functions: input, rule, output

EXERCISE 1. It means the table is not a function of its first column. One input has two outputs, so the table cannot answer the question "what does this give for that input?".

Say it plainly. The table is a record of what happened, not a rule for working something out. Both rows may be perfectly correct as records.

What to ask for next: the thing that differs between those two rows. Something not in the first column moved between them. Either it belongs in the table as a second input column, or one of the two rows is an error.

Then ask how many other repeats there are. One repeat in three hundred rows is a question about one row. Forty repeats is a question about the whole table, and it usually means the first column was never the input the rule was built on.

What you cannot do is average the two outputs and carry on. That produces a number that appears in neither row and matches nothing anybody recorded.

EXERCISE 2. Ten minutes, for somebody who expects a mechanism rather than a rule.

Draw a box on the board with an arrow going in and an arrow coming out. Say that the box is the rule, the arrow in is the input, and the arrow out is the output. Ask them to name a rule they use, and write its input and output on the two arrows with their units.

Now break it in front of them. Write a short table with four rows, and make two rows share an input and carry different outputs.

people	kilograms
4	20
6	30
4	35
5	25

Ask what the rule gives for four people. Let them sit with it until somebody says the table gives two answers. That is the moment, and it lands because they found it.

Then give them the test in one sentence: the same input must give the same output every time. Have them apply it to the rule they named at the start.

Close with the direction it runs in. Two inputs may share an output, and that breaks nothing. A resident who learns the test backwards starts rejecting good rules. That habit is harder to undo than never having had the test.

1. Multiply first, then add. Take them one at a time.

```
3 times 0 = 0
0 plus 2 = 2
```

```
3 times 1 = 3
3 plus 2 = 5
```

```
3 times 5 = 15
15 plus 2 = 17
```

```
3 times 10 = 30
30 plus 2 = 32
```

So the outputs are 2, 5, 17 and 32.

Notice that the input 0 gave the output 2. An input of nothing does not mean an output of nothing, because the rule adds after it multiplies.

2. The first is a function. Each value in the first column appears once, so no input has two outputs.

The second is not. The input 1 appears twice, once with 4 and once with 9. Ask it for the output at 1 and it gives you two answers.

The third is a function. Three different inputs all give 4, and that breaks nothing. The rule only forbids one input having two outputs. It says nothing about outputs being shared.

The third table is the one people get wrong. A column of identical outputs looks broken and is not.

3. The rule adds 7, so going backwards means taking 7 away.

```
12 minus 7 = 5
19 minus 7 = 12
40 minus 7 = 33
```

The inputs were 5, 12 and 33.

Check one of them forwards, which is how you find a slip.

```
33 plus 7 = 40
```

It comes back to 40, so the third one holds.

4. The domain is what the rule will accept. The input counts people in a household, so the inputs it has an answer for are the whole numbers from 1 upwards. There is no household of nought people and no household of four and a half.

The range is what the rule can give back. Multiply each of those inputs by five kilograms a month and you get 5, 10, 15, 20 and so on, upwards in steps of five.

```
5 times 1 = 5
5 times 2 = 10
5 times 3 = 15
```

So the range is the whole numbers of kilograms a month that are multiples of five, from five upwards.

What the office has done wrong with n = 4.5: it has put an input into the rule that the rule does not take. The arithmetic will still produce a number.

```
5 times 4.5 = 22.5
```

Twenty-two and a half kilograms a month, and nothing about that figure looks wrong on a page. That is the whole difficulty. A rule run outside its domain does not stop and does not warn you. It answers, and the answer is about no household that exists.

Where a 4.5 comes from is usually an average household size that somebody has fed into a rule built for one household at a time. The fix is to multiply after the rule rather than inside it: count the people in the district, then apply the rule to that count.

5. The rule takes a number of people in and gives kilograms a month out. Run it forwards twice.

```
5 times 3 = 15
```

Fifteen kilograms of foodgrains a month.

```
5 times 7 = 35
```

Thirty-five kilograms of foodgrains a month.

The third part runs the rule backwards. You are given the output and asked for the input, so undo the multiplying by dividing.

```
45 divided by 5 = 9
```

Nine people.

Check it forwards before you write it down.

```
5 times 9 = 45
```

So the first two parts put an input in and read an output. The third put an output in and worked back to the input, which only works because each output here comes from one input.

6. First the rule. Call the number of months m and the total t.

```
t = 25 times m
```

The input is months, the output is kilograms, so the 25 carries kilograms per month.

Four months, running it forwards.

```
25 times 4 = 100
```

A hundred kilograms of foodgrains.

Now backwards, from an output of 150 kilograms to the input that gives it.

```
150 divided by 25 = 6
```

Six months.

Read the unit of the answer off the division to be sure of it. Kilograms divided by kilograms per month leaves months standing, which is the check you met in the section on dimensions.

7. The step that broke is the third one. Everything after it is arithmetic on a decision that should never have been taken.

Two rows with the same input and different outputs mean the register is not a function of household size. That is the finding. Averaging does not repair it, because the average is an output that neither row records and that no rule produced.

Look at what the answer now claims. It says a household of four is counted for 27.5 kilograms a month, and no household in the register is counted for that.

What to do instead: find out what differs between the two rows. Either a second input is missing from the table, or one of the rows is wrong.

Why it looks reasonable: averaging two numbers that disagree is a move that is right in other places. The arithmetic in lines four and five is correct as well. A correct calculation sitting under a wrong decision reads as a finished piece of work.

8. What to compute: whether the rule runs backwards, and then whether the register was actually built from it.

Take the rule first. Five kilograms a person a month means the output is the input times five.

```
5 times 4 = 20
20 divided by 5 = 4
```

So one output comes from one input, and dividing by five does recover the household size. The arithmetic in the proposal is sound.

Now test it on the register itself. The four-person household counted for 35 kilograms is the row that settles it.

```
35 divided by 5 = 7
```

Seven people, for a household the register elsewhere records as four. So the proposal turns that row into an invented household of seven, and nothing downstream will show that it happened.

What this does not establish. It does not tell you whether the 35 is the error, or the 4 is, or neither. If the household is under the Antyodaya Anna Yojana, both figures are right. The register then holds two rules, and dividing by five is the wrong rule for that row. The arithmetic cannot say which of the three is true.

It does not tell you how many other rows disagree, because you have looked at one. And it says nothing about whether any of these households were counted correctly to begin with. Recovering an input from an output tells you what the rule would have needed, never what happened.

C5 · Graphs: axes, scale, slope and intercept

EXERCISE 1. Both figures show the same seven numbers. Neither is false.

The left figure shows a quantity that barely moves against the full height of the axis. A change of 4 sits in a height of 100.

```
64 minus 60 = 4
4 divided by 100 = 0.04
```

So the line uses four per cent of the height, and the picture reads as flat.

The right figure shows the same change of 4 in a height of 8.

```
66 minus 58 = 8
4 divided by 8 = 0.5
```

Half the height. The same rise now crosses the middle of the picture and reads as a climb.

What neither shows: whether a change of 4 matters. That question is settled by what the quantity is, how precisely it was measured, and what counts as a difference worth acting on. None of those three is in either drawing.

Which to print: the one whose axis you are willing to defend out loud. Printing the full-height version and saying the change is small is honest. Printing the short-axis version is also honest, as long as the caption says the axis starts at 58. The defect is not a short axis. It is a short axis that the caption does not mention.

EXERCISE 2. What is wrong: the speaker has read the steepness as though it measured the size of the change. It does not. The steepness was set when somebody chose the side axis.

Work out what the axis actually holds.

```
20.4 minus 19.8 = 0.6
```

Six tenths of a kilogram, across the whole height of the slide. So a line that climbs from the bottom of the picture to the top is a change of at most 0.6 kg over twelve months.

Now the slope, with its unit taken off the axes.

```
0.6 divided by 12 = 0.05
```

Five hundredths of a kilogram a month. Say it with the unit, because the unit is the part that makes it checkable.

What to ask for. First, the same figure with the side axis starting at zero, which takes one minute and settles the argument. Second, what change would have been large enough to act on, decided before the figure was drawn. Third, the spread of the underlying values, since a line drawn through scattered points hides how scattered they were.

What not to say: that the speaker is wrong about the numbers. They are not. They are wrong about what a picture can measure.

EXERCISE 3. Answer first, because that is what the page is for.

Both figures are correct. They plot the same numbers. They look different because the two groups chose different heights for the side axis, and nothing stops either of them doing that.

Then the one thing to do about it. Ask for both figures redrawn with the side axis starting at zero and ending at the same value. Any two figures of one set of numbers then look identical, and the argument moves to where it belongs.

Then the question that actually decides the matter. How big a change is worth acting on here, and who decided that? Until somebody answers it, no figure of any shape settles anything, because the picture cannot tell you what counts as big.

Then one sentence on what to watch for in future submissions. A side axis that does not start at zero is not a fault, and a caption that does not say so is.

No methods section. The secretary does not need to know how a slope is computed. They need to know that steepness is a choice and that the size of the change is not.

1. The rise is the change in the second number of each pair.

```
19 minus 7 = 12
```

The run is the change in the first number of each pair.

```
6 minus 2 = 4
```

The slope is the rise divided by the run.

```
12 divided by 4 = 3
```

The slope is 3. There are no units here, because the two quantities were not named.

2. Take each one as rise divided by run.

The first.

```
13 minus 5 = 8
4 minus 0 = 4
8 divided by 4 = 2
```

The second.

```
10 minus 10 = 0
6 minus 1 = 5
0 divided by 5 = 0
```

A slope of zero. The line is flat, which is what a slope of zero means: the y quantity does not change as the x quantity does.

The third.

```
5 minus 20 = -15
7 minus 2 = 5
-15 divided by 5 = -3
```

A slope of minus 3. The minus sign says the line falls as you go right, and the 3 says how fast.

3. The slope first.

```
27 minus 12 = 15
5 minus 0 = 5
15 divided by 5 = 3
```

The slope is 3.

The intercept is the value of y where x is zero, and one of the two points given has x equal to zero. So the intercept is 12, read straight off that point.

The intercept is the value the y quantity takes when the x quantity is nothing at all. It is not where the line starts, and it is not the smallest value on the line. Lines run in both directions.

4. The slope of 4 means y goes up by 4 for every 1 across. The intercept of 6 is the value at x equal to zero, so start there and add 4 each step.

```
y at 0 = 6
6 plus 4 = 10
```

So y is 10 at x equal to 1.

```
4 times 3 = 12
12 plus 6 = 18
```

So y is 18 at x equal to 3.

```
4 times 10 = 40
40 plus 6 = 46
```

So y is 46 at x equal to 10.

The last part is already answered by the line above, and it is worth seeing why. You were asked where y reaches 46, and the working ran the other way: take the intercept off first, then undo the multiplying.

```
46 minus 6 = 40
40 divided by 4 = 10
```

At x equal to 10.

5. Rise first, from the side axis.

```
110 minus 30 = 80
```

Then the run, from the bottom axis.

```
5 minus 1 = 4
```

Then the slope.

```
80 divided by 4 = 20
```

Now the unit, taken off the axes. Kilograms up the side and months along the bottom give kilograms divided by months, which is kilograms per month. So the slope is 20 kilograms a month.

For the intercept, walk back from the point at one month by one month's worth of grain.

```
30 minus 20 = 10
```

The line meets the side axis at 10 kilograms. That is the grain in the house at month zero.

6. Work out the two points from the rule first.

```
5 times 3 = 15
5 times 7 = 35
```

So the figure has a point at three people and 15 kilograms, and one at seven people and 35 kilograms.

Rise.

```
35 minus 15 = 20
```

Run.

```
7 minus 3 = 4
```

Slope.

```
20 divided by 4 = 5
```

The unit comes off the axes. Kilograms a month up the side, people along the bottom, so the slope is 5 kilograms per person per month. That is the figure section 3(1) states, recovered from the drawing.

The intercept is at zero people and zero kilograms. As arithmetic it is correct. As a statement about the world it means the grain for a household with nobody in it, and there is no such household. So the intercept here is a property of the line and not a fact you can check.

7. The change is the same in both figures.

```
64 minus 60 = 4
```

Four. Now the height each axis offers.

```
100 minus 0 = 100
4 divided by 100 = 0.04
```

Four hundredths of the height in the first figure.

```
67 minus 59 = 8
4 divided by 8 = 0.5
```

Half the height in the second.

What it means: the same change occupies four per cent of one picture and half of the other. A reader who looks at the shape sees a flat line and a steep climb. A reader who reads the axis numbers sees a change of 4 in both, because there is a change of 4 in both.

8. The unit of a slope is always the side unit divided by the bottom unit. Take them in order.

Kilograms divided by months is kilograms per month.

Kilocalories divided by grams is kilocalories per gram, which is an energy in every gram of something.

Rupees divided by kilograms is rupees per kilogram, which is a price.

People divided by square kilometres is people per square kilometre, which is a density of people.

None of those four came from looking at the line. They came from reading the two axis labels, which is why a figure with an unlabelled axis has a slope you cannot state.

9. The step that broke is the fourth. The rise and the run were worked out correctly and then divided the wrong way round.

A slope is the rise divided by the run, not the run divided by the rise.

```
60 divided by 6 = 10
```

The slope is 10.

Check it against the picture. Between the two points the line goes up 60 while it goes across 6, so it climbs steeply. A slope of 0.1 describes a line that is nearly flat, which is the opposite of the line described.

The units catch it too, when the axes are labelled. Put kilograms up the side and months along the bottom: the broken answer is in months per kilogram, and nobody asked for that.

Why it looks reasonable: both subtractions are right, the division is right, and the answer is a tidy number. The one wrong step is the one with no arithmetic in it.

10. The step that broke is the fourth. Steepness was read as though it measured the change, and the last two lines are the evidence that it does not.

Work out what each axis can hold.

```
20.4 minus 19.8 = 0.6
```

Figure A's whole height is six tenths of a unit. A line crossing all of it has changed by at most 0.6.

```
200 minus 0 = 200
```

Figure B's whole height is 200 units. A line staying near the bottom, say a fifth of the way up, has changed by 40.

```
200 divided by 5 = 40
```

So the figure that looks flat carries the larger change, by a wide margin.

The order to work in is the one this section fixed. First what each axis measures and its unit, then the numbers on the scale, and only then the line. The broken answer read the line first and never revisited it.

Why it looks reasonable: lines three and four sound like the same statement said twice, and the step between them is the whole error. The numbers on both axes are even printed in the working, so the answer had what it needed and did not use it.

11. What to compute: the size of the change in the unit of the axis, and the slope with its unit. The steepness is not computable as anything, because it is a choice.

The change first.

```
5.02 minus 4.95 = 0.07
```

Seven hundredths of a kilogram, across six months.

Now the slope.

```
0.07 divided by 6 = 0.0117
```

About 0.012 kilograms a month. Say the unit, because the number on its own is what let the press line happen.

And the reason the line looks vertical.

```
5.10 minus 4.90 = 0.2
0.07 divided by 0.2 = 0.35
```

The change uses about a third of the height of a picture whose whole height is two tenths of a unit.

What this does not establish. It does not establish that a change of 0.07 is small. Small and large are decided by what the quantity is, and by who set a threshold. Arithmetic does not decide it. It does not establish that the change is real either, since two readings six months apart carry whatever the weighing itself varies by. And it does not establish that anything shot up, because a drawing cannot carry that claim. What you have shown is that the word "vertical" describes the axis and not the data.

12. What to compute: what the entitlement rule says the intercept should be, and what an intercept of 12 would have to mean.

Section 3(1) of the National Food Security Act, 2013 entitles a person in a priority household to five kilograms of foodgrains per person per month. So the rule gives zero for zero people.

```
5 times 0 = 0
```

The rule's own intercept is zero.

Now take the note's line seriously for one step. At one person it would give 12 plus 5.

```
12 plus 5 = 17
```

Seventeen kilograms for one person, against the five the Act states. At four people it gives 32 against 20.

```
5 times 4 = 20
20 plus 12 = 32
```

So if the fitted line were true of the blocks, every household would be over-counted, not just the empty ones. The 12 is not a finding about households with nobody in them. It is a statement about the whole line.

What this does not establish. It does not establish that any grain went missing. An intercept at a household size nobody has is a number the fitting produced. Zero people is outside every household size that was fitted.

It does not tell you why the line sits high. That could be a handful of wrong rows, a second entitlement under another provision, or an error in the household-size column. And it does not establish that a straight line was the right thing to fit at all.

Before recovering anything from anybody, go and read the rows at the small household sizes. That is where the claim is supposed to live, and there may be no rows there at all.

C6 · Recognising shapes: linear, exponential, saturating

EXERCISE 1. It looks saturating. The steps up are getting smaller while the count is still climbing. The bend alone does not settle it, because a curve that climbs for ever by ever smaller steps bends the same way. Read values off the curve at equal steps along the bottom, and subtract each from the next. Then divide each difference by the one before it. Ratios that hold at, or stay under, some number below one make it saturating. Ratios creeping up towards one leave the shape unnamed.

What the flat part means in the arithmetic: each equal step along the bottom is adding less than the step before it added. The differences are shrinking towards zero. The count is not falling, and saying that it is falling is the commonest misreading of this shape.

What the figure does not tell you. It does not tell you the ceiling, unless the curve has been carried on far enough for the flattening to settle. It does not tell you why anything flattened, because a shape carries no mechanism. And it does not tell you that the early part was exponential, which is a claim you would have to test by dividing successive values.

One thing to check before saying any of this out loud: whether the side axis is an ordinary one or one where each step multiplies. The answer above assumes an ordinary axis, and the figure has to say which it is.

EXERCISE 2. Three minutes, no jargon, and they need something they can print.

Start with what the word means, in one sentence they can use. Exponential means every equal step multiplies by the same amount. It is a claim about the ratio between one month and the next. It is not a claim about the rise looking dramatic.

Then show them the test with their own numbers. Take the values a month apart, and divide each by the one before. If those ratios are about the same, the word holds. If they are falling, the rise is slowing, and there is a different and equally printable story in that.

Then give them the careful statement. Say that the numbers rose, and say by how much. Then say whether the word exponential fits, and give the ratios as the reason. If you only have three points, say that three points cannot settle it, because they cannot.

The quotable sentence: “It is rising, and the rise is slowing down. Each month adds less than the month before, which is the opposite of exponential.”

What not to do: agree that it is exponential because the line looks steep. Steepness is a choice made by whoever drew the axis, and you told them that last time.

1. A is linear. Subtract each from the next and the difference is the same every time.

```
8 minus 4 = 4
12 minus 8 = 4
20 minus 16 = 4
```

B is exponential. The differences are not equal, so divide instead.

```
8 divided by 4 = 2
16 divided by 8 = 2
64 divided by 32 = 2
```

The same factor every step.

C is the one to stop at. Subtract, and the differences are not the same, so it is not linear.

```
12 minus 4 = 8
16 minus 12 = 4
19 minus 16 = 3
21.5 minus 19 = 2.5
```

Divide, and the ratios are not the same either, so it is not exponential.

```
12 divided by 4 = 3
21.5 divided by 19 = 1.13
```

The differences shrink while the values climb. So divide each difference by the one before it.

```
4 divided by 8 = 0.5
3 divided by 4 = 0.75
2.5 divided by 3 = 0.83
```

Those ratios are creeping up towards one. So the test shows no ceiling, and there is no ceiling to quote. The tests name no shape for C. It may be flattening. These five values do not show that it is, and saying so is the answer.

2. D is exponential. Divide each value by the one before it.

```
50 divided by 100 = 0.5
25 divided by 50 = 0.5
6.25 divided by 12.5 = 0.5
```

The same factor every step, and the factor is less than one, so the values fall. Exponential describes the multiplying, not the direction.

E is linear. Subtract.

```
130 minus 100 = 30
160 minus 130 = 30
220 minus 190 = 30
```

F is exponential. Divide.

```
6 divided by 2 = 3
18 divided by 6 = 3
162 divided by 54 = 3
```

D and F are the pair to look at twice. One falls and one climbs steeply, and both are the same shape, because in both cases each step multiplies by a fixed number.

3. The first rule, adding 6.

```
8 plus 6 = 14
14 plus 6 = 20
20 plus 6 = 26
26 plus 6 = 32
```

So 8, 14, 20, 26, 32. Equal steps add the same amount, so this is linear.

The second rule, multiplying by 3.

```
8 times 3 = 24
24 times 3 = 72
72 times 3 = 216
216 times 3 = 648
```

So 8, 24, 72, 216, 648. Equal steps multiply by the same factor, so this is exponential.

Look at the two end values. The adding rule reached 32 and the multiplying rule reached 648, from the same start and in the same four steps.

4. Work out one month first.

```
5 times 4 = 20
```

Twenty kilograms a month. Now the running totals.

```
20 times 1 = 20
20 times 2 = 40
20 times 3 = 60
20 times 4 = 80
```

So 20, 40, 60 and 80 kilograms.

Name the shape by subtracting.

```
40 minus 20 = 20
60 minus 40 = 20
80 minus 60 = 20
```

Equal steps add the same amount, so it is linear. Drawn with months along the bottom and kilograms up the side, the slope is 20 kilograms per month.

What would make it exponential: the monthly amount would have to multiply rather than repeat. The second month would have to bring more grain than the first, in proportion to the total already received. Nothing in section 3(1) does that, because the entitlement is a fixed amount for each person for each month.

5. Subtract each value from the next.

```
44 minus 20 = 24
56.0 minus 44 = 12
62.0 minus 56.0 = 6
65.0 minus 62.0 = 3
66.5 minus 65.0 = 1.5
```

The values climb and the steps shrink. That is not yet enough, so divide each difference by the one before it.

```
12 divided by 24 = 0.5
6 divided by 12 = 0.5
3 divided by 6 = 0.5
1.5 divided by 3 = 0.5
```

The ratio holds at 0.5, which is below one, so it is saturating.

Now the ceiling. Divide the first difference by one minus the ratio, and add the answer to the starting value.

```
1 minus 0.5 = 0.5
24 divided by 0.5 = 48
20 plus 48 = 68
```

Check it from a later value. At 65.0 the next difference is 1.5.

```
1.5 divided by 0.5 = 3
65.0 plus 3 = 68
```

So the ceiling is about 68. Say “about”, because you have worked it out from the pattern of six values and not from anything that told you what the ceiling was.

6. Name the shapes first, because the shapes decide the answer.

G rises by the same amount each step.

```
35 minus 5 = 30
65 minus 35 = 30
```

So G is linear, adding 30 a step.

H multiplies by the same factor each step.

```
10 divided by 5 = 2
20 divided by 10 = 2
```

So H is exponential, doubling each step.

Now run both to step 12. G has taken 12 steps of 30 from a start of 5.

```
30 times 12 = 360
360 plus 5 = 365
```

H has doubled 12 times from a start of 5. Doubling twelve times multiplies by 2^{12}.

```
2^12 = 4096
5 times 4096 = 20480
```

Twenty thousand four hundred and eighty, against 365. H is far larger.

Notice that H was the smaller of the two at steps two, three and four. A rising exponential can start behind a linear and still pass it. Where it passes tells you nothing about how far ahead it ends up.

7. The step that broke is the fourth. Rising fast is not what exponential means, and the test that settles it was never run.

Divide each value by the one before it.

```
30 divided by 10 = 3
42 divided by 30 = 1.4
49.2 divided by 42 = 1.17
```

The ratios are falling, not holding. So nothing here is multiplying by a fixed factor.

Now subtract instead.

```
30 minus 10 = 20
42 minus 30 = 12
49.2 minus 42 = 7.2
```

The steps are shrinking while the values climb. That is not yet enough, so divide each difference by the one before it.

```
12 divided by 20 = 0.6
7.2 divided by 12 = 0.6
```

The ratio holds at 0.6, below one. If it keeps holding, this is saturating, which is the opposite of the shape claimed. The last line is the damaging part. This curve is flattening rather than about to take off.

Why it looks reasonable: the first difference really is 20, and the first ratio really is 3. Read only the start of a saturating curve and it looks like something taking off. The answer stopped at the first step, and the first step is the one place where the two shapes look alike.

8. The step that broke is the third. The line is straight, and the axis is not an ordinary one.

Read the side axis again. Each label is ten times the one before it, so this is the multiplying ruler from the section on logarithms.

```
100 divided by 10 = 10
1,000 divided by 100 = 10
```

On an axis like that, equal distances up the page mean equal multiplications rather than equal additions. A straight line therefore means a constant ratio, which is exponential.

Read the values off the axis instead of the line, and the difference is plain. If the line crosses 10 at the first month and 10,000 at the fourth, the quantity has multiplied by ten a month.

```
10 times 10 times 10 times 10 = 10000
```

A thousandfold rise over three months is the opposite of needing no further comment.

Why it looks reasonable: on an ordinary axis, a straight line really is a linear relationship. That is the first thing anybody learns about graphs. The axis labels are printed right there, and reading the line is quicker than reading them.

9. What to compute: the differences and the ratios between the three values, because those are what the word exponential is a claim about.

Differences first.

```
400 minus 200 = 200
600 minus 400 = 200
```

The same amount is added each month.

Now ratios.

```
400 divided by 200 = 2
600 divided by 400 = 1.5
```

Two, then 1.5. The ratio is falling, so this is not multiplying by a fixed factor. On these three values it is linear and not exponential, and the right sentence is that registrations rose by 200 a month.

What this does not establish. Three values cannot settle a shape, so this does not establish that the fourth month will add 200 either. It does not establish that the note is wrong about anything except the word, since 200 a month may be exactly what was expected. And it says nothing about whether registrations measure anything worth measuring. Naming a shape is not the same as knowing what is being counted.

10. What to compute: the running total of coverage, and what the shrinking steps are heading towards.

Add the gains up as they come.

```
12 plus 6 = 18
18 plus 3 = 21
21 plus 1.5 = 22.5
```

Coverage has risen by 22.5 points in four years, and it is still rising.

Now look at the steps themselves.

```
6 divided by 12 = 0.5
3 divided by 6 = 0.5
1.5 divided by 3 = 0.5
```

Each year's gain is half the year before. That is the signature of a saturating curve, not of a programme failing. The steps shrink because there is less room left to cover, which is what running towards a ceiling looks like from inside.

Work out roughly where it is heading. Divide the first gain by one minus the ratio.

```
1 minus 0.5 = 0.5
12 divided by 0.5 = 24
```

About 24 points above where it started, which is only 1.5 above where it stands now.

What this does not establish. It does not establish that the programme is working, because coverage could have risen for reasons that have nothing to do with it. It does not establish that 24 points is the true ceiling, since four values and an assumed halving are a thin basis for that. And it does not settle the money question at all. A saturating curve tells you that the next rupee buys less than the last one did. Whether that rupee is better spent elsewhere depends on what it would buy there. That is nowhere in these numbers.

C7 · Rate of change, average and instantaneous, without the machinery

EXERCISE 1. The average rate over the twelve months is the change between month zero and month twelve, divided by twelve. It is the slope of one straight line laid across those two points, and that line is a chord.

What it describes: the whole year taken as one block. If you want to know how much was added across the year, it is exactly the right number.

What it does not describe: any moment inside the year. The curve is steep at the start and nearly flat at the end. Its steepness passes through the average somewhere in the middle. At either end it sits nowhere near it.

The two numbers to ask for: the rate over the first three months, and the rate over the last three months. Each is a change divided by three months.

Why those two. They are the two ends of the story. Comparing them answers the question the average hides, which is whether the thing is still rising as fast as it was. A single figure for the year cannot answer that, however carefully it is computed.

One more thing to check before you say any of it. Read what one step on each axis stands for. A curve that bends over on one scale can look straight on another.

EXERCISE 2. The first sentence is arithmetic and it is nearly right. Add the four gains and divide by four.

```
12 plus 6 = 18
18 plus 3 = 21
21 plus 1.5 = 22.5
22.5 divided by 4 = 5.625
```

About 5.6 points a year, which the note has rounded to 6.

What is wrong with the second sentence: it takes an average over four years and uses it as the rate now. The most recent year's rate is the gain in the fourth year, which is 1.5 points a year.

Look at what the yearly gains are doing.

```
6 divided by 12 = 0.5
3 divided by 6 = 0.5
1.5 divided by 3 = 0.5
```

Each year's gain is half the year before it. The rate is not holding at 6 and it is not holding at anything. It is falling, and it halves every year.

Now the note's own target. Coverage stands 22.5 points above baseline and the next year's gain looks like being about 0.75 points.

```
1.5 divided by 2 = 0.75
```

Three more years at gains that keep halving add well under two points. So 40 is not reached in three years. Nothing in these four numbers says it is reached at all.

What to put in its place. Say that coverage rose 22.5 points over four years. Say that the gain in the most recent year was 1.5 points. Say that the gains have halved each year so far. Then say what would have to change for the trend to be different, and that nothing in these numbers tells you whether it will.

EXERCISE 3. Ten minutes, a whiteboard, and somebody who expects a mechanism rather than a rule.

Draw a curve that climbs steeply and then flattens. Mark the start and the end. Do not say anything about rates yet.

Ask them how fast the quantity grew. They will give you a number. Ask them how they got it, and they will tell you they took the change and divided by the time. Write that down.

```
the rate of change = the change, divided by the time it took
```

Now lay a ruler across the two marked points and draw the line. Tell them that is the number they just gave you, drawn: the average rate is the slope of that line. The line is a chord.

Then lay the ruler flat against the far end of the curve, where it has flattened, and draw that line too. Ask them whether the two lines have the same steepness. They can see that they do not. That is the moment the idea lands, and it lands because they looked rather than listened.

Name the second one. The steepness at a single point is the rate at that moment, and its line is a tangent.

Now give them numbers, because a picture alone will not survive the week. Use six values a month apart, work the whole-run average with them on the board, then work the last month on its own. Let them see the two answers differ by a factor of several.

Close with the question you want them to carry. Not "what is the rate", but "over what interval". Tell them to ask it out loud in the next meeting where somebody says a number is rising at so much a month.

1. The change first.

```
38 minus 14 = 24
```

Then divide by the time it took.

```
24 divided by 6 = 4
```

The average rate of change is 4 for each unit of time. There is no unit on it here, because neither quantity was named.

2. The first.

```
90 minus 50 = 40
40 divided by 8 = 5
```

Five for each unit of time.

The second.

```
50 minus 90 = -40
-40 divided by 8 = -5
```

Minus five. The minus sign says the quantity fell, and the 5 says how fast.

The third.

```
7 minus 7 = 0
0 divided by 5 = 0
```

Zero. The quantity did not change over the interval. That is a real rate and it is the answer. It does not mean the quantity sat still the whole time, only that it ended where it started.

3. Each step takes one unit of time, so each rate is just the change across that step.

```
10 minus 4 = 6
20 minus 10 = 10
34 minus 20 = 14
52 minus 34 = 18
```

So the four step rates are 6, 10, 14 and 18.

Now the whole run. It covers four steps.

```
52 minus 4 = 48
48 divided by 4 = 12
```

An average rate of 12.

Look at the four step rates against that 12. The average sits between the second and the third, and it matches neither. Here the rate is rising, so the average understates the end and overstates the start.

4. The rate is the change divided by the time. So the change is the rate multiplied by the time.

```
7 times 9 = 63
```

A change of 63 over 9 units of time.

The second part runs the other way. You have the change and the rate, and you want the time, so divide the change by the rate.

```
91 divided by 7 = 13
```

Thirteen units of time.

Both parts are the same relation read in different directions, which is worth seeing once. A rate ties three quantities together, and knowing any two gives you the third.

5. The unit of a rate of change is always the quantity's unit divided by the time unit. Take them in order.

Kilograms divided by months is kilograms per month.

Kilocalories divided by days is kilocalories per day.

People divided by years is people per year.

Rupees divided by weeks is rupees per week.

None of those came from the numbers, because there were none. They came from reading the two labels, which is the whole of the unit check from the section on dimensional analysis.

6. Work out the monthly entitlement first.

```
5 times 6 = 30
```

Thirty kilograms a month. Now the running values, starting from 12.

```
12 plus 30 = 42
42 plus 30 = 72
72 plus 30 = 102
```

So 42 kilograms at month one, 72 at month two and 102 at month three.

The average rate over the three months.

```
102 minus 12 = 90
90 divided by 3 = 30
```

Thirty, and the unit comes off the two quantities. Kilograms divided by months is kilograms per month, so it is 30 kilograms a month.

The second month alone.

```
72 minus 42 = 30
30 divided by 1 = 30
```

Thirty kilograms a month again.

They agree because the same amount is added every month, so the points sit on a straight line. A straight line has one steepness and it has it everywhere, so every interval gives the same rate.

7. Five readings a month apart cover four months, because the intervals are one fewer than the readings.

```
425 minus 200 = 225
225 divided by 4 = 56.25
```

An average of 56.25 a month.

The first month.

```
320 minus 200 = 120
```

A hundred and twenty a month.

The last month.

```
425 minus 410 = 15
```

Fifteen a month.

Which to quote to somebody asking about now: the 15. The question was about the present, and the last month is the only one of the three that is about the present.

The 56.25 answers a different question, which is how much was added across the whole run for each month of it. Say that out loud when you give it, or it will be heard as the rate now.

And say the interval with every one of the three figures. Without it, 120 and 15 are the same kind of claim about the same quantity, and a reader cannot tell them apart.

8. Subtract the first value from the last, keeping the order.

```
300 minus 900 = -600
```

Then divide by the time.

```
-600 divided by 12 = -50
```

Minus fifty for each month.

What the sign means: the quantity fell. A rate of change is negative when the end is below the start, and the size still says how fast.

What it does not tell you. It does not tell you that the quantity fell every month. It could have dropped 600 in the first month and sat still for eleven. It could have risen and then fallen further. Every one of those gives the same average of minus 50, because the average uses only the two end values.

To find out which happened, ask for the monthly readings. There is no way to recover them from the average, and no amount of care with the arithmetic will change that.

9. Average rate for P.

```
220 minus 100 = 120
120 divided by 4 = 30
```

Thirty a month.

Average rate for Q.

```
256 minus 100 = 156
156 divided by 4 = 39
```

Thirty-nine a month.

On the averages alone, Q is growing faster than P.

Now the last month. For P.

```
220 minus 190 = 30
```

Thirty a month.

For Q.

```
256 minus 248 = 8
```

Eight a month.

What the pair shows: the two comparisons point opposite ways. Over the whole run Q outgrew P. Over the most recent month P is growing nearly four times as fast as Q.

Neither comparison is wrong and they answer different questions. P is a straight line, so its average and its monthly rate are the same 30. Q is flattening, so its average is held up by months that are over.

The lesson is the one to take into a meeting. Ask which interval a comparison used, because with a flattening quantity the choice of interval decides the winner.

10. The step that broke is the third. The change and the time were divided the wrong way round.

A rate of change is the change divided by the time, not the time divided by the change.

```
60 divided by 5 = 12
```

Twelve a month.

The units catch it as well as the arithmetic does. Divide months by kilograms and you get months per kilogram, which is how long each kilogram takes. That is a real quantity and it is not a rate of change.

A size check finishes it off. The quantity rose by 60 in five months, so it gained more than 10 in a typical month. An answer under a tenth of one unit a month cannot describe that rise.

Why it looks reasonable: the subtraction is right, the division is right, and the answer is given to three decimals, which reads as care. The one broken step is the one with no arithmetic in it.

11. The step that broke is the fifth. Every line before it is correct, and the word that breaks it is “currently”.

The 9.22 is the average rate over the five months. It is built from the first value and the last value and uses nothing in between.

Work out the rate over the most recent month instead.

```
56.1 minus 53.5 = 2.6
```

Two point six a month. That is the most recent month’s rate, and it is less than a third of the average.

Look at all five monthly rates and the reason is plain.

```
30 minus 10 = 20
42 minus 30 = 12
49.2 minus 42 = 7.2
53.5 minus 49.2 = 4.3
56.1 minus 53.5 = 2.6
```

The rate has fallen at every step, so 9.22 describes none of the five months on its own. It describes all five taken together, which is a different claim and not the one the fifth line makes.

The last line inherits the error. At 2.6 a month, five more months add about 13.

```
2.6 times 5 = 13
56.1 plus 13 = 69.1
```

About 69, against the 102.2 the broken working projects. Even 69 is an upper bound, because the rate has fallen every month and nothing says it has stopped. If the ratio between its steps keeps holding, the section before this one puts the ceiling at 60.

Why it looks reasonable: four correct lines in a row buy the fifth a great deal of trust, and the fifth changes only one word. An average rate and a rate now are written down identically, so nothing on the page marks the switch.

12. The step that broke is the fourth. A rise divided by a run between two points is the average rate over the interval between them. The answer has labelled it as the rate at one of the ends.

The line laid across month 2 and month 8 is a chord. Its slope is the average rate across those six months.

```
36 divided by 6 = 6
```

Six kilograms a month over the interval from month 2 to month 8. That number is right and its label is wrong.

The rate at month 8 is a different thing. It is the steepness of the curve at that single point, which is the slope of the tangent there. Getting it needs either the tangent laid on the drawing, or two readings close together around month 8.

Notice that the answer cannot be rescued by argument. If the curve is flattening, the rate at month 8 is below 6. If it is steepening, the rate at month 8 is above 6. Nothing in the two readings given says which, so the working has no way to reach the number it claims.

Why it looks reasonable: the arithmetic is right and both readings come off the drawing. The phrase “rise divided by run” is exactly the right phrase. Only the word “at” is wrong, and it is the word that turns a statement about six months into a statement about one moment.

13. What to compute: the average rate of change over the twelve months, and what that rate would take to clear the remaining 300.

The change first, keeping the order.

```
300 minus 900 = -600
```

Then the average rate.

```
-600 divided by 12 = -50
```

Fifty off the list each month, on average across the year.

Now the note’s own projection, using that rate.

```
300 divided by 50 = 6
```

Six months. So the arithmetic in the note is right, and that is the whole trap.

What this does not establish. It does not establish that the rate is steady. Two readings a year apart are consistent with a list that fell fast and stopped, and with one that is falling faster now than ever. The word “steady” is not in the numbers, and the twelve monthly readings that would settle it have not been given.

It does not establish that the same rate will continue. A list may clear its easy cases first. That would make the rate fall exactly when a projection assumes it will not. Nothing in two readings says whether it happened here.

And it does not establish that the list falling is the thing worth tracking. A list can shorten because people were seen, and it can shorten because people stopped joining it. Those are different events with the same arithmetic.

What to ask for: the twelve monthly figures, and the number joining the list each month alongside the number leaving it.

14. What to compute: the average the note quotes, and the rate in the most recent year. Then what the four yearly rates are doing to each other.

The average first. Add the gains and divide by four.

```
12 plus 6 = 18
18 plus 3 = 21
21 plus 1.5 = 22.5
22.5 divided by 4 = 5.625
```

About 5.6 points a year, so the note's average is right.

The rate in the most recent year is the fourth gain, which is 1.5 points a year. That is less than a third of the average.

Now look at what the yearly rates are doing.

```
6 divided by 12 = 0.5
3 divided by 6 = 0.5
1.5 divided by 3 = 0.5
```

Each year's gain is half the year before. On that pattern the next year's gain is about 0.75 points.

```
1.5 divided by 2 = 0.75
```

Compare the 5.625 with the most recent year's 1.5, and with the 0.75 the pattern points to.

```
5.625 divided by 1.5 = 3.75
5.625 divided by 0.75 = 7.5
```

So budgeting for 5.6 points next year is budgeting for nearly four times what the most recent year delivered. It is seven and a half times what the halving pattern points to.

What this does not establish. It does not establish that next year's gain will be 0.75. Four values and an assumed halving are a thin basis, and the pattern is read off the same numbers it is being used to project.

It does not establish that the programme is failing. Gains that shrink are what running towards a ceiling looks like from inside. A programme close to full coverage has little room left to gain in.

And it does not settle the budget question, because none of these numbers says what the money would buy anywhere else. What the arithmetic settles is narrower and still worth saying. The average is about the past four years. It is not a forecast of the fifth.

C8 · Accumulation, a running total, without the machinery

EXERCISE 1. The area under the line is the total number of litres over the twelve months. It is the running total, drawn.

The unit comes off the two axes. Litres a day up the side and months along the bottom, so the area is in litres a day multiplied by months.

That is not yet a usable unit, and the fix is a conversion from the section on converting units. Turn the months into days, and litres a day multiplied by days leaves litres.

What the figure does not tell you. It does not tell you the rate now. For that you read the height of the line at the right-hand end, rather than the area under it. Those are two different readings of the same picture, and only one of them answers "how fast".

It does not tell you how the rate moved inside each month. A line drawn between monthly readings is an assumption about the days between them.

And it does not tell you whether a total of that size matters. The picture carries no threshold and no comparison, so nothing in it says whether the number is large.

EXERCISE 2. The arithmetic is right. Fifty-two weeks at 400 bags is 20,800.

```
400 times 52 = 20800
```

What has been assumed: that a rate measured across one week held across fifty-two. The file has one rectangle, one week wide, and has stretched it to cover a year.

That assumption is doing all the work and it is nowhere stated. Ask whether issue rates move with the season or with the month of the entitlement cycle, and get the weekly figures that would show it.

What to ask for: the weekly issue figures for as many past weeks as exist. Fifty-two readings give fifty-two rectangles and a total that assumes almost nothing. Twelve monthly readings give twelve, which is far better than one.

What to write in its place. Say that the depot issued 400 bags in the first week of April. Then say that a yearly figure needs the rate across the year. The 20,800 is what the year would come to if every week matched that one. Give the 20,800 as a conditional and never as a count.

One more line worth adding. Ask whether the first week of April is a typical week, or whether the entitlement cycle makes it the busiest.

EXERCISE 3. Answer first, because that is what the page is for.

The two reports may both be right. A rate and a total are different quantities. One says how fast something is happening and the other says how much has happened. Neither can be checked against the other without the time it applied over.

Then the one line of arithmetic that settles it. Multiply the monthly rate by twelve months. If that lands near the yearly total, the two reports agree.

```
the yearly total = the monthly rate, times twelve months
```

Then the question to send back if the two do not agree. Ask how many times the rate was measured across the year. A single month stretched to twelve is a forecast, and a figure built from twelve monthly readings is a count. The two arrive on the page looking identical.

Then one sentence on what to require in future. Every total should say what readings it was built from. Every rate should say the interval it covers.

No methods section. The secretary does not need rectangles. They need to know that a rate and a total are different things, and that the multiplication between them hides an assumption.

1. The amount is the rate multiplied by the time.

```
7 times 9 = 63
```

```
7 times 20 = 140
```

```
7 times 0.5 = 3.5
```

So 63, then 140, then 3.5.

The last one is worth a second look. A time shorter than one unit gives an amount smaller than the rate, which is what multiplying by a number below one always does.

2. Each amount is the rate multiplied by the time it held for.

```
12 times 4 = 48
5 times 6 = 30
20 times 2 = 40
```

So 48, then 30, then 40.

Now add them.

```
48 plus 30 = 78
78 plus 40 = 118
```

A total of 118.

Notice the third stretch. It had the highest rate of the three and it contributed less than the first, because it lasted two units of time instead of four. A rate on its own does not tell you how much a stretch contributed.

3. Each interval is one unit of time long, so each amount is the rate itself.

Add them one at a time.

```
9 plus 9 = 18
18 plus 4 = 22
22 plus 4 = 26
26 plus 4 = 30
30 plus 2 = 32
```

So the running total reads 9, 18, 22, 26, 30, 32.

Two things to notice. The running total never falls, because every rate here is positive. And it climbs in steps of different sizes, because the rate changed while the intervals did not.

4. Each stretch first.

```
10 times 5 = 50
6 times 5 = 30
6 times 10 = 60
```

Now the total.

```
50 plus 30 = 80
80 plus 60 = 140
```

A total of 140.

Now the average rate. The whole time is 5 plus 5 plus 10.

```
5 plus 5 = 10
10 plus 10 = 20
```

Twenty units of time. The average rate is the total divided by the time.

```
140 divided by 20 = 7
```

Seven for each unit of time.

That 7 is the height of a single rectangle twenty units wide that would hold the same area as the three real ones. It is a useful summary and it matches the rate over none of the three stretches.

5. Start with the monthly rate for the household.

```
5 times 7 = 35
```

Thirty-five kilograms a month. The unit comes from the Act's own rate: kilograms per person per month, multiplied by seven people, leaves kilograms per month.

Now multiply by the time.

```
35 times 12 = 420
```

Four hundred and twenty kilograms.

Check the unit before accepting it.

```
kilograms per month, times months
= kilograms
```

The months cancel, so the answer is a mass and not a rate. Four hundred and twenty kilograms a year for the household.

Which figure is whose. The five kilograms per person per month is the Act's. The seven people, the twelve months, the 35 and the 420 are all yours. Say so whenever you quote the 420. A figure that came out of your calculator reads exactly like one that came out of a statute.

6. Each month is one interval, so each amount is that month's rate multiplied by one month.

```
300 plus 300 = 600
600 plus 450 = 1050
1050 plus 450 = 1500
1500 plus 200 = 1700
1700 plus 100 = 1800
```

So the running total reads 300, 600, 1,050, 1,500, 1,700 and 1,800 litres.

The total over the six months is 1,800 litres.

Check the unit. Litres per month multiplied by months leaves litres, so the answer is a volume and not a rate.

The assumption. Each figure was read once and then treated as the rate for that whole month. Where the rate moved inside a month, the rectangle for that month is too tall or too short. Nothing in the six readings shows which, and nothing in the total warns you about it.

7. Each reading stands for a three-month interval, so each amount is the rate multiplied by three months.

```
40 times 3 = 120
60 times 3 = 180
60 times 3 = 180
20 times 3 = 60
```

Now add the four amounts.

```
120 plus 180 = 300
300 plus 180 = 480
480 plus 60 = 540
```

Five hundred and forty units across the year.

Now the other sum, which is what people write down when they are hurrying.

```
40 plus 60 = 100
100 plus 60 = 160
160 plus 20 = 180
```

A hundred and eighty. Its unit is units a month, because adding four rates gives a rate and never an amount.

So 180 is not a wrong total. It is not a total at all. It is four rates added together, and the unit says so before the number does.

8. This one runs backwards from the total, which is the direction you will need more often than the other.

The average rate over the whole stretch is the total divided by the time.

```
96 divided by 8 = 12
```

Twelve kilograms a month.

Now the first two months. Their amount is the rate multiplied by the time.

```
18 times 2 = 36
```

Thirty-six kilograms. So the remaining six months hold the rest.

```
96 minus 36 = 60
```

Sixty kilograms over six months. Their average rate is that amount divided by that time.

```
60 divided by 6 = 10
```

Ten kilograms a month.

Look at the three figures together. The average over the whole stretch is 12, the early rate was 18 and the later average is 10. The whole-stretch average sits between them and describes neither part.

9. The step that broke is the third. Four rates were added together, and a sum of rates is not a total.

The units say it first. Litres a minute added to litres a minute gives litres a minute. The answer is labelled litres, so somewhere a unit was dropped rather than cancelled.

An amount needs a time. Each reading has to be multiplied by the length of the interval it stands for, and the working never says what that length is. Take the readings as five minutes apart and each rectangle is five minutes wide.

```
12 times 5 = 60
9 times 5 = 45
6 times 5 = 30
3 times 5 = 15
```

Now add the four amounts.

```
60 plus 45 = 105
105 plus 30 = 135
135 plus 15 = 150
```

A hundred and fifty litres, if the readings are five minutes apart.

Notice what that means. The broken answer cannot be corrected from what it gives you, because the interval was never recorded. Without it there is no total to be had.

Why it looks reasonable: adding a column of numbers is what a total feels like, and the four readings are right there in a column. The arithmetic is correct. The step that never happened is the one that leaves no trace on the page.

10. The step that broke is the third, and it broke on the side axis rather than on the arithmetic.

Read the axis label again. It carries kilograms in the store, which is a quantity, not kilograms a month, which would be a rate.

The area under a line is a total only where the side axis carries a rate. Here the area is in kilograms multiplied by months, which is not a quantity of grain and not a quantity of anything else anybody uses.

What the figure does say is worth stating, because it is useful and it is not what was claimed. The store held about 300 kilograms at every month of the year. The line is flat, so the amount in the store did not change.

What the figure does not say: how much passed through. A store can hold a steady 300 kilograms while thousands of kilograms move in and out. It can also hold a steady 300 kilograms with nothing moving at all. The picture cannot tell those apart, and that is the next section.

Why it looks reasonable: the phrase “area under the line” is exactly right for a rate graph. Here it has been carried across to a graph of a quantity, and nobody reread the axis. The number 3,600 is plausible, has a unit attached, and answers a question nobody could have answered from this figure.

11. What to compute: the daily amount from the rate, and then the yearly total the note is claiming, so you can see what it assumed.

The daily amount is the rate multiplied by the hours.

```
8 times 6 = 48
```

Forty-eight litres a day. Check the unit: litres an hour multiplied by hours leaves litres.

Now a year of that.

```
48 times 365 = 17520
```

About 17,520 litres, so the note’s arithmetic holds.

What this does not establish. It does not establish that the vehicle runs every day of the year. Three hundred and sixty-five is the note’s own assumption and it is nowhere stated. At 250 working days the figure is different.

```
48 times 250 = 12000
```

Twelve thousand litres, which is about a third less.

It does not establish that 8 litres an hour is the rate on other days. One measurement was stretched across a year, which is one rectangle drawn a year wide.

And it does not establish that six hours is typical. Every one of the three numbers is a single reading doing the work of a year of readings.

What to ask for: the fuel actually drawn over the last twelve months. A count beats a total built from one day, and the depot already holds it.

12. What to compute: the change between the two totals, and what a monthly rate would have to be to produce each of them.

Write both figures out in full first.

```
24 lakh = 24 times 10^5 = 2400000
20 lakh = 20 times 10^5 = 2000000
```

Now the change.

```
2400000 minus 2000000 = 400000
```

Four lakh kilograms more, which as a share of the earlier year is this.

```
400000 divided by 2000000 = 0.2
```

A fifth more, or twenty per cent.

Now turn the later total back into a rate, since a total on its own hides the year it came from.

```
2400000 divided by 12 = 200000
```

Two lakh kilograms a month, averaged across the twelve months.

What this does not establish. It does not establish that more people were reached. A total of grain divided among fewer people, at a larger amount each, gives exactly the same number. Kilograms are not people, and the note has swapped one for the other with no working in between.

It does not establish that the rise happened evenly. A yearly total is one rectangle. Two good months and ten flat ones give the same total as twelve steady ones.

And it does not establish that either year met what was owed. That question needs the entitlement, the number of people entitled, and a comparison against those. None of the three is in the sentence.

What to ask for: the monthly distribution figures, and the number of people who collected in each month. The second is the one the claim was actually about.

C9 · Stocks and flows

EXERCISE 1. The 4,000 kilograms is a stock. Its unit is kilograms with no time on the bottom. It is also attached to a date, which is what a quantity existing at a moment needs.

The 12,000 kilograms a month is a flow. Its unit has a month on the bottom, so it describes an amount for each stretch of time rather than an amount at a moment.

What the stock answers: how much was on the floor of the store on one day. That is the figure you check by counting sacks.

What the flow answers: how fast grain is moving through. Multiply it by a length of time and you get the amount that moved over that time.

What neither answers. Neither tells you whether the store is running short, because that needs the inflow and the outflow separately rather than a single figure for traffic. Neither tells you whether anybody received what they were owed, because kilograms moving is not people served. And the stock on one day says nothing about the days on either side of it.

One thing to ask for: the two flows written separately, month by month, beside the stock at the end of each month. Three columns make the balance line checkable. One figure of each kind does not.

EXERCISE 2. What is wrong: a stock has been read as though it measured its flows. It cannot.

Six counts of the same stock, one a month apart, establish one thing. Over each of those months, what came in matched what went out. Write the balance line and the reason is plain.

```
the stock at the end = the stock at the start, plus the total in, minus the total
out
```

If the two ends are equal, the two totals are equal. That is all the arithmetic gives you, and it holds whether the flows were 10 kilograms a month or 10,000.

The word “static” is doing the damage. A steady level and a still scheme are different claims, and only one of them has been measured.

What to ask for. The issue records, which give the outflow to households month by month. The receipt records, which give the inflow from the depot the same way. The two together make the balance line checkable, and they answer the question the note was actually asking.

One more thing worth asking. Whether the six counts were taken at the same point in the distribution cycle. A store counted on the first of each month may be at the same point of the cycle every time. A steady figure is then the least surprising result available.

EXERCISE 3. Three minutes, no jargon, and they need a line they can print.

Start with a picture they already have. A tank with a tap running in and a drain running out. The level in the tank is one thing. The water moving through it is another.

Then the point. If the tap and the drain run at the same rate, the level does not move. It does not move whether both are trickling or both are roaring. The level tells you the two matched. It tells you nothing about how big either one was.

Then make the swap, in one sentence. Body weight is the level. What goes into the body and what leaves it are the tap and the drain.

Then the thing they are about to get wrong, said before they get it wrong. A weight that has not changed is not a body where nothing is happening. It is a body where the two have been matching, which is an entirely different statement and a far harder thing to do.

The quotable sentence: “A steady weight does not mean nothing is going on. It means two flows, which may be large, have been matching each other, and that is not the same as being still.”

What not to offer: a figure for either flow. You do not have one here, and a number invented to fill a pause is the part that will get printed.

1. Use the balance line. The stock at the end is the stock at the start, plus what came in, minus what went out.

```
500 plus 120 = 620
620 minus 90 = 530
```

The stock ends at 530.

The net flow is the inflow minus the outflow.

```
120 minus 90 = 30
```

Plus 30 over the period. It is positive, and the stock rose by exactly that amount.

2. Take each row through the balance line.

The first.

```
800 plus 200 minus 200 = 800
```

Eight hundred. The stock did not move, because the two flows matched.

The second.

```
800 plus 200 minus 350 = 650
```

Six hundred and fifty. More went out than came in, so the stock fell.

The third.

```
800 plus 400 minus 150 = 1050
```

One thousand and fifty. More came in than went out, so the stock rose.

Look at the first row against the other two. Its flows are not the smallest of the three, and it is the only row where nothing appears to have happened.

3. Read the unit of each one and look for a time on the bottom.

Grain in the block store is a stock. Kilograms, with no time in the unit, so it is an amount existing at a moment.

Grain arriving at the store is a flow. Kilograms a month has a month on the bottom.

People on a waiting list is a stock. People, with no time in the unit.

People joining the list is a flow. People a week has a week on the bottom.

Energy spent is a flow. Kilocalories a day has a day on the bottom.

Notice the pattern down the list. Each flow names an action, and each stock names a thing sitting somewhere. That is a useful second check and the unit is the one that settles it.

4. This one runs the balance line backwards, which is how most real questions about a stock arrive.

Start with what the line says.

```
the stock at the end = the stock at the start, plus the total in, minus the total
out
```

Put in what you know. The start plus the inflow comes to this.

```
1400 plus 2600 = 4000
```

Four thousand would be the stock if nothing had gone out. It ended at 1,100, so the difference is what left.

```
4000 minus 1100 = 2900
```

Two thousand nine hundred flowed out.

Check it forwards.

```
1400 plus 2600 minus 2900 = 1100
```

Eleven hundred, which is the figure given. The stock fell by 300 while 2,900 went out of it, which is the point of the whole section in two numbers.

5. Work out the entitlement for one household first.

```
5 times 5 = 25
```

Twenty-five kilograms a month for each household. Now for all 300.

```
25 times 300 = 7500
```

Seven thousand five hundred kilograms a month go out. The same amount comes in.

Now the balance line for one month.

```
1500 plus 7500 minus 7500 = 1500
```

Fifteen hundred kilograms after one month. The two flows matched, so the stock did not move.

After six months it is the same figure, because the same thing happens each month. The stock is 1,500 kilograms.

Now the size of the traffic against the size of the stock.

```
7500 divided by 1500 = 5
```

Five times the contents of the store room went through it every month, and the count on the shelves never changed.

Which figures are whose. The five kilograms per person per month is the Act's. The 300 households, the five people in each and the 1,500 kilograms are yours. So is every number after them.

6. The inflow is a rate, so turn it into an amount by multiplying by the time it ran.

A day holds this many minutes.

```
24 times 60 = 1440
```

Now the amount that came in.

```
15 times 1440 = 21600
```

Twenty-one thousand six hundred litres in, and the same amount out, since the two rates are equal.

Check the unit, because a rate multiplied by a time has to leave the quantity's own unit.

```
litres per minute, times minutes
= litres
```

Now the replacements. Divide what went through by what the tank holds.

```
21600 divided by 6000 = 3.6
```

Three point six times over.

So a tank whose level did not move by one litre passed more than three times its own contents in a day. The level and the traffic are two different measurements, and only one of them was taken.

7. The total issued is a rate multiplied by a time.

```
9000 times 6 = 54000
```

Fifty-four thousand kilograms out over the six months.

The stock ended where it started, so the totals must match.

```
4000 plus 54000 minus 54000 = 4000
```

So 54,000 kilograms also came in. The monthly inflow is that total divided by the time.

```
54000 divided by 6 = 9000
```

Nine thousand kilograms a month, the same as the outflow.

Now the other case. At 8,500 a month the total in is this.

```
8500 times 6 = 51000
```

Fifty-one thousand in against 54,000 out.

```
4000 plus 51000 minus 54000 = 1000
```

The July count would have read 1,000 kilograms.

That shortfall is 500 kilograms a month, which is under six per cent of the flow. It empties three quarters of the store in half a year. A small difference between two large flows moves a small stock a long way.

8. The step that broke is the third. A stock that does not change has a net flow of zero, which is not the same as having no flows.

Write the balance line and see what the two counts actually fix.

```
the stock at the end = the stock at the start, plus the total in, minus the total
out
```

Both ends are 2,000, so the total in equals the total out. Nothing in that says how large either total was.

Take two cases that both fit the measurements exactly. In the first, nothing moved.

```
2000 plus 0 minus 0 = 2000
```

In the second, ten thousand kilograms moved each way.

```
2000 plus 10000 minus 10000 = 2000
```

Both end at 2,000 kilograms, and the two describe completely different months. The measurement cannot separate them, so the conclusion cannot be drawn from it.

What would separate them: the issue records and the receipt records, which measure the flows directly.

Why it looks reasonable: the first two lines are a correct reading of two correct measurements. The third line sounds like arithmetic and is an assumption. It is also the assumption anybody makes who has looked at a shelf rather than at a ledger.

9. Two things went wrong and the units find both of them.

The step that broke is the fifth. The 2,850 is a quantity of grain, in kilograms, and it has been read as a number of months.

Check what the fourth line produced.

```
kilograms, minus kilograms per month times one month
= kilograms
```

Kilograms. So 2,850 is the stock after one month, which is a useful figure with the wrong label on it.

To get a time, divide the stock by the rate it falls at rather than subtracting. That rate is 150 kilograms a month: the size of the net flow, without its minus sign. Dividing by the minus 150 itself would give minus 20, and a number of months cannot be negative.

```
3000 divided by 150 = 20
```

Twenty months, and the unit confirms it.

```
kilograms, divided by kilograms per month
= months
```

The second error hides inside the first. Line four subtracted a flow from a stock without multiplying by a time. It came out right only because the time happened to be one month, and nothing on the page says so.

Why it looks reasonable: the net flow is computed correctly and carries the right sign. The subtraction is arithmetic anybody would accept. A unit check on the last line catches it. So does a size check, because 2,850 months is a long enough answer to stop anybody.

10. What to compute: what the two counts fix, and what they leave open. The list is a stock. People joining and people leaving are its two flows.

Write the balance line.

```
the list in October = the list in April, plus the total joining, minus the total
leaving
```

Both ends are 1,200, so the number joining over the six months equals the number leaving.

Now take two cases that fit the measurements equally well. In the first, nobody joined and nobody was seen.

```
1200 plus 0 minus 0 = 1200
```

In the second, 300 joined each month and 300 were seen each month.

```
300 times 6 = 1800
1200 plus 1800 minus 1800 = 1200
```

Eighteen hundred people joined and 1,800 were seen, and the list ends where it began.

What this does not establish. It does not establish that nobody was cleared. The second case has 1,800 people cleared in six months and produces the same two counts as the first.

It does not establish the size of either flow. A stock at two dates fixes the difference between the two totals and never the totals themselves.

And it does not establish that the list is the thing to watch. A list can hold steady while the wait on it lengthens, if the people leaving are not the people who have waited longest.

What to ask for: the number added and the number seen, month by month. Those are the flows, and the note's claim is about them rather than about the level.

11. What to compute: whether a steady stock fixes the size of its flows. Weight is the stock. What goes into her body and what leaves it are the two flows.

Take any stock and call it 100 units, so that the arithmetic is about the shape rather than about a person. Run the balance line over a year for three different pairs of flows.

Nothing in and nothing out.

```
100 plus 0 minus 0 = 100
```

Nine hundred in and 900 out.

```
100 plus 900 minus 900 = 100
```

Twelve hundred in and 1,200 out.

```
100 plus 1200 minus 1200 = 100
```

All three end at 100. The stock is the same in every case and the flows differ by as much as you like. So a steady stock fixes the difference between the two totals at zero and fixes neither total.

That is the answer to your colleague. The steadiness is evidence that the two matched over the year. It is no evidence at all that either was small.

Her energy balance has the same shape: a steady energy stock would mean her energy in and energy out matched, at any size. But a steady weight is a statement in kilograms. Whether it means her stored energy was steady is the step this book does not take. So do not let "her energy in matched her energy out" slip into the answer as though the scales had shown it.

What this does not establish. It does not tell you how large either flow was, and nothing in this course so far gives you a figure for one. It does not tell you that the two matched at every moment of the year, only that the totals came out equal across it.

And it does not tell you what would happen if one flow changed. In a store room the inflow does not care what the shelves are doing. Cut it by a hundred units a year and the balance drops by a hundred units a year. Now take a system where one flow moves when the other moves, or moves with the level of the stock. That subtraction does not survive. The balance line cannot tell you which kind of system is in front of you, because the same line describes both. Which kind a person is, and what follows from it, is the work of the booklet on energy balance.

D1 · What a probability is

EXERCISE 1. Start with what is right in the sentence. Seventy-five per cent is a real figure from the Act, and it is a probability, at most, not odds.

Explain odds in one move. Odds are how many times more likely the event is than its opposite. A probability of seventy-five per cent gives odds of three to one, because three-quarters divided by one-quarter is three.

```
0.75 divided by 0.25 = 3
```

Say plainly that "odds of seventy-five per cent" is not a sentence anyone can act on. Odds are not written as a percentage. When somebody does that, they are quoting the probability and calling it odds by mistake. The two are close only for rare events. At one half they are already a factor of two apart.

What to hand them: "Under the law, up to three in four of the rural population can be covered. That is a ceiling set by the Act, not a count of how many actually are."

EXERCISE 2. Odds are not expressed as a percentage. A percentage is a probability's natural home, because a probability is bounded between 0 and 1 and a percentage is just that scaled by 100. Odds have no upper bound — odds of 99 are a real number, for a probability of 0.99. Writing them as a percentage silently assumes they behave like a probability. That assumption is the whole of the mistake.

Two honest rewrites, depending on which figure was actually meant.

If 60% is the probability of coverage, say so, and give the odds only if the audience needs them.

```
0.6 divided by 0.4 = 1.5
```

"Sixty per cent of the block is covered. That is odds of three to two in favour, for anyone who needs the odds form."

If 60% was meant as the odds themselves — an odds of 0.6, or three to five — convert it back to the probability. Do this before printing anything.

```
0.6 divided by 1.6 = 0.375
```

"Odds of three to five in favour of coverage is a probability of 37.5 per cent, not sixty."

Either way, the fix is the same: name which figure you have, in words, before the number.

1. Count the outcomes in the event, then the outcomes in the whole sample space.

```
5 divided by 8 = 0.625
```

Red: 0.625.

```
3 divided by 8 = 0.375
```

Blue: 0.375.

```
0.625 plus 0.375 = 1
```

They add to 1, as they must: red and blue between them are every outcome there is.

2. Odds are the probability divided by its complement.

```
1 minus 0.2 = 0.8
```

```
0.2 divided by 0.8 = 0.25
```

Odds of 0.25, which is one to four.

Going the other way, probability equals odds divided by 1 plus odds.

```
1 plus 1.5 = 2.5
```

```
1.5 divided by 2.5 = 0.6
```

A probability of 0.6.

3. The complement first, since the odds are built from it.

```
1 minus 0.4 = 0.6
```

Now divide the probability by the complement.

```
0.4 divided by 0.6 = 0.6666…
```

Odds of about 0.667, which is two to three. Notice that the probability, 0.4, is smaller than the complement, 0.6. The odds come out smaller than 1 to match. Odds below 1 always mean the event is less likely than its opposite.

4. The Act's own figure is the probability of coverage, read as a ceiling.

```
P(covered) is at most 0.75
```

The complement is the probability of not being covered.

```
1 minus 0.75 = 0.25
```

At most 0.75 covered, at least 0.25 not covered. Both figures are ceilings and floors taken from the Act's own "up to" wording, not counts of anyone actually reached.

5. Divide the probability by its complement.

```
1 minus 0.75 = 0.25
```

```
0.75 divided by 0.25 = 3
```

Odds of three to one, at most. Here the odds figure and "three in four" happen to describe the same fact in a way that is easy to eyeball. That will not hold at the urban ceiling, which is the next problem.

6. The urban complement first.

```
1 minus 0.5 = 0.5
```

```
0.5 divided by 0.5 = 1
```

Odds of one to one — "evens" — for the urban ceiling, against three to one for the rural ceiling.

The rural probability is only 1.5 times the urban one.

```
0.75 divided by 0.5 = 1.5
```

But the odds are three times as large, not 1.5 times.

```
3 divided by 1 = 3
```

The comparison shows that odds do not scale the way probability does. The stretching grows as the probabilities approach 1. Near 0 it almost vanishes.

7. The step that broke is the second line, where odds are read straight across as a probability.

Odds and probability are different numbers built from the same fact. Odds and probability are equal only at zero. At every other value the odds are the larger number.

```
0.25 divided by 1.25 = 0.2
```

A probability of 0.2, not 0.25. One household in five is covered, not one in four.

Why it looks reasonable: both odds and probability are written as plain decimals between roughly zero and one. This happens whenever the event is more likely to fail than succeed. A reader has nothing on the page to tell the two apart. The only fix is to ask, every time, whether the number was labelled odds or probability. The label is the only thing that survives being printed.

8. The formula on the second line is wrong. Odds are the probability divided by its complement — one minus the probability — not by one plus the probability.

```
1 minus 0.6 = 0.4
```

```
0.6 divided by 0.4 = 1.5
```

Odds of 1.5, not 0.375.

The two formulas look alike. That is exactly why this slip is common: “1 minus p” and “1 plus p” share three of their four characters. A quick check catches it without redoing the division. A probability above 0.5 means the event is more likely than not, so its odds must be above 1. And 0.375 is below 1. That already says the working is wrong, before you even find the broken line.

9. What to compute: whether “three in four” matches the Act’s own figure, and what kind of figure it is.

Section 3(2) sets the rural ceiling at up to 75 per cent, which is the same fraction as three in four.

```
3 divided by 4 = 0.75
```

So the note’s figure matches the Act’s ceiling exactly.

What this does not establish. The Act’s word is “up to”. A ceiling states the most that can be covered, and the note has silently turned it into a statement of fact about this place. It does not tell you how many people are actually covered here.

And section 3(2) names no State or district. The Act leaves each State’s share to the Central Government under section 9, so it says nothing about this place in particular. Nor does it tell you whether the people entitled under the ceiling have actually been identified and issued a ration card. That is a separate administrative step, and the Act does not guarantee it happens up to the limit it sets.

D2 · Counting outcomes; independence

EXERCISE 1. What is missing is any statement about independence. Multiplying is only correct if knowing that the first resident is covered tells you nothing about the second. The sentence does not say whether that holds.

If the two are from different, unrelated households, the multiplication is a reasonable approximation, at most.

```
0.75 times 0.75 = 0.5625
```

If the two are from the same household, coverage is decided once for the household. So the second resident’s status is fixed the moment the first one’s is known. The correct figure is not 0.5625. It is the same 0.75 the household itself carries.

```
P(both covered, same household) is at most 0.75
```

So the answer 0.5625 is only right for one of the two cases the sentence could have meant. The sentence gives no way to tell which.

EXERCISE 2. Start from something they already trust: two rolls of a fair die. Ask whether the first roll changes anything about the second. They will say no. Write that down as the test in words: does knowing one change the chance of the other?

Then break the habit on purpose. Describe two people from the same household under the food security law's entitlement, where the household is covered or not as one unit. Ask what happens to the second person's chance once you know the first is covered. Let them work out that it becomes certain, not 0.75 again.

Write the test as arithmetic once they have felt it. A and B are independent exactly when P(A and B) equals P(A) times P(B), and dependent otherwise. Have them check the household case at the ceiling itself. If coverage is 0.75, P(both covered) is 0.75. P(A) times P(B) would have been 0.5625. So the two do not match. That mismatch is the proof, not just the arithmetic slip.

Close with the standing rule: when you do not know whether two events are independent, assume they are not, until you can show it. A resident who defaults to "assume independent" will eventually be wrong in a way that reads as confident.

1. Each toss has two outcomes, and there are three tosses, so list every combination.

HHH	HHT	HTH	THH
HTT	THT	TTH	TTT

Eight outcomes.

```
2 times 2 times 2 = 8
```

Each extra toss doubles the list, which is 2 to the power of the number of tosses: three tosses give 2 cubed.

2. A and B are independent, so multiply.

```
0.6 times 0.5 = 0.3
```

P(A and B) is 0.3.

C and D are mutually exclusive, so add.

```
0.3 plus 0.4 = 0.7
```

P(C or D) is 0.7.

Same shape of question, two different words, two different operations. Multiplying for the second pair, or adding for the first, would both be wrong.

3. Go by the complement: the probability that neither happens, subtracted from 1.

```
1 minus 0.2 = 0.8
```

```
0.8 times 0.8 = 0.64
```

```
1 minus 0.64 = 0.36
```

A probability of 0.36 that at least one happens. Working this out directly would mean adding the probability of "just the first", "just the second" and "both" separately. The complement reaches the same answer in three short steps.

4. The test is whether P(A and B) equals P(A) times P(B).

```
0.5 times 0.4 = 0.2
```

That matches the given P(A and B) of 0.2, so A and B are independent.

Notice that this is a check, not an assumption. If P(A and B) had been given as, say, 0.3, the same test would have told you the two events are dependent. No amount of the sentence sounding plausible would change that.

5.

```
HH    HT    TH    TT
```

Four outcomes. A is {HH, HT}, so P(A) is 2 divided by 4.

```
2 divided by 4 = 0.5
```

B is {HH, TH}, so P(B) is 0.5 as well, by the same count.

A and B together is {HH}, one outcome out of four.

```
1 divided by 4 = 0.25
```

Check the independence rule.

```
0.5 times 0.5 = 0.25
```

P(A and B) is 0.25, which matches P(A) times P(B). The two tosses are independent, exactly as common sense says they should be. Now you have the arithmetic to prove it, rather than assert it.

6. Different households, so treat the two as independent and multiply.

```
0.75 times 0.75 = 0.5625
```

At most 0.5625 that both are covered. Note the "at most" carries through from the ceiling in the Act itself. The multiplication does not make the figure any more exact than it started.

7. Coverage under section 3(1) is decided for the household as a whole: every person belonging to a priority household is entitled together. If the household is covered, both people are; if it is not, neither is. There is one draw, not two. So the two people's covered-or-not status is not independent.

```
P(both covered, same household) is at most 0.75
```

Not 0.5625. Multiplying 0.75 by itself would have treated the second person's status as a fresh, independent draw. But it is fixed the moment the first person's status is known.

8. Go by the complement: the chance none of the three is covered, subtracted from 1.

```
1 minus 0.75 = 0.25
```

```
0.25 times 0.25 = 0.0625
```

```
0.0625 times 0.25 = 0.015625
```

```
1 minus 0.015625 = 0.984375
```

At most, about 98.4 per cent. With a ceiling this high on a single person, it takes very few independent draws before "at least one covered" becomes close to certain. This is worth noticing on its own. It is easy to assume "at least one" always needs a large group.

9. The step that broke is the second line. Adding is the rule for "or" only when the two events are mutually exclusive, and independent is not the same claim as mutually exclusive. In fact two independent events with positive probabilities can always happen together. So they are never mutually exclusive.

The correct rule for "or" subtracts the overlap.

```
0.7 times 0.6 = 0.42
```

```
0.7 plus 0.6 = 1.3
```

```
1.3 minus 0.42 = 0.88
```

P(A or B) is 0.88, not 1.3.

There is a check that would have caught this without any of the correct arithmetic. A probability can never exceed 1, and 1.3 already says the working is wrong. Independent events can and usually do overlap. Treating "independent" as though it meant "cannot both happen" is the confusion behind the broken line.

10. The step that broke is the third line, where the two people are treated as independent.

Coverage is decided for the household as a whole under section 3(1), so the two people in the same household are not two separate draws. If one is covered, so is the other, with certainty.

```
P(both covered, same household) is at most 0.75
```

At most 0.75, not 0.5625.

Why it looks reasonable: the arithmetic of multiplying two 0.75s is the same move that was correct for two different households in an earlier problem. Nothing in "two rural people" on its own tells you which case you are in. The fix is always the same question: what was the decision actually made about? Here it was made about the household, not the person.

11. The step that broke is the addition. "At least one" is not found by adding the individual probabilities. The answer itself gives the game away: a probability can never exceed 1. And 2.25 already cannot be a probability, before you even read the conclusion drawn from it.

Go by the complement instead.

```
1 minus 0.75 = 0.25
```

```
0.25 times 0.25 = 0.0625
```

```
0.0625 times 0.25 = 0.015625
```

```
1 minus 0.015625 = 0.984375
```

At most about 98.4 per cent, not 2.25. The conclusion in the broken answer happens to point the right way — it is indeed close to certain. But it is close to certain because of the correct figure, 0.984, not because of the 2.25.

The 2.25 is not meaningless, though. Adding the three gives the most people you would expect covered on average, 2.25. That is a real number, and it answers a different question. It is not the chance that at least one is covered.

Why it looks reasonable: "at least one" sounds like it should be built by combining the three individual chances. Addition is the most familiar way to combine numbers. The combination that is actually needed runs through the complement. A total above 1 is the signal that adding was the wrong move here.

12. What has to be true: the two residents are treated as independent, which means, in this setting, that they come from two different, unrelated households.

```
0.75 times 0.75 = 0.5625
```

The arithmetic is right on that assumption.

What this does not establish. If the two residents happen to be from the same household, the correct figure is 0.75, not 0.5625, because coverage is decided once per household. The note does not say which case applies. So 0.5625 could be the right answer, or it could be badly wrong for the actual pair being discussed. It also says nothing about how many people are actually covered anywhere real. Every figure here is still built from the Act's "up to" ceiling rather than a count.

13. The tempting route treats the four household members as four independent draws and finds the chance at least one is not covered through the complement.

```
0.75 times 0.75 = 0.5625
```

```
0.5625 times 0.75 = 0.421875
```

```
0.421875 times 0.75 = 0.31640625
```

```
1 minus 0.31640625 = 0.68359375
```

That route gives about 68 per cent, and it is wrong, because it repeats the same mistake as the same-household problem earlier in this set. Coverage is decided once for the whole household under section 3(1), not once per member. A household of four is covered or not covered as one draw, and every member shares that single outcome.

The question "at least one member is not covered" is therefore the same question as "is the household not covered". So it is one chance, not four: the chance this one person is not covered. The Act's 75 per cent is a ceiling on all rural people together. For a rural person drawn with nothing else known, it gives a floor on that chance.

```
1 minus 0.75 = 0.25
```

But this person is known to live in a household of four. A ceiling on all rural people puts no bound on any one group within them, such as people in households of four. The section has no figure for that group. So the report's 'high' has nothing under it, and nothing here rules it out either.

What this does not establish. A district report cannot quote section 3(2) as a district ceiling. Section 3(2) names no State or district. Section 9 leaves each State's own share to the Central Government, so nothing here is a fact about this particular district.

It also does not say how many households actually sit on the wrong side of the ceiling. That needs a count, not a probability. And it says nothing about a household reached only partly in practice. The "covered as one unit" model is the Act's own structure. Any real slippage inside a household would need data this section does not have.

D3 · Conditional probability through the two-way table

EXERCISE 1. Answer first. Eighty-six per cent of what? "Accurate" could mean three different figures.

Suppose it is the sensitivity, the share of people with the condition that the test catches. Then it says nothing yet about how often a positive result is right.

The reason in one line: how many of the test's positive results are actually right depends on the group tested, not only on the test. The more common the condition is there, the more of the positive results are right.

Here is the number that matters, worked from a real study's own figures. Among these medical students, about 32 per cent of the 131 men are in the band. A positive result there was right about 61 times in 100. In a group where one in five is in the band, the same test's positive results would be right about 45 times in 100.

What to ask before approving anything: what share of the population actually targeted has the condition. Without that figure, "86 per cent accurate" cannot be turned into a number of people. It cannot say how many a positive result would wrongly reassure or wrongly worry.

EXERCISE 2. Nothing taught so far settles it by arithmetic. A later section will give a way.

What can be checked is how many places in the paper repeat each number. The methods section, Table 2 and Table 3 all give the women's cut-off as 28 per cent. Only the abstract's one sentence gives 30 per cent instead.

Three places agree on 28, and only one place states 30. That makes 28 the figure the study's own numbers were almost certainly built on, and 30 in the abstract the more likely slip. Report both figures, and say which three places agree, rather than assuming a headline abstract figure is always the right one.

1. Start from what you are given. Fifteen of the 20 with the condition test positive, so 5 of the 20 test negative.

```
20 minus 15 = 5
```

Ten of the 80 without the condition test positive, so 70 test negative.

```
80 minus 10 = 70
```

Add across for the two row totals.

```
15 plus 10 = 25
5 plus 70 = 75
```

	condition present	condition absent	total
test positive	15	10	25
test negative	5	70	75
total	20	80	100

Every number in the finished table is invented, to practise the move of filling one in.

2. P(test positive given condition present) divides by the column of people who have the condition, which totals 20.

```
15 divided by 20 = 0.75
```

P(condition present given test positive) divides by the row of people who test positive, which totals 25.

```
15 divided by 25 = 0.6
```

Same top number, 15. Two different bottom numbers, the column total of 20 and the row total of 25, give 0.75 and 0.6. The two questions are not the same question.

Separately, check whether the test and the condition are independent, using last section's test. Does P(test positive and condition present) equal P(test positive) times P(condition present)?

```
15 divided by 100 = 0.15
25 divided by 100 = 0.25
20 divided by 100 = 0.2
0.25 times 0.2 = 0.05
```

P(test positive and condition present) is 0.15. P(test positive) times P(condition present) is 0.05. They do not match, so the test and the condition are not independent — which is exactly the point of running a test at all.

3. This divides by the column of people without the condition, which totals 80.

```
70 divided by 80 = 0.875
```

A proportion of 0.875. This, and the P(test positive given condition present) from the previous problem, are the two proportions a table like this is usually built to report.

4. Sensitivity is the share of people with the condition that the test catches. It is the test-positive cell in the column of people who have the condition, divided by that column's own total.

```
15 divided by 20 = 0.75
```

Specificity is the share of people without the condition that the test correctly clears. It is the test-negative cell in the column of people who do not have it, divided by that column's total.

```
70 divided by 80 = 0.875
```

The positive predictive value is the share of positive results that are right. It is the same condition-present cell as before, but now divided by the row total for everyone who tested positive.

```
15 divided by 25 = 0.6
```

Three questions. Three different totals. The same handful of cells does all the work each time.

5.

```
52 divided by 131 = 0.3969…
```

About 39.7 per cent have an index of 25 or more, among the 131 men.

```
131 minus 52 = 79
```

```
79 divided by 131 = 0.6031…
```

About 60.3 per cent are under 25. The two add to 100 per cent, as a group and its complement must.

6.

```
151 minus 46 = 105
```

Try the nearest whole numbers of 46 against 88 per cent.

```
40 divided by 46 = 0.8695…
41 divided by 46 = 0.8913…
```

Forty gives 87 per cent, and 41 gives 89. Neither is 88. No whole number of 46 reproduces the sensitivity exactly.

Now try 105 against 83 per cent.

```
87 divided by 105 = 0.8285…
```

Eighty-seven of 105 rounds to exactly 83 per cent. This side does reproduce the label's own total. That shows a check can pass by coincidence on one side and still fail on the other. One passing side is not enough to trust the label.

7.

```
151 minus 41 = 110
```

```
0.88 times 41 = 36.08
36 divided by 41 = 0.8780…
```

Thirty-six of 41 rounds to exactly 88 per cent, the sensitivity the paper reports.

```
0.83 times 110 = 91.3
91 divided by 110 = 0.8272…
```

Ninety-one of 110 rounds to exactly 83 per cent, the specificity. Both round back to the printed figure, the same way the men's 42-against-89 split did.

```
41 minus 36 = 5
110 minus 91 = 19
36 plus 19 = 55
```

```
36 divided by 55 = 0.6545…
```

About 65 per cent of the women the arm measurement flags are actually in the 25-to-29.9 band. The women's row appears to carry the same mislabelling problem the men's row did, and the same fix.

8. The row of cleared men totals 72.

```
6 plus 66 = 72
```

```
66 divided by 72 = 0.9166…
```

About 92 per cent. Being cleared by the tape is a stronger signal, in this group, than being flagged by it.

9. The step that broke is calling 40.1 per cent "close enough" to 38.7. A gap of about 1.4 percentage points is not rounding noise. It is a real mismatch, on a figure printed to one decimal place.

Try the paper's other count for the same measurement, 109, from its own results text and Table 2.

```
109 divided by 282 = 0.3865…
```

That rounds to 38.7 per cent exactly, matching the printed figure. The results text and Table 2's own 109 is the count the paper's arithmetic actually supports. The abstract's 113 is the slip.

Why it looks reasonable: 40.1 and 38.7 both round to about 40. A reader skimming past two similar percentages assumes they are the same claim written two ways. Dividing back and comparing the decimal is the only way to catch that they are not.

10. The step that broke is the conclusion. Both counts do round to 66 per cent, but the paper's own addition settles the question without needing to round anything.

```
71 plus 114 = 185
```

Table 2 gives 71 men and 114 women with high PBF, and they add to 185, not 186.

```
97 plus 185 = 282
```

The normal-range row, 97, plus the high row, 185, gives exactly the study's total of 282.

```
97 plus 186 = 283
```

Adding 186 instead gives 283, one more person than the study actually has. That is impossible, so 185 is the paper's own figure and 186 in the abstract is the slip.

Why it looks reasonable: both percentages round to the same whole number, so a check that stops at rounding finds nothing wrong. The paper's own row-plus-row arithmetic is a second, independent check, and it is the one that actually decides it.

11. The step that broke is rounding 34.85 down to 34. The nearest whole number is 35. Divide both back to see which one fits.

```
34 divided by 41 = 0.8292…
35 divided by 41 = 0.8536…
```

Thirty-four gives 83 per cent, not 85. Thirty-five gives 85, the printed figure. The last line claimed a match it never computed.

Why it looks reasonable: 34.85 sits between 34 and 35, and cutting it down feels cautious. The last line then states the result it expects instead of dividing. Checking the answer means dividing back and comparing the result with the number you started from.

12. What to compute: the positive predictive value at that one-in-three share. Use the same sensitivity and specificity as before, in a group of 900 men, to keep the arithmetic in whole people.

```
900 divided by 3 = 300
900 minus 300 = 600
```

```
0.86 times 300 = 258
0.74 times 600 = 444
```

```
600 minus 444 = 156
258 plus 156 = 414
```

```
258 divided by 414 = 0.6231…
```

About 62 per cent, close to the 61 per cent found among the medical students.

What this does not show. The two numbers are close because one in three is close to the students' 32 per cent. At a different share they would not be. The one-in-three figure is supposed, not measured, and nothing here checks it. The sensitivity and specificity are still the students' own figures, used here as if they hold true in a new district too.

13. What to compute: which split of the 131 men reproduces 81 and 71 as whole counts. Then the positive predictive value each split gives.

```
0.81 times 52 = 42.12
42 divided by 52 = 0.8076…
```

Forty-two of 52 rounds to exactly 81 per cent.

```
0.71 times 79 = 56.09
56 divided by 79 = 0.7088…
```

Fifty-six of 79 rounds to exactly 71 per cent. Now try the band's split, 42 against 89.

```
0.81 times 42 = 34.02
34 divided by 42 = 0.8095…
0.71 times 89 = 63.19
63 divided by 89 = 0.7078…
```

Both 52 against 79 and 42 against 89 round back to 81 and 71, so this row alone cannot say which group the paper used.

```
52 minus 42 = 10
79 minus 56 = 23
42 plus 23 = 65
```

```
42 divided by 65 = 0.6461…
```

On the 52-against-79 split, about 65 per cent of the men the neck measurement flags are at 25 or more, among these medical students.

```
89 minus 63 = 26
34 plus 26 = 60
34 divided by 60 = 0.5666…
```

On the band's split, 34 of the 60 men flagged are in the band, about 57 per cent. The figure moves with the group.

```
0.20 times 1,000 = 200
1,000 minus 200 = 800
```

```
0.81 times 200 = 162
0.71 times 800 = 568
```

```
800 minus 568 = 232
162 plus 232 = 394
```

```
162 divided by 394 = 0.4111…
```

About 41 per cent in a stated group where one in five is at 25 or more.

What the claim does not establish. None of 65, 57 or 41 per cent is "most of the time" in a way that supports "everywhere". Each belongs to one group, the students or a stated one, not to the measurement on its own.

The paper's own running text gives a different specificity for this same row, 82 rather than 71. This section has not settled which of the two the paper actually used. Replacing one measurement with another is also not a decision that turns on sensitivity and specificity alone. Cost, ease of use and what the measurement is needed for all matter too. And nothing here establishes that either measurement is a good screening tool in the first place. This section has deliberately not tried to answer that.

D4 · Variation: what differs and by how much

EXERCISE 1. The range only tells you the two most extreme waiting times out of the 500. It says nothing about where the other 498 patients fall between them. A single patient waiting 340 minutes sets the top of the range alone; everybody else could be clustered anywhere below it.

"Most patients wait somewhere between 2 and 340" is true only in a trivial sense. Every value lies between the smallest and the largest. That is not the same claim as most patients waiting somewhere in the middle of that stretch. Nothing given so far tells you which is true.

What to ask for instead: a frequency table of waiting times in useful bands. Or ask how many of the 500 patients waited more than an hour. Either one describes where patients actually sit, rather than where the two extremes happen to be.

EXERCISE 2. Ten minutes, for somebody who expects a mechanism.

Draw three labelled boxes on the board: between people, within a person, between measurements. Ask the resident for a patient's two weight readings that disagreed, taken at the same visit.

Ask the first question: were the two readings taken on two different patients by mistake? If so, the difference is between people, and it says nothing about either patient's own weight.

Ask the second: were the two readings taken on the same patient, but hours or days apart? If so, part of the gap can be a real change in that one patient. Food, fluid and the time of day can each move it, and that is a difference within a person.

Ask the third: were the two readings taken minutes apart, on the same scale, with nothing about the patient changing? If so, any gap left over is a difference between measurements. It comes from the scale or from how the reading was taken, not from the patient.

Close by having them re-sort their own mismatched readings into the three boxes, out loud. The box a disagreement lands in decides what they do next: trust the first reading, the second, their average, or neither. That decision is the whole reason the three boxes exist.

1. Divide each count by the total, 50.

```
10 divided by 50 = 0.2
25 divided by 50 = 0.5
10 divided by 50 = 0.2
5 divided by 50 = 0.1
```

Add the four relative frequencies to check them.

```
0.2 plus 0.5 = 0.7
0.7 plus 0.2 = 0.9
0.9 plus 0.1 = 1.0
```

They add to 1, as every set of relative frequencies covering a whole count must.

2. Find the largest and the smallest, then subtract.

```
largest = 42
smallest = 3
42 minus 3 = 39
```

The range is 39.

3.

```
set A: 50 minus 10 = 40
set B: 50 minus 10 = 40
```

Both sets have a range of 40.

What it does not tell you: how the six values inside each set are actually spread out. Set A sits bunched between 10 and 14, with one value, 50, sitting a long way out on its own. Set B is spread almost evenly across the whole distance from 10 to 50. Two sets with the same range can have completely different shapes, because the range only ever looks at the two most extreme values.

4. (a) Between people. Two different patients were weighed, so the 13-kilogram gap is a real difference between them. Nothing here says either reading is wrong.

1. Mostly within a person. The same patient gave two different numbers days apart. What they ate, what they drank and the time of day can each move a weight reading. Part of the 1.2 kilograms could still come from the scale, and nothing here separates the two.
2. Between measurements. The same patient, the same scale, one minute apart, with nothing about the patient changing. The 0.2-kilogram gap came from the act of measuring, not from the patient.

5. Divide the count in that band by the total counted.

```
15 divided by 282 = 0.053191489…
0.053191489 times 100 = 5.3191489
```

That rounds to about 5.3 per cent, matching the percentage the paper itself prints beside the count.

6. Add the four counts first.

```
5 plus 74 = 79
79 plus 42 = 121
121 plus 10 = 131
```

They add to 131, which matches the stated number of men, so this row of the table passes the check.

Now the relative frequency of the band at 30 or more, among the men.

```
10 divided by 131 = 0.076335878…
0.076335878 times 100 = 7.6335878
```

About 7.6 per cent of the 131 men are in that band.

7. One observer removes differences between observers. Two different people measuring the same student can place the tape or read the scale a little differently. That gap is one kind of difference between measurements. One observer, measuring everyone the same way, removes it.

It does not touch a real difference between the people being measured. Two students still really differ in weight and height, whoever measures them.

It does not touch a real difference within one person from one occasion to the next. If a student weighs more in the evening than in the morning, one observer does not change that.

And it does not remove every difference between measurements, either. The same observer, with the same tool on the same person, can still get two slightly different readings a few minutes apart. One observer only removes the part of the gap that comes from who holds the tool.

8. Add the two bands first, since both are at 25 or more.

```
41 plus 5 = 46
```

Now divide by the total number of women.

```
46 divided by 151 = 0.304635762…
0.304635762 times 100 = 30.4635762
```

About 30.5 per cent of the 151 women have a body-mass index of 25 or more.

9. The division itself is correct arithmetic. The step that broke is the choice of denominator: 282 is the total number of students, not the total number of men.

There are 131 men, not 282, and the 42 is a count of men. Divide by the count it actually came out of.

```
42 divided by 131 = 0.320610687…
0.320610687 times 100 = 32.0610687
```

About 32.1 per cent of the men are in that band, not 14.9 per cent.

Why it looks reasonable: 282 is a number from the very same table. It is also the biggest number on the page, which makes it look like the obvious total to divide by. The table has more than one total in it: one for everybody, one for each sex. Only one of them matches the count in the numerator.

10. Two bands being close in size is not the same claim as the whole distribution being symmetric. Symmetry is about how far every value sits from the middle, on both sides. It is not about how many people fall into two chosen bands at the edges.

The flaw is that band counts depend on where the band edges fall. The band "under 18.5" and the band "30 or more" are both open-ended, and the table does not say how far either one actually reaches. Comparing their sizes tests nothing about shape. A wide-reaching open band and a narrow one can hold similar counts for reasons that have nothing to do with symmetry.

The comparison also rests on very little of the sample.

```
24 plus 15 = 39
282 minus 39 = 243
```

The two bands being compared hold 39 students between them. The other 243, in the two middle bands, never enter the comparison at all.

Why it looks reasonable: 8.5 and 5.3 really are close numbers, and "the two ends look similar" sounds like exactly what symmetry means. It checks only the two ends, and it never looks at where the other 243 students, out of 282, actually are.

11. What to compute: where the 282 students actually sit across the four bands, rather than just which bands are occupied.

```
24 plus 160 = 184
184 plus 83 = 267
267 plus 15 = 282
```

Well over half of the group, 160 of 282, sits in one single band, from 18.5 to 24.9. Only 24 students, 8.5 per cent, are below it. Add the two bands above it.

```
83 plus 15 = 98
```

So 98 students, about a third of the group, are above it.

What this does not establish. Naming the two extreme bands says nothing about how many people are actually in them. Here it is a small minority on the low side and a larger minority on the high side. Of the 282, 160 sit in the one band from 18.5 to 24.9.

It also does not establish the true range of body-mass index in this sample. Once every value has been sorted into one of four bands, the single largest and single smallest values are gone. No band boundary can give them back. "Varies enormously" needs the individual numbers this table no longer holds.

And a spread of body-mass index bands is not a spread of weight. The table sorts body-mass index, not weight. Two students with the same index can differ in weight by many kilograms if their heights differ. The note's own last sentence swaps one quantity for another without saying so.

D5 · Average and spread, and why the average is not the person

EXERCISE 1. The mean, 6,200, sits well above the median, 4,100. The patients above the median reach far enough up to pull the mean past it. At least half of all patients walk at or below 4,100 steps, which is already under the report's own target of 5,000.

"Comfortably above the target" describes the mean, not the typical patient. The typical patient, by the median, sits below it.

What to ask for instead: the proportion of patients whose own step count sits below 5,000. Work that out directly, rather than inferring it from the average. That number, not the mean, answers the question the report is actually trying to answer.

EXERCISE 2. Ten minutes, for somebody who expects a mechanism.

Write two numbers on the board. One is a mean of 24.2, from a real study of 131 men. The other is 25, the WHO cut-off that paper used. Both are body-mass index, in kilograms per square metre. Ask the resident what the mean means for the group.

They will likely say the group is under the cut-off. Write a second real fact from the same study underneath: 52 of the 131 men individually have a body-mass index of 25 or more.

```
52 divided by 131 = 0.396946565…
0.396946565 times 100 = 39.6946565
```

About 39.7 per cent, close to two in five. Let the resident sit with that fact. The mean is under the line. Nearly two in five of the actual men are at it or over.

Ask what would have to be true for the mean to sit below 25 while some men sit above it. The answer is that other men sit far enough below 25 to balance them out. Nothing about the mean tells you how many are on which side, or by how much.

Close with the rule to carry into the next chart they read. A mean below a line is a fact about the mean. Whether most of the group is below that same line is a different question. Whether the one patient in front of them is below it is a third. Both need the group's spread, not just its average.

1. The mean first.

```
4 plus 9 plus 2 plus 15 plus 7 plus 9 plus 3 = 49
49 divided by 7 = 7
```

For the median, order the numbers first.

```
2    3    4    7    9    9    15
```

Seven numbers, so the middle one is the fourth. That is 7.

The mean and the median both come out at 7 here. They do not have to agree, and the next problem shows a case where they do not.

2. The mean.

```
6 plus 9 plus 11 plus 14 plus 18 plus 50 = 108
108 divided by 6 = 18
```

1.

The median. The numbers are already ordered. With six of them, average the two in the middle.

```
11 plus 14 = 25
25 divided by 2 = 12.5
```

12.5.

The mean, 18, sits well above the median, 12.5. The single value 50 is pulling the mean upward while the median stays close to where five of the six values actually are.

3. The second quartile is the median of the whole set. With eight numbers, average the two in the middle.

```
9 plus 11 = 20
20 divided by 2 = 10
```

The second quartile is 10.

The lower half is 3, 5, 5, 9. Its middle two are the second and third values.

```
5 plus 5 = 10
10 divided by 2 = 5
```

The first quartile is 5.

The upper half is 11, 11, 15, 19.

```
11 plus 15 = 26
26 divided by 2 = 13
```

The third quartile is 13. The interquartile range is the third quartile minus the first.

```
13 minus 5 = 8
```

The interquartile range is 8.

4. Seven is odd, so the middle value, the fourth one, is the second quartile on its own. Leave it out of both halves before you take their medians.

```
second quartile = 9
```

The lower half, with the middle value left out, is 2, 5, 7. Its own middle value is the first quartile.

```
first quartile = 5
```

The upper half, with the middle value left out, is 12, 15, 20. Its own middle value is the third quartile.

```
third quartile = 15
```

The interquartile range is the third quartile minus the first.

```
15 minus 5 = 10
```

The interquartile range is 10. Software sometimes computes quartiles by a different rule and can print a slightly different number from this one. That is not a sign either calculation is wrong.

5. The mean first.

```
8 plus 6 plus 10 plus 4 plus 12 plus 8 = 48
48 divided by 6 = 8
```

The deviation of every value from the mean.

```
8 minus 8 = 0
6 minus 8 = -2
10 minus 8 = 2
4 minus 8 = -4
12 minus 8 = 4
8 minus 8 = 0
```

Square every deviation.

```
0 times 0 = 0
-2 times -2 = 4
2 times 2 = 4
-4 times -4 = 16
4 times 4 = 16
0 times 0 = 0
```

Add the squares.

```
0 plus 4 = 4
4 plus 4 = 8
8 plus 16 = 24
24 plus 16 = 40
40 plus 0 = 40
```

Divide by one less than the count, which is five.

```
40 divided by 5 = 8
```

The variance is 8. Take its square root for the standard deviation.

```
square root of 8 = 2.828427125
```

The sample standard deviation is about 2.83.

6. Square each deviation.

```
-2 times -2 = 4
4 times 4 = 16
-1 times -1 = 1
3 times 3 = 9
-4 times -4 = 16
```

Add them.

```
4 plus 16 = 20
20 plus 1 = 21
21 plus 9 = 30
30 plus 16 = 46
```

Divide by one less than the count, which is four.

```
46 divided by 4 = 11.5
```

The variance is 11.5. The standard deviation is its square root.

```
square root of 11.5 = 3.391164992
```

The standard deviation is about 3.39.

7.

```
24.2 minus 23.8 = 0.4
```

The gap is 0.4 kg/m^2.

The mean sits a little above the median. That pattern happens when values above the median pull the mean upward more than values below it pull it down. The gap here is small next to the spread of the whole group. So this is only a slight sign of a tail on the high side, not a firm one.

8.

```
26.8 minus 21.2 = 5.6
```

The interquartile range is 5.6 kg/m^2.

Check the median against the two quartiles. 23.8 sits between 21.2 and 26.8. So this row of the table is internally consistent, unlike the weight row worked through in the illustration above.

9.

```
23.8 minus 21.2 = 2.6
26.8 minus 23.8 = 3.0
```

The first quartile sits 2.6 kg/m^2 below the median, and the third sits 3.0 above it. So the quarter of men just above the median is spread a little wider than the quarter just below it. That points the same way as the mean sitting above the median in the level 4 problem: a tail on the high side. The two distances differ by only 0.4, so this is a slight sign, not a firm one.

10. Find the deviation first.

```
30 minus 24.2 = 5.8
```

Divide by the standard deviation to express it in standard deviations.

```
5.8 divided by 3.9 = 1.487179487
```

That reading would sit about 1.49 standard deviations above the men's mean.

11.

```
39.2 minus 28.1 = 11.1
```

The interquartile range is 11.1 percentage points.

Check the median against the two quartiles. 34.0 sits between 28.1 and 39.2, so this row is internally consistent.

Here the mean, 33.8, sits below the median, 34.0, the other way round from the men's rows worked through above. A mean below the median happens when the values below the median reach further down than those above it reach up. If this pattern holds, the tail here points toward the low side rather than the high side.

12. The mean, the deviations and the squares are all correct, and so is their sum, 90. The step that broke is the division for the variance.

These five patients are a sample from the clinic's larger list, and the question asked for the sample standard deviation. Its variance divides the sum of squared deviations by one less than the count, which is four here, not five.

```
90 divided by 4 = 22.5
```

The variance is 22.5, not 18. Take its square root.

```
square root of 22.5 = 4.743416490
```

The correct standard deviation is about 4.74, not 4.24.

Why it looks reasonable: dividing by the count of values, five, is exactly what the mean itself does two lines earlier. Re-using the same 5 feels consistent. The sample rule uses one less than the count only at this one step. It appears nowhere else in the working, which is exactly what makes it easy to miss.

13. The median has to sit between the two quartiles printed beside it. Here the median, 57.4, sits below both 155.0 and 163.0, not between them, so the three numbers cannot all describe the same row.

The height row printed just below it gives a median of 159, with 51.3 and 63.9 printed beside it. Swap the two rows' quartiles and check both medians again. The weight row's median, 57.4, sits between the height row's printed numbers, 51.3 and 63.9. The height row's median, 159, sits between the weight row's printed numbers, 155.0 and 163.0. Both checks pass with the quartiles swapped. The numbers fit if the two rows' quartiles were printed in each other's places, the same pattern as in the men's rows above.

What to check before quoting any pair of quartiles: does the paired median actually sit between the two quartiles printed with it. That check takes a moment, and it is the one this line fails.

14. The conclusion has the direction backwards. By the median's own definition, at least half of the men are at or below 20.3. And 20.3 sits below the mean of 22.1. So at least half of the men are below the mean, not above it.

The men above the median reach far enough up to pull the mean past it. At least half sit at or below the median.

Why it looks reasonable: "the mean is higher, so more men must be higher" sounds right. It is the wrong direction. A higher mean is the sign of a tail on the high side. Some readings go far up. It is not a sign that most readings sit above the mean.

15. What to compute: where the women's own quartiles sit against 25, rather than resting on the group's average. Table 1 prints the women's body-mass index as 22.7 (20.5, 25.6). That is a median of 22.7, with quartiles of 20.5 and 25.6.

```
25.6 minus 25 = 0.6
```

The third quartile, 25.6, sits 0.6 above the cut-off. About a quarter of the women sit at or above 25.6. Since 25.6 is above 25, at least about a quarter of the 151 women are at 25 or above, even though the mean is 23.0.

What this does not establish. Body-mass index does not measure body fat directly. It cannot tell fat from lean mass. So this figure does not establish anything about any one woman's health.

Nor does it show that 25 is where risk starts for this group. The figure 25 is the WHO cut-off the paper used. The same paper notes that some Asian groups show metabolic risk at a lower body-mass index than that. And it says nothing about students elsewhere. This figure is for one college in one year.

16. What to compute: what the quartiles in the same row of Table 1 say about where the men sit. That is better than guessing a narrow band around the mean.

Men's body fat is printed as a median of 20.3 per cent, with quartiles of 16.7 and 28.8.

```
28.8 minus 16.7 = 12.1
```

By definition, about a quarter of the men sit at or below 16.7 and about a quarter sit at or above 28.8. So about half the men lie between 16.7 and 28.8. The colleague's 20 to 24 sits inside that stretch, so at most about half can lie in it. The mean, 22.1, also sits above the median, 20.3, by 1.8 points.

```
22.1 minus 20.3 = 1.8
```

The median says at least half of the men sit at or below 20.3. The men above it reach far enough up to pull the mean past it.

What this does not establish. Neither the mean nor the quartiles tell you what fraction of men actually fall inside 20 to 24 specifically. That needs a frequency table of the individual readings, the way Table 2 gave one for body-mass index. No such table is given here for body fat. And the quartiles already show that "most men" cannot be packed into a band as narrow as 20 to 24.

D6 · Sampling: how a part can tell you about a whole

EXERCISE 1. It is right that 4,000 is a large number. A genuinely random sample of 4,000 would have a small spread around whatever population it was drawn from. Size does buy precision against sampling variation, and the claim is not wrong about that part.

What it leaves out is how the 4,000 were chosen. A hospital's social media followers are a particular kind of person. They may be more engaged, more online, younger or more urban than the hospital's whole patient population. That is a convenience sample, whatever its size.

So "extremely reliable" is doing two different jobs in that sentence, and only one of them is supported. The 60 per cent is a precise estimate of what these 4,000 followers think. It is not shown to be a precise estimate of what the hospital's patients, or the public it serves, think. Enlarging the survey to 40,000 followers would sharpen the first claim and do nothing for the second.

EXERCISE 2. Ten minutes, for someone who expects a mechanism rather than a slogan.

Draw two circles on the board. Label the first "population" and put a hundred dots in it. Circle a random handful of dots inside it and label that circle "sample". Ask what decided which dots got circled. The honest answer is: nothing about the dots themselves, only chance.

Now draw a second population circle. This time circle a cluster of dots sitting together in one corner, leaving the rest of the circle untouched. Ask the same question. This time the answer is: something about those dots, namely being easy to reach, decided the circling.

Write the square-root law on the board.

```
spread of a sample average = spread of the population, divided by square root of
sample size
```

Point at the first picture. Say that this formula describes exactly how far the circled dots' average will typically sit from the true average of the hundred dots. Say too that a bigger circle makes that distance smaller, in a way you can calculate.

Then point at the second picture. Ask what the formula has to say about the gap between the corner cluster's average and the true average of all hundred dots. The honest answer is nothing, because the formula was never about that gap. Let the resident sit with that before moving on. The move you want them to keep is: ask how a sample was drawn before asking how big it was.

1. The square-root law: divide the population's spread by the square root of the sample size.

```
square root of 4 = 2
10 divided by 2 = 5
```

The spread of the sample average is 5.

2. Size 16 first.

```
square root of 16 = 4
8 divided by 4 = 2
```

A spread of 2. Now size 64.

```
square root of 64 = 8
8 divided by 8 = 1
```

A spread of 1. Sixty-four is four times sixteen, and the spread has gone from 2 to 1: exactly halved. That is the square-root law again — four times the sample, half the spread — on a different pair of numbers.

3. Run the law backwards. The spread of the sample average is the population's spread divided by the square root of the sample size. So the square root of the sample size is the population's spread divided by the target spread.

```
15 divided by 3 = 5
```

The square root of the sample size is 5, so the sample size itself is 5 times 5.

```
5 times 5 = 25
```

A sample of 25 gives a spread of 3. Check it forwards: square root of 25 is 5, and 15 divided by 5 is 3, which matches.

4. Divide the spread by the square root of the sample size.

```
square root of 25 = 5
3.9 divided by 5 = 0.78
```

The spread of the sample average is 0.78 kg/m². Sampled at 25 people, a population this variable would typically give an average within a kilogram per square metre of its own true average.

5. First the 131 men.

```
square root of 131 = 11.4455
3.9 divided by 11.4455 = 0.3407
```

A spread of about 0.34 kg/m² around the population's own average.

Now 524 men. 524 is four times 131. The square-root law says four times the sample halves the spread, so there is no need to work out a second square root.

```
0.3407 divided by 2 = 0.17035
```

About 0.17 kg/m². Four times the sample bought exactly half the spread, as the law predicts.

6. Run the law backwards, as before.

```
3.9 divided by 0.1 = 39
39 times 39 = 1521
```

A sample of 1,521 people would be needed. Notice how much bigger this jump is than the jump from a spread of 0.34 to 0.17 in the previous problem. The sample size itself is never squared here. What gets squared is the factor by which the spread has to shrink.

```
0.3407 divided by 0.1 = 3.407
3.407 times 3.407 = 11.607649
131 times 11.607649 = 1520.602
```

Shrinking the previous problem's spread of about 0.34 down to 0.1 is a shrink of about 3.4 times. That shrink has to be squared to get the growth in sample size: about 3.4 times 3.4, which is roughly 11.6. Multiplying 131 by that comes out close to 1,521, matching the direct working above.

7. Check the arithmetic first.

```
24.2 minus 3.9 = 20.3
24.2 plus 3.9 = 28.1
```

Both sums are right. The step that broke is calling that interval a range for "the true mean".

The standard deviation, 3.9, describes how the individual men's own body-mass index values are spread out around 24.2. It answers how far from the mean one man typically sits. A different question is how far the sample's own mean, 24.2, might sit from the true mean of the wider population. Answering that needs the spread of the sample average. Suppose, for the moment, that these 131 men were a random sample from a population whose spread is 3.9. The square-root law then gives that spread from the same two numbers.

```
square root of 131 = 11.4455
3.9 divided by 11.4455 = 0.3407
```

A spread of about 0.34 kg/m² for the sample's own mean, not 3.9. That 0.34 describes random samples. These men volunteered, so it says nothing about how far 24.2 sits from any wider population's mean.

Why it looks reasonable: both numbers, 24.2 and 3.9, are real figures straight from the same table. Adding and subtracting a spread from a mean looks like a natural move. The flaw is using the spread of the people in place of the spread of the average. Those are two different numbers describing two different questions.

8. The first two lines are right: the square root of 100 is 10, and 50 divided by 10 is 5.

The step that broke is the third line. The square-root law does not halve when the sample size doubles. Halving the spread needs the square root of the sample-size ratio to equal 2. That needs the sample size itself to be multiplied by 4, not by 2.

Work it out properly for a sample of 200.

```
square root of 200 = 14.142
50 divided by 14.142 = 3.5355
```

The correct spread at 200 is about 3.54, not 2.5. Doubling the sample only takes the spread down to about 71 per cent of what it was, not to half.

Why it looks reasonable: "double the sample, halve the spread" sounds like the same idea as "four times the sample, halve the spread". The two are easy to blur together. The law scales with the square root of the sample size, not with the sample size itself. That is exactly the distinction this section exists to fix.

9. The first line is worked out from Table 2 and is right: 83 plus 15 is 98, and 98 of 282 is 34.8 per cent. The second line claims more than the paper does. The paper gives 19 and 21 per cent for men and women. It reports these as NFHS-4's figures for India, without saying which ages or which people they cover. The authors rounded them to 20 for planning. The third line is correct arithmetic. The square-root law really does say that quadrupling a random sample's size halves the spread of its average around the population it was drawn from.

The step that broke is the fourth line, and it is not an arithmetic error. It applies a fact about random-sample spread to a gap that random-sample spread never produced. Line 4 also calls 20 per cent the true national figure. It is the authors' rounded planning number, not a true value.

The 282 medical students were not drawn at random. They were volunteers from one college who came forward after an announcement. Nothing in this arithmetic can say why the gap between 34.8 and 20 per cent is there. It could be that. It could be the years between NFHS-4 and this study. It could be how each drew its own line for "overweight". The rounding in the second line moves it by at most a point either way.

Quadrupling the invitation would shrink the spread around whatever these particular medical student volunteers answer. It does nothing to settle which of those explanations is right.

Why it looks reasonable: the square-root law is applied correctly right up to the conclusion. A reader who checks only the arithmetic finds nothing wrong. The mistake is in which quantity the arithmetic was ever entitled to say something about.

10. What to compute: the study's own numbers, and how far they sit from the comparison figure the claim leans on. Table 2 puts 83 students at 25 to 29.9 and 15 at 30 or more.

```
83 plus 15 = 98
98 divided by 282 = 0.3475177305
```

Ninety-eight of 282 students, about 34.8 per cent, had a body-mass index of 25 or more. The researchers themselves used 20 per cent to plan the study. They rounded that from 19 and 21 per cent for men and women. The paper reports these as NFHS-4's figures for India, without saying which ages or which people they cover. The sample's 34.8 and the planning figure's 20 differ by about fifteen percentage points.

What it is not is a gap the square-root law can explain or explain away. The law describes how a random sample's own answer scatters. These 282 were not a random sample of anything. So the law has nothing to say about why their figure differs from the authors' planning figure.

So the arithmetic shows a difference between this sample's figure and the planning figure. What it does not establish is the sentence as written.

"Medical students are far more overweight than India's young adults generally" makes a claim about two groups: medical students, and young Indian adults. Said carefully, about 34.8 per cent of these 282 students had a body-mass index of 25 or more. That is more than the 20 per cent the authors rounded from the paper's NFHS-4 figures.

The 282 students were volunteers from one college. They came forward after an announcement. That is not a random sample of medical students anywhere, still less of young adults generally. People who choose to come forward for a study may differ from those who do not. That difference need not have anything to do with being a medical student. They may be more health-conscious, more curious, or more worried about their weight — or the opposite.

Nothing in this arithmetic can tell "medical students, as a group, have a higher body-mass index" apart from a narrower claim. That narrower claim is "the kind of medical student who volunteers for a study like this has a higher body-mass index". A bigger sample of volunteers from more colleges would narrow the estimate of what such volunteers look like. It would still not answer the question the newsletter line is actually asking.

11. What to compute: how the spread of a sample average changes between a sample of 2,000 and a sample of several million. Use the square-root law on a stated spread, not a real one, since no real figure for this quantity is given.

Say the quantity being measured has a spread of 40, in whatever units it is recorded in.

```
square root of 2000 = 44.7214
40 divided by 44.7214 = 0.8944
```

A sample of 2,000 gives a spread of about 0.89 around the true population average. Now a sample of 2 million, a thousand times as large.

```
square root of 2000000 = 1414.2136
40 divided by 1414.2136 = 0.0283
```

A spread of about 0.028. Going from 2,000 to 2 million is a thousandfold increase. It only shrinks the spread by about thirty-two times. That is because the square-root law shrinks spread with the square root of the sample size, not with the sample size itself. The claim that "several million" is needed does not follow from how sampling spread behaves. A sample of 2,000 already has a fairly tight spread relative to the population's own variation. The colleague's number is not doing the arithmetic work they think it is.

Notice that the country's size appears nowhere in this working. The spread of a random sample's average depends on the sample's own size, not on the size of the population it was drawn from. "A whole country" sounds like it should demand a bigger sample. The square-root law has no term for it at all.

What this does not establish. It does not establish that 2,000 is always enough. The right sample size depends on how variable the quantity is, and how tight an answer needs to be. Neither of those was stated here.

It also does not establish that any sample of 2,000 is trustworthy, regardless of how it was chosen. Say those 2,000 people were a convenience sample rather than a random one. Then no sample size, however large, closes the gap between who was asked and who the answer is supposed to be about. This arithmetic only answers "how big a random sample needs to be". It never answers "was this particular sample drawn that way".

D7 · Random error and systematic error as different things

EXERCISE 1. The figure describes precision, not accuracy. It is about how closely repeated readings agree with each other, here on the same person, taken close together in time. That is a real property worth stating. A tight one is a genuine strength of the device.

It leaves unanswered whether those readings sit close to the person's true blood pressure. A monitor could read within 1 mmHg of itself on every repeat. It could still read 15 mmHg high every single time, if it carries a systematic error. Repeating the same check on the same faulty device will not catch that, no matter how many times it is repeated.

What would answer it: compare the monitor's readings against a reference device already shown to be accurate. Use it on a set of people whose true blood pressure by that method is known. Only that comparison speaks to accuracy. The manufacturer's own figure, however it is worded, speaks only to precision.

EXERCISE 2. Ten minutes, for someone who expects a mechanism rather than reassurance.

Draw a small target on the board. Mark a cluster of shots tightly grouped in the top-right corner, well away from the bullseye. Say that this is what "the same number every time" looks like: tight, repeatable, and a whole target's-width away from the truth.

Ask the trainee what would fix this pattern. Let them try "shoot more times and average the positions."

Mark the average of the cluster: it lands in almost the same spot, still off the bullseye. Repetition of a systematic error changes nothing about it.

Now draw a second target with shots scattered loosely all around the bullseye itself. Mark that this one is far less repeatable, shot to shot, and average its positions. The average lands close to the bullseye, because the scatter is random and partly cancels.

Write the two words on the board next to their targets. The tight, off-target cluster is precise, not true. The loose, centred scatter is true, not precise. Close with the question you want the trainee to ask from now on, about every "fine because it's consistent" instrument. Is it consistent with itself, or consistent with the truth?

1. Error is the measured value minus the true value.

```
194 minus 200 = -6
```

The error is -6. The negative sign says the reading came out below the true value, by 6.

2. Subtract 75 from each reading.

```
78 minus 75 = 3
77 minus 75 = 2
79 minus 75 = 4
76 minus 75 = 1
80 minus 75 = 5
```

The five errors are 3, 2, 4, 1 and 5.

3. Set P first.

```
3 plus 2 plus 4 plus 1 plus 5 = 15
15 divided by 5 = 3
5 minus 1 = 4
```

Average error 3, range 4. Every error is positive and none strays far from the average: this looks systematic, pushed one way by about the same amount each time.

Now set Q.

```
-3 plus 4 plus -5 plus 2 plus -1 = -3
-3 divided by 5 = -0.6
4 minus -5 = 9
```

Subtracting a negative number adds its size. On a number line, the distance from -5 up to 4 is 5 plus 4, which is 9.

Average error -0.6, close to zero. The range is 9, far wider than set P's own range. The errors cross zero and spread out a long way either side. This looks random, with little or no consistent push in one direction.

4. Subtract 20.0 kg from each reading.

```
20.3 minus 20.0 = 0.3
20.2 minus 20.0 = 0.2
20.4 minus 20.0 = 0.4
20.3 minus 20.0 = 0.3
20.3 minus 20.0 = 0.3
```

Errors of 0.3, 0.2, 0.4, 0.3 and 0.3 kg. Average them.

```
0.3 plus 0.2 plus 0.4 plus 0.3 plus 0.3 = 1.5
1.5 divided by 5 = 0.3
```

An average error of 0.3 kg. Every reading is high by close to the same amount, which is the pattern of a systematic error.

5. Subtract 37.0 °C from each reading.

```
36.4 minus 37.0 = -0.6
37.6 minus 37.0 = 0.6
36.9 minus 37.0 = -0.1
37.3 minus 37.0 = 0.3
37.0 minus 37.0 = 0.0
```

Errors of -0.6, 0.6, -0.1, 0.3 and 0.0 °C.

```
-0.6 plus 0.6 plus -0.1 plus 0.3 plus 0.0 = 0.2
0.2 divided by 5 = 0.04
0.6 minus -0.6 = 1.2
```

Subtracting a negative number adds its size, as the earlier problem on set Q showed: 0.6 minus -0.6 is 0.6 plus 0.6, which is 1.2.

An average error of about 0.04 °C, close to zero, and a range of 1.2 °C, wide compared with the average. This is the opposite pattern from the scale. The errors scatter both above and below zero and mostly cancel out, which is the pattern of random error, not systematic error.

6. Error is the measured value minus the true value.

```
68.3 minus 68.0 = 0.3
```

An error of 0.3 kg, or 300 g.

Knowing the scale's recording step, on its own, cannot answer that question. The step tells you the smallest change the display can show. It says nothing about the pattern of the scale's errors across repeated readings. Answering that needs several readings of the same known weight, not one reading and not the size of the display's steps.

7. The first two lines are fine as stated: a zero error of 0.3 °C, and an average reading of 100.3 °C.

Line 3 states a rule that is true of random error as if it held for all error. That is the break. Averaging reduces random error, because random error varies from reading to reading and partly cancels. A zero error is a systematic error. If it is the same at every temperature, it adds the same 0.3 °C to every reading of the boiling water too. So it is still fully present in the average.

```
100.3 minus 0.3 = 100.0
```

Subtracting the known zero error is a correction (VIM 2.53). Corrected for it, the best estimate is 100.0 °C. Twenty readings, or two thousand, would change nothing about that 0.3 °C error, until it is corrected for.

Why it looks reasonable: "averaging reduces error" is true of random error. The third line states it as if it were true of all error. The flaw is applying that rule to the wrong case. It only works for random error, and this case is systematic. Nothing in the arithmetic itself flags which kind of error is in play.

8. The first two lines are fine: an average 0.05 kg from the truth is good trueness (VIM 2.14).

```
40.05 minus 40.0 = 0.05
```

Line 3 is where it starts to break: a close average shows trueness, not accuracy. Line 4 then draws the conclusion that the mislabel invites. Trueness describes the average of many readings. It says nothing about how far any one reading

might sit from that average. A scale can have this exact same average and still scatter widely, reading by reading. Say five of its readings were 38.0, 42.0, 39.0, 41.0 and 40.25 kg.

```
38.0 plus 42.0 plus 39.0 plus 41.0 plus 40.25 = 200.25
200.25 divided by 5 = 40.05
```

The same average, 40.05 kg, true on average to within 0.05 kg. Yet no single reading sits anywhere near that close: they range from 38.0 to 42.0, four kilograms apart. Good trueness on average and poor precision, reading by reading, sit together without contradiction.

Why it looks reasonable: "accurate" is being used the way people use it in speech, for the whole instrument. The VIM keeps trueness apart from precision, because one can be good while the other is not.

9. The first line quotes the paper correctly. The second line is also right about what precision means in the VIM's sense (entry 2.15): how closely repeated readings agree with each other.

The step that broke is the third line. Showing that readings agree needs several readings of the same person, compared against one another. The paper's methods do not name the instrument used on the neck, and report no repeat measurement. "With 1 mm precision" most likely names the step the readings were recorded in. That is closer to resolution than to precision. Resolution is the "smallest change in a quantity being measured that causes a perceptible change in the corresponding indication" (VIM 4.14).

The fourth line inherits the error, and adds one of its own. Even a genuinely precise instrument is not automatically free of error. Precision says nothing about trueness, and "free of measurement error" would need both.

Why it looks reasonable: the paper's own sentence uses the word "precision". Taking a source at its own word feels like careful reading, not carelessness. A term of art can be used loosely by the very people who would be expected to use it precisely.

10. The first two lines are fine as stated: a zero error of 0.4 kg, and a reading of 62.1 kg.

The step that broke is the third line. The reading of 62.1 kg already carries the +0.4 kg zero error built into it, since the scale adds that much to everything it weighs. A correction (VIM 2.53) removes a known systematic error. It has to work against the direction the error pushed in: subtract it, not add it.

```
62.1 minus 0.4 = 61.7
```

The corrected estimate is 61.7 kg. Adding the zero error instead of subtracting it does not remove the error. It adds a second dose of the same push. The reported 62.5 kg sits 0.8 kg away from the corrected estimate. That is twice the size of the zero error itself, in the same direction the scale was already wrong in.

Why it looks reasonable: "correcting" sounds like it should add something back, the way you might add a missing item to a total. A correction for a known error runs the other way: it takes the error back out.

11. Compute the average reading and the average error for each scale.

```
50.1 plus 49.9 plus 50.2 plus 49.9 plus 50.0 = 250.1
250.1 divided by 5 = 50.02
50.02 minus 50.00 = 0.02
```

The old scale averages 50.02 kg, an error of 0.02 kg.

```
50.63 plus 50.57 plus 50.71 plus 50.55 plus 50.64 = 253.1
253.1 divided by 5 = 50.62
50.62 minus 50.00 = 0.62
```

The new scale averages 50.62 kg, an error of 0.62 kg, thirty-one times the old scale's.

This does not establish that the new scale is more accurate. It points the other way. Every new reading sits at least 0.55 kg from the truth, against at most 0.2 kg for the old scale. Its trueness (VIM 2.14) is far worse too, despite the extra decimal place.

The extra digit is a statement about resolution. That is the "smallest change in a quantity being measured that causes a perceptible change in the corresponding indication" (VIM 4.14). For a digital display, that is in practice the step of its last digit. It says nothing about how close the average sits to the truth, and here it sits further away. A finer display can dress up a worse instrument exactly as confidently as a better one.

Nor does this show the new scale is worse in every way. Its readings agree more closely with each other: a range of 0.16 kg against 0.3 kg. And five readings at 50 kg say nothing about either scale at other weights.

12. Compute each person's difference between the two observers, then the average difference.

```
34.6 minus 34.2 = 0.4
36.0 minus 35.6 = 0.4
34.3 minus 33.8 = 0.5
36.4 minus 36.1 = 0.3
35.2 minus 34.9 = 0.3
```

```
0.4 plus 0.4 plus 0.5 plus 0.3 plus 0.3 = 1.9
1.9 divided by 5 = 0.38
```

Observer 2 reads about 0.38 cm higher than observer 1. This is true on every person, not just on average. All five differences come out positive.

That is the shape of a systematic error: one observer's own habit. It shows up only once a second observer's readings exist to check against.

This does not prove the one-observer study had no measurement error.

It shows that two observers, measuring the same necks, can disagree by an amount that looks systematic.

A study using only one of these two observers would carry that same habit through every reading. Nothing in its own data would reveal it.

Nor does this say which observer is closer to the truth. Comparing two observers to each other is not the same as comparing either one to a known correct value.

E1 · Atoms, molecules and bonds — enough to see where energy is stored

EXERCISE 1. Breaking a bond takes energy in. You have to supply it. Pulling two bonded atoms apart always costs energy, and a stronger bond costs more.

Forming a bond gives energy out. The energy is released as the bond is made.

The rule for the whole reaction. Add up the energy needed to break every bond in the starting materials. Add up the energy released by making every bond in the products. Take the second away from the first.

If more comes out than went in, the reaction releases heat. That is the exothermic case, and it is what burning a food is.

If more went in than came out, the reaction takes heat in. That is the endothermic case.

One thing to check in your own answer. Did you write both halves, or only the breaking half? Half a sum is the commonest way this gets remembered. That half is what makes a food look like a container with energy in it.

EXERCISE 2. Take the two sentences apart, because they fail in different ways.

The first sentence uses strong to mean full. A strong bond is one that takes a lot of energy to break. The word describes what it costs to pull the bond apart, and nothing at all about what the bond will pay out.

The second sentence then runs the sum backwards. Energy comes out of a reaction when the new bonds formed are stronger than the old bonds broken. Starting with unusually strong bonds makes the breaking half of the sum bigger, which makes the release smaller, not larger.

So the brochure has described a drink that would give you less, and sold it as giving you more.

There is a third problem underneath both. The claim talks about energy locked inside a bond, as though a bond were a small container. It is not. There is no energy in a bond to be let out. There is only a difference between two arrangements, and the difference only exists once you say what the second arrangement is.

What to ask for. First, what the molecules turn into, because a release is a difference between a before and an after and they have named only the before. Second, the two totals separately: energy to break, energy released on forming. Third, whether the reaction they describe happens in a person at all. The size of a possible release says nothing about whether anything occurs.

EXERCISE 3. Ten minutes, a whiteboard, and somebody who has been told the answer without being told the reason.

Draw two dots on the board and a line between them. Say the line is a bond, and that a bond is just two atoms attracting each other.

Now ask them what it would take to pull the two dots apart. They will say effort. Write energy in beside it. Then rub the line out, draw it again, and ask what happens on the way back. Write energy out.

Stop there and let that sit, because it is the whole section. Breaking costs. Making pays.

Now draw four dots: two of one kind, two of another, in two joined pairs. Rub out both lines and draw two new lines across the pairs, so each atom is now joined to the other kind. Say out loud what you did. Two bonds broken. Two bonds made.

Write the sum on the board in words. Energy in to break, minus energy out to make. Ask them what has to be true for the answer to be a release. They will get it: the new bonds have to be stronger than the old ones.

That is the moment to say the sentence you want them keeping. The energy is in the arrangement, not in the atoms.

Close with the test they can use on Monday. Any time somebody says a substance releases energy, ask what it turns into. If they cannot say, they have described a before with no after and there is no sum to do.

E2 · Chemical energy, combustion, and the bomb calorimeter

EXERCISE 1. The two steps of the instrument.

First, calibrate. Burn something whose energy is already known and measure the rise it produces. Divide the known energy by that rise. What you get is the calorimeter's energy equivalent, in joules for each degree, and it belongs to that one machine.

Second, measure. Burn the unknown sample, measure its rise, and multiply the rise by that same energy equivalent. That is the energy released.

What each step needs: the first needs a substance of known energy, the second needs nothing except the constant from the first.

The cascade. The food that is eaten holds its gross energy, also called ingested energy, which is what a bomb calorimeter measures. Then come the losses.

Energy is lost in the faeces, from food that is not digested and absorbed. Energy is lost as gas, from unabsorbed carbohydrate fermented in the colon. Energy is lost in the urine, mostly as nitrogenous waste from protein that was not fully broken down. A small amount is lost from the body surface. What remains is metabolizable energy.

Below that the report carries one more step, net metabolizable energy. It takes off the energy the body itself spends on digesting the food and handling it.

One thing to check in your own answer. Did you write a digestible-energy step in the middle? It is not in the report's cascade, and the habit of putting it there is exactly what this section is for.

EXERCISE 2. There are three claims here and they fail in three different ways.

The first is the phrase delivers to the body. A bomb calorimeter burns the sample to nothing in pure oxygen. What it measures is the gross energy, which is the most the food could ever supply. It cannot measure what reaches a person, because the faecal, gaseous and urinary losses happen in a person and not in a steel pot.

The second is the comparison with a label figure. A calculated label figure uses the Atwater factors, and those factors already have average losses taken out of them. So the calculated figure is an estimate of metabolizable energy and the burning figure is gross energy. They are numbers about different things. The burning figure will be the larger of the two, and being larger is not being more accurate.

The third is the word confirms, doing work that one measurement cannot do. A single run in one calorimeter rests entirely on that machine's calibration constant on that day.

What to ask for. The calibration record for the machine, with the date and the substance of known energy that was burnt. The number of runs and the spread across them. Whether the fuse wire was weighed before and after, which is how anyone knows the sample burnt completely. And the composition of the serving in grams, so you can do the Atwater sum yourself and see how far the two figures sit apart.

And say the main point in one line, because it is the one that gets waved through. Burning a food tells you what is in the food. It does not tell you what the food gives to a person, and no amount of care with the instrument will make it do so.

EXERCISE 3. Answer first, one page, no methods.

The proposal has shown that the menu is designed to meet the standard. It has not shown that the scheme is meeting it. Those are two different claims and only the first has evidence behind it.

Three gaps, in the order they matter.

The first is between the menu and the plate. A menu is an intention. What a child receives depends on what was cooked, how much of it was served, and whether it was eaten. None of that is in an arithmetic total.

The second is between the plate and the child. The figure in the standard is a food-energy figure. Some of the energy in any food is lost in the faeces, some as gas, and some in the urine. So the energy available to a child is always less than the energy in the food. The proposal's total is the upper figure.

The third is between the child's intake and any outcome anyone cares about. What a child spends is not in this arithmetic at all, and nothing in a menu total establishes a result.

What would close the gaps, in order of cost. Weighed portions on a sample of days at a sample of schools, which turns the menu into a measurement. Plate waste on the same days, which turns what was served into what was eaten. Both are ordinary field measurements and neither needs a laboratory.

What not to spend money on: burning food samples in a calorimeter. It would answer, very precisely, the question the proposal has already answered. It would say nothing about the two that are open.

One line for the meeting. The menu arithmetic is right. It is arithmetic about a plan, and the question in front of you is about a plate.

1. Kilojoules to kilocalories means dividing, because a kilocalorie is the bigger unit and there will be fewer of them.

```
2,092 divided by 4.184 = 500
```

Five hundred kilocalories.

Check the direction before you move on. The answer came out smaller than the number you started with, and that is right. One kilocalorie is worth more than four kilojoules. So any amount of energy is a smaller number when it is counted in kilocalories.

2. The energy equivalent is a number of joules for each degree, so multiply it by the number of degrees.

```
9,500 times 3.4 = 32,300
```

Thirty-two thousand three hundred joules.

A kilojoule is a thousand joules, so divide.

```
32,300 divided by 1,000 = 32.3
```

Thirty-two point three kilojoules.

3. This is the calibration, and it runs the other way from the last problem. There you had the constant and wanted the energy. Here you have the energy and want the constant.

Divide the known energy by the rise it produced.

```
44,000 divided by 4.00 = 11,000
```

Eleven thousand joules for each degree.

Check the unit, because it is what tells you the answer is the right kind of thing.

```
joules, divided by degrees
= joules per degree
```

That number now belongs to this calorimeter and to no other.

4. Take each of the three on its own, then add.

```
12 times 4.0 = 48
```

```
8 times 9.0 = 72
```

```
40 times 4.0 = 160
```

Now add the three.

```
48 plus 72 plus 160 = 280
```

Two hundred and eighty.

Look at the three parts before you leave it. The fat is a fifth of the weight of the carbohydrate. It gives almost half as much energy. That is what a factor of nine does against a factor of four.

5. Take the rounded figure first.

```
17 divided by 4.184 = 4.063
```

Four point zero six kilocalories a gram.

Now the precise figure from the footnote.

```
16.7 divided by 4.184 = 3.991
```

Three point nine nine kilocalories a gram.

Which one the 4.0 came from: the second. Three point nine nine rounds to four point zero. Four point zero six does not, at least not without throwing away a digit you were given.

So the report's two columns are not exact conversions of one another. They were set down separately, and for protein the precise kilojoule figure does round to the printed kilocalorie one. Convert the rounded 17 instead and you land near the printed figure, not on it.

What to do about it in practice. Use whichever unit the source you are quoting used, and convert only when you have to. Every conversion of a rounded figure moves the answer. A move of a few parts in a thousand is enough to make two people think one of them has slipped.

6. Calibrate first. Known energy divided by the rise it caused.

```
52,500 divided by 3.50 = 15,000
```

Fifteen thousand joules for each degree, for this calorimeter.

Now the food. Rise times the energy equivalent.

```
15,000 times 2.80 = 42,000
```

Forty-two thousand joules. Into kilojoules.

```
42,000 divided by 1,000 = 42
```

Forty-two kilojoules from the whole pellet. The pellet weighed 2.00 grams, so divide.

```
42 divided by 2.00 = 21
```

Twenty-one kilojoules in each gram of that food.

Keep the label as well as the unit. This is twenty-one kilojoules of gross energy for each gram of the dried food. Dried is not the same as eaten, because the water was taken out. And gross energy is not the same as what a person would get from it.

7. The protein first. Grams times the factor for protein.

```
12 times 4.0 = 48
```

Forty-eight kilocalories from the protein standard.

Now the share. Divide the part by the whole, which here is the energy standard for the same meal.

```
48 divided by 450 = 0.1066666667
```

Keep every figure for now. Turn the proportion into a percentage by multiplying by a hundred.

```
0.1066666667 times 100 = 10.66666667
```

Round once, at the end: 10.7. About ten point seven per cent.

Read the unit on the 450 before you leave this. The column is headed Calories with Kcal in brackets, so the figure is kilocalories. Taken as plain calories it would be a meal of 450 calories. In joules that is this.

```
450 times 4.184 = 1,882.8
```

That is under two kilojoules, and no meal is that.

And read what the answer is about. It is the share of the Act's energy standard that the Act's protein standard accounts for. It is not the protein share of any meal a child ate.

8. The first laboratory. Rise times the energy equivalent.

```
8,000 times 2.50 = 20,000
```

```
20,000 divided by 1,000 = 20
```

Twenty kilojoules from a 1.25 gram sample.

```
20 divided by 1.25 = 16
```

Sixteen kilojoules for each gram. Into kilocalories.

```
16 divided by 4.184 = 3.824
```

Three point eight two kilocalories a gram.

The second laboratory. Each component times its own factor, then add.

```
0.10 times 4.0 = 0.4
```

```
0.05 times 9.0 = 0.45
```

```
0.70 times 4.0 = 2.8
```

```
0.4 plus 0.45 plus 2.8 = 3.65
```

Three point six five kilocalories a gram.

The difference:

```
3.824 minus 3.65 = 0.174
```

The burning answer is the larger, by about 0.17 kilocalories a gram.

Why that was the expected direction. The calorimeter measures gross energy: everything the food gives up when it is burnt to nothing. The Atwater factors were built from burning measurements and then corrected for what is lost in digestion, in absorption and in the urine. So the factors are already a step down the cascade. A figure worked out from them should come out below a burning figure for the same food.

What the size of the gap does not tell you. It is not a measurement of anybody's losses. The corrections inside the factors are averages, taken over foods and over people. So the 0.17 is the gap between one measurement and one average. It is not a finding about this food.

9. The step that broke is the fourth. The energy equivalent converts a rise, and 24.9 is not a rise. It is a reading.

Get the rise first, by taking one reading from the other.

```
24.9 minus 21.4 = 3.5
```

Three and a half degrees. Now use the constant on that.

```
10,000 times 3.5 = 35,000
```

```
35,000 divided by 1,000 = 35
```

Thirty-five kilojoules, not 249.

What went wrong underneath the arithmetic. A Celsius reading is counted from a zero that somebody chose, so the reading itself is not an amount of anything. Only the gap between two readings is. Start the same burn in the same machine from a colder room. You get the same rise and a completely different final reading. The broken method would then give a different energy for the same event.

Why the wrong answer looks reasonable. Every line of the arithmetic is correct. The constant is right and the multiplication is right. And 249 kilojoules is an ordinary size for a food-energy figure, so nothing about the answer looks strange. Only the units catch it: joules for each degree, times degrees, needs a number of degrees, and 24.9 is a position on a scale.

10. The step that broke is the second. The twelve grams of sugars are not a fourth item. They are part of the thirty grams of carbohydrate, and their energy has already been counted on the line above.

Do it again with three lines instead of four.

```
30 times 4 = 120
```

```
6 times 4 = 24
```

```
5 times 9 = 45
```

```
120 plus 24 plus 45 = 189
```

One hundred and eighty-nine kilocalories.

How far out the wrong answer was:

```
237 minus 189 = 48
```

```
48 divided by 189 = 0.253968254
```

Keep every figure for now, and multiply by a hundred.

```
0.253968254 times 100 = 25.3968254
```

Round once, at the end: 25.4. Twenty-five per cent too high, from one extra line.

Why it looks reasonable. Every multiplication is correct and every factor is the right factor. The error is not in the arithmetic at all; it is in reading the panel. A panel nests. The words of which mean the line is a part of the line above it. A part is never added to the whole it came out of.

The habit that catches it. Before you add anything from a panel, check whether the weights you are adding could come to more than the packet does. Thirty plus twelve plus six plus five is fifty-three grams of material. The packet holds thirty grams of carbohydrate in all. The sum has invented twelve grams of food.

11. What to compute. Two things. First, the same figure in the other unit, so you can see what size of quantity is being talked about. Second, where the 250 came from.

The conversion, using 4.184 kilojoules to the kilocalorie.

```
250 times 4.184 = 1,046
```

About a thousand kilojoules.

Where it came from. If the packet is sold in India, the figure was calculated rather than measured. Regulation 5(3)(e)(i) of the labelling regulations of 2020 says the energy to be listed is calculated using conversion factors. So somebody took the grams of carbohydrate, protein and fat, and multiplied each by its factor. Nothing was burned.

Now what the 250 does not establish, which is the part your colleague needs.

It does not establish what the person absorbs. The Atwater factors that produced it carry an average correction for losses in digestion, in absorption and in the urine. An average correction is not this person's losses on this food. The factors use one value for each of protein, fat and carbohydrate, whatever the food.

It does not establish that the packet holds exactly that. It is arithmetic on declared weights, and the weights are themselves rounded.

It does not establish anything about what the person spends. That is a different quantity entirely and nothing on a packet measures it.

And it does not establish anything about what would change in their body. Going from an intake figure to a change in a body needs one more number. You would need to know what a kilogram of that change is made of. This book does not give you that number, because it depends on what the change is made of. Do not supply it from memory.

The line to say out loud: 250 kilocalories is a property of the packet. It is not an event in a person.

12. What to compute. First, read the unit off the Schedule rather than off the sentence. The energy column of Schedule II is headed Calories with Kcal in brackets, so the figure is 700 kilocalories. The speaker said calories and meant kilocalories, and this time they happen to be right.

Second, work out the one thing the two figures in the Schedule can tell you together. That is how much of the energy standard the protein standard accounts for.

```
20 times 4.0 = 80
```

```
80 divided by 700 = 0.1142857143
```

Keep every figure for now, and turn it into a percentage.

```
0.1142857143 times 100 = 11.42857143
```

Round once, at the end: 11.4. About eleven per cent of the energy standard is accounted for by the protein standard. The other eighty-nine per cent has to come from fat and carbohydrate, and Schedule II sets no figure for either. That is worth saying in the meeting, because it is a real gap in what the Schedule fixes.

Now what none of this establishes.

It does not establish that any child received 700 kilocalories. Schedule II is a standard a meal must meet. A standard is a requirement, not a measurement, and the sentence has read one as the other.

It does not establish that a meal that met the standard was eaten, or eaten whole. Nothing in an Act measures a plate.

It does not establish that 700 kilocalories reached a child even if a meal met the standard exactly and was finished. Seven hundred is a food-energy figure. Between the energy in a food and the energy available to the body sit the faecal, gaseous and urinary losses. The figure that reaches the body is always the smaller one.

And the last clause is the one to stop. Taking it straight off what a child needs from home assumes the two flows are independent. It assumes that adding to one leaves the other where it was. The section on stocks and flows says why you cannot assume that. The balance line holds whatever the flows do. It never tells you what happens to one flow when another changes. Whether a school meal displaces food at home is a question about behaviour, and it has to be measured, not assumed.

What to ask for. Weighed portions and plate waste on a sample of days, which turns the Schedule's requirement into a measurement of what children actually received.

E3 · Conservation of energy, and a system with a boundary

EXERCISE 1. The balance line, in words. Start with the energy inside the boundary at the start. Add everything that crossed the boundary inwards. Take away everything that crossed it outwards. What you are left with is the energy inside the boundary at the end.

What it rules out: any change in the energy inside that did not happen by something crossing the line. There is no other route, so a proposed mechanism that adds no crossing cannot change the balance.

What it cannot tell you: which route energy took, or why a crossing changed size. It also cannot tell you what a quantity of energy is worth in any unit other than a unit of energy.

One thing to check in your own answer. Did you write "inside the boundary" rather than just "inside"? The line is the part people drop, and it is the part that makes the sentence mean anything.

EXERCISE 2. Two separate things are wrong, and the first one is a word.

The word "lost" is doing damage. Energy does not leave the accounts. The third of the energy that did not reach the food went somewhere. The note has not said where. So it has reported that the writer stopped counting, dressed up as a finding.

Redraw the line and the sentence changes. Put the boundary round the food, and that third crossed out of it. Put it round the whole kitchen, and the flue is a hole in your line, so somebody has to say how much went up it. That third is in the air, in the steel of the pans, in the walls, and in whatever went up the flue. Name those routes and you have an account. Do not name them and you have a gap.

The second thing wrong is the conclusion. A fraction reaching the food is not by itself evidence of a fault, because no figure has been given for what a working burner does. A number becomes a finding only when it is compared with something.

What to ask for. First, the routes: how much went up the flue, how much into the pans, how much into the room, measured or at least estimated separately. Second, the same measurement on a burner known to be in good order, so that the two thirds has something to be worse than. Third, the time over which each figure was taken. A boundary treated as sealed is only sealed for a stated stretch of time.

And one thing to say out loud. The energy in the room is real energy and it is no use for cooking. That is a fair complaint about a kitchen. It is not the same complaint as energy going missing, and only one of the two is possible.

EXERCISE 3. Three minutes, no jargon, and they need a line they can print. Say yes and no in that order, because the yes is the easy half.

The yes. Energy really is never made and never destroyed. Draw a line round a person and the energy inside that line can only change by energy crossing it. That much is arithmetic and nobody serious argues with it.

Now the no, and keep it concrete. That sentence has two words in it that are not doing what they look like they are doing.

The first is “out”. Energy leaves a person by several routes at once. It leaves as heat. It leaves as work done on things around them. And it leaves in what passes through the body without being absorbed. “Out” is a list, not a number, and the list has to be measured route by route.

The second is “weight”. Weight is measured in kilograms and energy is measured in joules, and the balance line never converts one into the other. To do that you would have to say what a kilogram of change in a body is made of. That is a question about bodies, not about physics. So the physics does not hand them the sentence they read. Something else has to, and that something else is evidence.

The quotable sentence: “Energy in minus energy out is a ledger, not an explanation. It tells you the numbers have to add up. It never tells you why one of them moved.”

What not to offer: a figure for how many kilocalories make a kilogram. You do not have one, and a number given to fill a pause is the part that gets printed.

1. Use the balance line. The amount inside at the end is the amount inside at the start, plus what crossed in, minus what crossed out.

```
1200 plus 450 = 1650
1650 minus 610 = 1040
```

One thousand and forty inside at the end.

The net crossing is what came in minus what went out.

```
450 minus 610 = -160
```

Minus 160 over the interval. It is negative, in the way the section on counting showed you, and the amount inside fell by exactly that much.

Check it in one line.

```
1200 plus 450 minus 610 = 1040
```

2. The same line does all four, run forwards in the first row and backwards in the rest.

First row. Start, plus in, minus out.

```
900 plus 300 minus 100 = 1100
```

Eleven hundred inside at the end.

Second row. The end is given, so work out what must have crossed out. Start plus in comes to this.

```
900 plus 300 = 1200
```

Twelve hundred if nothing had left. It ended at 1,050, so the difference is what crossed out.

```
1200 minus 1050 = 150
```

A hundred and fifty crossed out.

Third row. Now the inflow is missing. The end plus what left is what the start plus the inflow must have been.

```
1100 plus 250 = 1350
1350 minus 900 = 450
```

Four hundred and fifty crossed in.

Fourth row. Nothing crossed either way, so nothing changed.

```
900 plus 0 minus 0 = 900
```

Nine hundred at the end. Notice what the fourth row does and does not say. The amount inside is unchanged because nothing crossed the line. That is a different situation from an amount unchanged because two large crossings matched, and the end figure alone cannot tell the two apart.

3. First the light, which is what the hour's energy did not leave in the room.

```
900000 minus 840000 = 60000
```

Sixty thousand joules leaves through the window as light.

Now take the three boundaries one at a time, and for each one ask what crossed it.

a) Round the kerosene. Over the hour the kerosene gives up 900,000 joules of chemical energy. All of it crosses out of that boundary, into the flame and from there into the air, the walls and the light. Nothing crosses in.

```
0 minus 900000 = -900000
```

The change is minus 900,000 joules.

b) Round the lamp, the air and the walls. The chemical energy of the kerosene starts inside this line, so it is not a crossing. Nothing crosses in. The only crossing is the light going out of the window.

```
0 minus 60000 = -60000
```

The change is minus 60,000 joules, and it is exactly the light. Check it as changes. The kerosene gave up 900,000 joules, and the air and the walls gained 840,000.

```
840000 minus 900000 = -60000
```

c) Round the room and everything the light reaches. Now the light is inside the line too, so nothing crosses it at all.

```
0 minus 0 = 0
```

The change is zero joules. Inside that line, 900,000 joules of chemical energy became 900,000 joules of warmth and light. The form changed and the total did not.

Read the three answers together. Minus 900,000, minus 60,000, and zero, for one hour of one lamp. Each is the correct answer to its own question, and the question is the line.

4. Run the balance line backwards. The change inside is what crossed in minus what crossed out, so what crossed out is what crossed in minus the change inside.

```
5000000 minus 4100000 = 900000
```

Nine hundred thousand joules crossed out of the tank over the day.

Now the fraction, which is the crossing out divided by the crossing in.

```
900000 divided by 5000000 = 0.18
```

Nought point one eight, which as a percentage is 18 per cent.

```
0.18 times 100 = 18
```

Check it forwards before you trust it.

```
5000000 minus 900000 = 4100000
```

Four million one hundred thousand joules, which is the figure given.

What this does not tell you. It does not tell you by which route the 900,000 joules left. It could have gone through the tank's walls into the air, or along the pipes. It could have gone back out from the hot surface, or in water drawn off and used. The balance fixes the total that crossed and says nothing about how many routes it was shared between.

It also does not tell you that 18 per cent is a lot or a little, because nothing here says what a well-made tank does. A fraction becomes a finding only when it is set against something.

5. The step that broke is the fourth. Nothing was destroyed. Eighteen thousand joules went somewhere the accounting did not look.

The trouble starts one line earlier, and it is a boundary that changed without anybody saying so. The 60,000 is counted round the burner and the gas. The 42,000 is counted round the water alone. Those are two different lines, so the difference between the two figures is not a shortfall. It is the energy that ended up between the two lines — in the steel of the pan, in the air, in the flue.

Write the third line out with its line named and the arithmetic stops looking alarming.

```
60000 minus 42000 = 18000
```

Eighteen thousand joules crossed out of the boundary that holds the burner and the water, and it is now in the surroundings. That is a finding about routes, not a violation.

What to do instead: put both numbers on the same boundary before subtracting. One way is to bring the pan and the air inside the line. Then nothing has crossed, and the two figures no longer need to match. The other way is to keep the line round the water. Then you need a figure for what crossed into the water, not a figure for what the gas released.

Why the wrong answer looks reasonable. The subtraction is correct arithmetic on two correct measurements, and the conclusion even appeals to a real law to check itself. Every part of it is careful except the part nobody wrote down, which is where each number's line was.

6. The step that broke is the third, and the word that broke it is "so".

Conservation says the total energy is constant. It does not say the energy inside any line you happen to draw is constant. Those become the same statement only when nothing crosses the line, and here something does: the mains wire.

Go and look at the boundary. The door is shut and the window is shut, and a cable runs through the wall. Electricity crossing that cable is energy crossing the boundary, exactly as much as anything carried through the door would be.

So the balance line reads like this.

```
0 plus 300000 minus 0 = 300000
```

The energy inside the room went up by 300,000 joules over the hour. The balance does not say what form it ends in. It does remove the only support for “cannot have got warmer”, and the next section takes up where such energy ends up.

The fourth line is arithmetic with nothing behind it. It subtracts the 300,000 from itself because line three assumed the answer. Subtracting a number from itself always gives zero, and tells you nothing at all.

Why the wrong answer looks reasonable. The first two lines are correct, the law quoted is the right law, and the conclusion has the shape of a conservation argument. The check that catches it is the one this section asks for every time. Draw the line, then walk round it naming every route that crosses. A cable through a wall is easy to miss precisely because it is not a door.

7. What to compute: the balance across a boundary drawn round a person, with every crossing named. The claim rests on conservation, so the way to test it is to use conservation properly.

Draw the line round the person, at the mouth and skin. Then list the routes by which energy crosses it. Inwards: food and drink. Outwards: heat leaving the body to the air and to whatever it touches. Also work done on things outside the body. Also the energy in everything that leaves the body unabsorbed.

Now put the balance line on that.

```
the change in the energy inside = the energy in, minus the energy out
```

Take a day where 9,000 kilojoules crosses in with food, 1,000 kilojoules leaves unabsorbed, and 7,600 kilojoules leaves as heat and work. All three figures are made up, to show the shape.

```
1000 plus 7600 = 8600
9000 minus 8600 = 400
```

Four hundred kilojoules stays inside. Not 9,000. The word “somewhere” in the claim was doing the work of four routes, three of which point outwards.

What this does not establish. It does not establish that the answer is 400 rather than some other figure, because the three numbers are invented and this section measures nothing. The shape is the finding. The size is not.

It does not establish anything about weight. The answer is in kilojoules. Weight is in kilograms. Nothing in the balance converts one to the other. Doing so would mean saying what a kilogram of change in a body is made of.

And it does not establish that the crossings are independent of each other. The balance holds whatever they do. Whether changing one of them leaves the others where they were is a question about bodies. The subject booklet on energy balance is where it is asked.

What to say instead of the page’s sentence. Energy in food that is not stored has left the person by a route you can name. Naming the routes is the entire content of the claim.

8. What to compute: whether the question the label’s figure answers is the same question the colleague is asking. That is the only question here, and it is answered by drawing both lines.

The label’s line. The figure was calculated with the Atwater general factors. The section before this one showed that those factors already have average losses taken out. These are what is lost in digestion, in absorption and in the urine. So 1,200 kilojoules is an estimate of what an average person gets from the packet. It is not everything the packet holds.

The colleague’s line. This one is drawn round a person, and the sentence is about this person. This person’s losses need not be the average ones. Call the gap between this person’s losses and the average D, because nobody in the room has a figure for it.

Draw the person's line at the mouth, and the whole packet crosses in and whatever is not absorbed crosses back out. Draw it at the gut wall instead, and the unabsorbed part never crosses in. Either way the net crossing comes to the same thing.

```
the energy crossing into the person = 1200 minus D
```

Now the only honest computation available. If D is zero the colleague's figure is right for this person, and if D is anything else it is not. Nobody in the meeting has a value for D. So the correct output of this computation is a sentence, not a number: we do not have that figure.

What this does not establish. It does not establish that 1,200 is wrong. It is very likely the right answer to the question the label was answering.

It does not establish how big D is, or even which way it points. This section gives you no way to find out. It only shows you that the two questions differ, so a figure is needed.

And it does not establish anything at all about the person's weight. That would need a second balance, in kilograms. It would also need a claim about what any change is made of, and the ground floor does not carry that claim.

What to say in the meeting. The number is fine and the sentence is not. The label estimates what an average person gets from the packet. The claim is about what crosses into this person. Those are two different questions, and the difference between them is a quantity somebody has to measure rather than assume away.

E4 · Heat and temperature; how heat moves

EXERCISE 1. Heat is the transfer of energy from a hotter thing to a colder thing, and it happens because the two are at different temperatures. It is an amount of energy and its unit is the joule.

Temperature is how hot something is. It is a level rather than an amount, and its unit is the degree Celsius or the kelvin.

The test that separates them: temperature does not care how much of the thing there is, and heat does. A spoonful and a bucket of boiling water are at the same temperature and hold very different amounts of energy.

The three ways heat moves.

Conduction is transfer through matter that is staying put, by contact. A steel tumbler of hot chai warming your fingers.

Convection is transfer by a fluid moving and taking the energy with it. The hot air rising off a tawa onto your palm held above it.

Radiation is transfer as electromagnetic radiation, emitted or absorbed, with nothing needed in between. The warmth on your face from an open tandoor across the room.

One thing to check in your own answer. Did you write a unit for each of the first two? If heat and temperature came out with the same unit, or with no unit, the distinction has not been made yet.

EXERCISE 2. There are three claims stacked here and they get weaker as you go along.

The first claim is about a rate. A double wall slows conduction, because the gap between the two walls has less matter in it to conduct through. It slows convection too, if the air in the gap cannot circulate. That is a real mechanism and it is checkable. Note that it says nothing about radiation, which is the third route and crosses a gap perfectly well.

The second claim does not follow from the first. "Keeps food hot for six hours" is a statement about how fast energy leaves. "No energy escapes" is a statement that none does. Those are different claims, and only the weaker one can be true. If the food is still hotter than the room after six hours, energy is still crossing. Energy crosses whenever there is a temperature difference. What the container has done is make the crossing slow, not stop it.

The third claim is a change of subject. Calories in food are chemical energy sitting in the food's molecules. The energy leaving a hot container is thermal energy, and it is leaving because the food is hotter than the room. Letting the food go cold does not remove the chemical energy from it, and keeping it hot does not add any. A cold roti and a hot roti hold the same chemical energy. The sheet has run two different energies together under one word.

What to ask for. First, the rate: how many degrees does the food fall in six hours, starting from what temperature, in a room at what temperature. That is the only measurable version of the first claim. Second, which of the three routes the wall acts on, and what it does about radiation. Third, nothing at all for the third claim, because it should be withdrawn rather than supported.

EXERCISE 3. Ten minutes, a whiteboard, and somebody who is not wrong yet — they are right about the instrument and about to over-apply it.

Start by agreeing with them, out loud, and write the calorimeter sentence on the board. Twice the rise, twice the energy. Tell them it is correct and you are not going to take it away.

Then draw two circles and label them: one kilogram of water, one kilogram of steel. Ask the room how far each goes up if you put the same energy into both. Let them answer before you write anything; most will say the same amount.

Now write the two numbers from the table. Water 4186 joules per kilogram per degree. Steel 452. Then bracket the steel's rise on the board, so they watch it happen rather than watch a calculator.

```
452 times 9 = 4068
452 times 10 = 4520
```

Four thousand one hundred and eighty-six joules sits between those two. So the tawa goes up by between nine and ten degrees, while the water goes up by one. Same energy, one goes up nine times as far. That is the moment to stop and say nothing for a few seconds. The sentence they walked in with has just broken in front of them.

Now rebuild it, and this is the part they will keep. Write the equation, naming every letter as you write it. Q is the quantity of heat. m is the mass. c is the specific heat of the material. ΔT is the change in temperature. Then point at c and say that this is the thing that was hiding.

Inside one calorimeter, m and c never change from run to run. So the rise is the energy divided by a number that never moves, and their original sentence is exactly right. Give that number its name on the board: the heat capacity of the instrument, joules per degree.

Change the instrument and the number changes. Change the object and the number changes. Then degrees from one are not degrees from the other, and a comparison of rises is not a comparison of energies.

Close with the question you want them asking on Monday. When anybody reports an energy that came from a temperature, ask what heat capacity was used and where it came from. Tell them that "we used the calorimeter's energy equivalent" is a good answer. "We compared the temperature rises" is not an answer at all.

1. Put the three numbers in place of the three letters and multiply.

```
2 times 500 times 30 = 30000
```

Thirty thousand.

Take the multiplication in two steps if that is easier, and check you get the same.

```
2 times 500 = 1000
1000 times 30 = 30000
```

2. Both parts run the same relationship backwards. Q is the product of the other three, so any one of them is Q divided by the product of the two you have.

a) Multiply the two you have.

```
2 times 500 = 1000
```

Then divide.

```
30000 divided by 1000 = 30
```

ΔT is 30.

b) Same move, with a different letter missing.

```
3 times 6 = 18
9000 divided by 18 = 500
```

c is 500.

Check each one forwards before you leave it.

```
2 times 500 times 30 = 30000
3 times 500 times 6 = 9000
```

3. First the change in temperature, which is the same for both.

```
100 minus 28 = 72
```

Seventy-two degrees Celsius.

Now the water. Mass times specific heat first.

```
2 times 4186 = 8372
```

Eight thousand three hundred and seventy-two joules for every degree. Now times the 72 degrees.

```
8372 times 72 = 602784
```

Six hundred and two thousand seven hundred and eighty-four joules into the water.

Now the steel, the same way.

```
1.2 times 452 = 542.4
542.4 times 72 = 39052.8
```

Thirty-nine thousand and fifty-two point eight joules into the steel.

The total.

```
602784 plus 39052.8 = 641836.8
```

Six hundred and forty-one thousand eight hundred and thirty-six point eight joules altogether.

Which took more, and why. The water, by a long way — about fifteen times as much. Two reasons stack. There is more of it, 2 kilograms against 1.2. And water needs more than nine times as many joules per kilogram per degree as steel does. The vessel is almost an afterthought in the arithmetic. That is why a pan of water takes so long to heat, and the empty pan takes almost no time at all.

4. The glass first. Mass times specific heat.

```
0.18 times 840 = 151.2
```

A hundred and fifty-one point two joules for each degree. Now times 55 degrees.

```
151.2 times 55 = 8316
```

Eight thousand three hundred and sixteen joules into the glass tumbler.

Now the steel.

```
0.18 times 452 = 81.36
81.36 times 55 = 4474.8
```

Four thousand four hundred and seventy-four point eight joules into the steel tumbler.

Which takes more. The glass. Now put a size on how many times more, and bracket it rather than divide. Try 1.8 times the steel's figure.

```
4474.8 times 1.8 = 8054.64
```

Eight thousand and fifty-four, and the glass took 8,316. So the factor is more than 1.8. Now try 1.9.

```
4474.8 times 1.9 = 8502.12
```

Eight thousand five hundred and two, which is more than the glass took. So the factor sits between 1.8 and 1.9. A calculator puts it at 1.858.

Now check that against the two specific heats, by multiplying 452 by the same figure and seeing whether 840 comes back.

```
452 times 1.858 = 839.816
```

Eight hundred and thirty-nine point eight, which is 840 to the nearest whole number. The ratio of the energies is the ratio of the specific heats. It has to be. The two tumblers have the same mass and the same temperature change, and differ only in the material.

What it means for the chai. Every joule that goes into warming the tumbler came out of the chai. So the glass tumbler takes about 1.86 times as much energy out of it. With the same amount of chai poured into each, the two tumblers could not both rise the full 55 degrees. The glass would stop lower, and so would the chai in it.

And one thing this does not settle. How fast the chai then goes on cooling depends on the routes out of the tumbler. How well each material conducts, how thick the wall is, how much surface meets the air. This calculation is about the energy taken up in warming the vessel, and it is only the first part of the story.

5. The step that broke is the fourth, and the arithmetic in it is correct. What broke is a unit.

The specific heat is quoted per kilogram. The mass was put in as grams. Multiplying grams by a per-kilogram figure gives an answer a thousand times too big. Nothing on the page says so, because the units were never written beside the numbers.

Convert the mass first.

```
500 divided by 1000 = 0.5
```

Half a kilogram. Now the same calculation with the mass in the unit the specific heat is quoted in.

```
0.5 times 4186 times 50 = 104650
```

A hundred and four thousand six hundred and fifty joules, which is about 105 kilojoules.

A size check would have caught it. The two answers are a thousand times apart, which is the kind of gap the section on orders of magnitude exists to make visible.

Why the wrong answer looks reasonable. Every number in it is a real number. The subtraction is right and the multiplication is right. The specific heat is the correct one for water. The only thing wrong is that two of the quantities were measured in units that do not go together. That never shows up in the digits.

6. The step that broke is the last one. The two temperature rises are very different and the two quantities of heat are almost the same.

Work out each one, using the specific heats from the table: water 4186 joules per kilogram per degree Celsius, steel 452.

The tawa first.

```
39.3 minus 30.0 = 9.3
1 times 452 times 9.3 = 4203.6
```

About 4,204 joules.

Now the water.

```
31.0 minus 30.0 = 1.0
1 times 4186 times 1.0 = 4186
```

About 4,186 joules.

Put the two side by side.

```
4203.6 minus 4186 = 17.6
```

Seventeen point six joules apart, out of roughly four thousand two hundred each. That is well under one per cent, which is smaller than the error in reading either thermometer. The two burners delivered the same energy, which is what the second line of the working said they did.

What the fourth line should have led to instead. A temperature rise is the heat divided by the object's mass times its specific heat. The two objects have the same mass and very different specific heats, so the same heat has to produce very different rises. The rises differ because the objects differ, not because the energies differ.

Why the wrong answer looks reasonable. Lines one to four are all correct, and the last line is the sentence anybody carries away from using a calorimeter, where it is true. The missing step is the one that asks whether the two things being compared need the same energy per degree. Inside a single instrument that question never has to be asked.

7. What to compute: the energy needed to warm four kilograms of water through the temperature range the poster names. The prompt gives the mass as four kilograms. That figure is the only quantity the poster's claim actually pins down, and it is worth knowing before arguing about anything else.

The change in temperature first.

```
37 minus 5 = 32
```

Thirty-two degrees Celsius. Now the mass times the specific heat of water.

```
4 times 4186 = 16744
```

Then times the change.

```
16744 times 32 = 535808
```

Five hundred and thirty-five thousand eight hundred and eight joules, which is about 536 kilojoules. Now put it in the unit food labels use. One kilocalorie is 4,184 joules, from the section on energy units.

That 4,184 is the calorie's definition, not water's specific heat. The two are different quantities.

So ask how many kilocalories fit in 535,808 joules. Try 128.

```
4184 times 128 = 535552
```

Five hundred and thirty-five thousand five hundred and fifty-two, which is just short of what you have.

```
535808 minus 535552 = 256
```

Two hundred and fifty-six joules short, which is well under one kilocalorie. So the answer is a shade over 128 kilocalories a day, if every part of the poster's set-up holds.

What this does not establish, and there are four things.

It does not establish that the body spends that energy on top of what it was already spending. The balance from the section on conservation and the boundary says the numbers must add up across that boundary. It says nothing about whether the other crossings stay put when you change one of them. Whether they do is a question about bodies, and it is the subject of the booklet on energy balance rather than of this section.

It does not establish anything about body weight. The answer is in kilojoules. Weight is in kilograms. Nothing here converts one into the other.

It does not establish the two temperatures. They are the poster's, and they were taken as given so that the arithmetic could be done. A different pair gives a different answer in proportion.

And it does not establish that all four litres arrive at 5 degrees and are warmed through the full 32 degrees inside the person. Water sitting in a glass in an Indian kitchen does not stay at fridge temperature for long.

What is left after all that is a number: 128 kilocalories a day. It is a ceiling on the size of the effect the poster describes, not a measurement of it. That is still worth having, because it stops the claim being either dismissed or believed without a figure.

8. What to compute: the energy behind each rise, using the bomb's own heat capacity, and then the ratio of the two. The heat capacity is in joules per degree, so multiply it by the degrees. Note what that figure is and is not. It is the bomb, not the whole calorimeter.

The first food.

```
893 times 4.2 = 3750.6
```

About 3,751 joules absorbed by the bomb.

The second.

```
893 times 2.1 = 1875.3
```

About 1,875 joules.

Now the ratio.

```
3750.6 divided by 1875.3 = 2
```

Exactly two, which it was always going to be, because the same heat capacity multiplies both rises. On the conditions stated, the colleague's arithmetic is right.

What this does not establish, and the conditions are where all of it sits.

It does not establish anything if the two runs were not done in the same calorimeter. A degree is worth 893 joules in this bomb and some other figure in another one. So rises from two instruments cannot be compared at all until both heat capacities are in hand.

It does not establish that one food holds twice the energy of the other unless the two samples weighed the same. Energy content is quoted per gram. Burn twice as much of an identical food and you get twice the rise, and nothing about the food has changed.

It does not account for everything that absorbed the energy. The 893 joules per degree is the bomb itself. A bomb calorimeter sits in water, and the water takes up energy too. So the total released by the food is larger than either figure computed here.

And it does not establish what reaches a person. The boundary round the bomb is not the boundary round a body, and what crosses the two is not the same quantity.

What to say in the meeting. The ratio is sound and it is a ratio of temperature rises in one instrument. Before it becomes a statement about two foods, someone has to confirm three things. The same calorimeter, the same sample mass, and the water as well as the bomb.

E5 · The cell, membranes, proteins and receptors

EXERCISE 1. The water-hating signal first. It dissolves in grease, so it crosses the middle of the membrane on its own. Inside the cell it meets its receptor, which is waiting in the jelly. It fits into it. The receptor changes shape. The new shape does a job the old shape could not.

Now the water-loving signal. It cannot cross the greasy middle, so it never enters. Its receptor is built right through the membrane, with one end outside and one end inside. The signal lands on the outside end and stays there. The receptor changes shape. The inside end, now a different shape, starts something happening inside the cell.

Why the two go different ways. The middle of the membrane is greasy. So what dissolves in grease goes through, and what dissolves in water does not.

One thing to check in your own answer. Did you write the shape change in both versions? That is the event. A version that stops at "it binds" has left out the part that does the work.

EXERCISE 2. Take the two sentences separately, because the first one may well be true and the second does not follow from it.

The first sentence is a binding claim. It says the molecule fits into that receptor. On its own that is checkable and it may be right.

The second sentence is an action claim, and it is a different claim. A ligand binds a receptor, and in some cases delivers a signal in the process. In some cases. Three things can happen when a molecule sits in a receptor. It can change the shape the way the usual ligand does, and the usual thing happens. It can change it some other way. Or it can sit there, change nothing, and keep the usual ligand out, so less happens than before.

The sheet has picked the first of those three and reported it as though binding settled it. Nothing in a binding result chooses between them.

The last clause is a third claim again. Side effects come from everywhere else the molecule goes and everything else it binds. A sheet that has measured one receptor has measured nothing about the rest. “No side effects” from that evidence is a claim about what nobody looked for.

What to ask for. First, what happened downstream in a cell that carries the receptor, not just whether binding occurred. Second, what else the molecule binds. Third, at what amount the binding was seen, and whether a person taking the capsule reaches it.

And say the size point out loud, because it is the one that gets waved through. Binding at some concentration in a dish tells you the shapes fit. It does not tell you the effect is big enough to matter in a person.

EXERCISE 3. Ten minutes, a whiteboard, and somebody who wants a mechanism rather than a rule.

Draw the membrane first, and draw it as two rows of lollipops with the sticks pointing at each other. Heads out on both sides, sticks meeting in the middle. Then write two words on the board: wet outside, greasy inside. Do not say anything about drugs yet.

Ask them which of two molecules gets through on its own: one that dissolves in water, or one that dissolves in oil. They will get it right, because you have drawn the answer. That is the moment to stop and let them see they have just derived something rather than been told it.

Now put a receptor on the board. Draw a shape with a notch in it and a small molecule that fits the notch. Say the two names once: the small molecule is the ligand, the notched protein is the receptor.

Then draw the same protein again with the ligand sitting in the notch, and draw it a different shape. That second drawing is the whole lesson. The event is not the arrival. The event is the shape change.

Now put the two ideas together and let them do it. Ask where the receptor has to be for an oil-loving signal, and where it has to be for a water-loving one. They will place the first inside the cell and the second through the wall. They have now built the general rule themselves.

Place the one they came in with. Insulin’s receptors sit on the cell membrane, so insulin is a signal of the second kind: it lands outside and never needs to enter.

Close with the thing you want them using on Monday. When they meet a new drug, three questions, in order. What is the molecule. What does it bind. What changes when it binds. Tell them the third question is the one that is usually missing. A drug they cannot answer it for is a drug they do not yet understand.

E6 · Metabolism as chemistry happening in a body

EXERCISE 1. The rice first. Rice is mostly starch, which is a long chain of sugar molecules. The gut cuts the chain into single sugars. Glucose is the one that matters.

It crosses the wall of the small intestine into the blood, and the blood carries it to the liver first. From there it goes round the body.

A cell that takes glucose in breaks it down, and the energy that comes out goes into ATP. Whatever is not needed at once is put away, as glycogen in the liver and muscles, or turned into triglyceride in adipose tissue.

Now the oil. Oil is triglyceride. A whole triglyceride molecule cannot cross the gut wall, so it is broken into fatty acids first. Those cross. On the far side they are put back together into triglyceride.

That triglyceride is either used for energy or stored in adipose tissue. To get energy out of it, it is pulled apart again into fatty acids and glycerol. Those are then broken down to make ATP.

What is true of both. Neither crosses the gut wall in the form it was eaten in. The energy from both goes into ATP before it does any work.

What is different: the sugar goes to the small store as well as the large one, and the fat only goes to the large one.

One check on your own answer. Did you write ATP into both?

If your version has energy going from food straight to the job that used it, go back. You have left out the step this section is mostly about.

EXERCISE 2. Three separate things are wrong, and it is worth keeping them apart, because they need different fixes.

The first is the one this section is about. A food table gives an average estimate for a food. It does not give what crossed these women's gut walls, and nothing in the note measured that. Some of what was eaten was never absorbed at all. No human makes an enzyme for most plant fibre, so fibre is eliminated rather than taken up. So the number computed here is a number about plates. The sentence written under it is a claim about bodies.

The second is that nothing was measured. Somebody said what they ate, and somebody else looked it up. Two estimates were stacked and the result was reported as an observation.

The third is the word therefore. Even if the intake figure were exact, "intake is not the problem" is a claim about a cause. An intake figure on its own establishes nothing about a cause.

What to ask for. First, say what the figure is a figure of. It is a table estimate for the food recorded, not a measure of energy absorbed, and that belongs on the same line as the number. Second, ask how the food record was taken and over what days. Three days of reporting is a measurement with an error of its own, and that error belongs beside the estimate. Third, ask what the comparison figure is and where it came from.

And say the last thing out loud, because it is the one that will otherwise be waved through. Nothing in this note measured what crossed a gut wall. Nothing in this course lets you get from the plate to the blood by arithmetic.

EXERCISE 3. Three minutes, no jargon, and they want to know whether there is anything there.

Start with what the word means, because the answer sits inside it. Metabolism is not an organ and not a setting. It is a name for all the chemical reactions going on in a body at once, added together. Thousands of them, going in both directions.

Then the point. If you add up thousands of reactions and call the total metabolism, then "boosts your metabolism" has not named anything. It has not said which reaction, or which direction, or in which part of the body, or by how much.

Give them the questions to put to the company, because that is what they can use. Which reaction. In which tissue. Measured how, and in whom. And how big was the change.

Then tell them what a real answer would look like, so they can tell one from this. A real claim names a measured quantity in a named group of people and gives a size. This claim names nothing, so there is nothing for the evidence to be evidence about.

The quotable sentence: "Metabolism is not a dial. It is the name for every chemical reaction in your body added together. Saying a drink boosts it is like saying a lorry boosts traffic."

What not to offer: a figure for how much anything changes anybody's metabolism. You do not have one, this section does not carry one, and a number invented to fill a pause is the part that gets printed.

E7 · Genes, DNA and inheritance in outline

EXERCISE 1. The four letters are A, G, C and T. They stand for adenine, guanine, cytosine and thymine.

A pairs with T. C pairs with G. Bases that go together in this way are called complementary.

What the pairing lets a cell do: because each letter has exactly one partner, the order along one strand fixes the order along the other. Read one strand and you can write the second. That is how a cell makes a copy of its DNA, and it is how a copy is passed on.

A genotype is what a person carries. It is the alleles in their DNA, counting the ones that are not showing as well as the ones that are.

A phenotype is what shows — the traits you can observe.

One check on your own answer. If your sentence for phenotype used the word gene, go back. The point of having two words is that one of them is about what you can see and the other is about what is there.

EXERCISE 2. Three things are wrong and they are three different kinds of wrong.

The first sentence is an observation about one household. It may be accurate. It is also four people, which is not a number you can conclude anything from, and it says nothing about why.

The second sentence is a claim about the world and the note gives no source for it. Nothing in this section establishes it. This section tells you what a gene is, what an allele is, and that the same phenotype can sit on more than one genotype. It gives you no way to move from a family pattern to a cause, and no figure for how strongly anything is inherited. A sentence beginning "is known to be" with no citation attached is the sentence to stop at.

The third sentence is the one that does harm, and it does not follow even if the second were true. A gene fixes the order of parts in a protein. It does not fix when that protein is made. A signal reaching the cell can set the making of one going. And whether the protein works depends on the conditions around it. Several steps sit between an allele and a person, and each answers to something.

There is also a fourth problem, which is not about genetics. The note moves from a claim about causes to a decision about who gets a service. That move needs its own justification and it has none here.

What to do with it. Remove the third sentence entirely. Change the second to say what was actually observed, which is that the trait appears in three of four adults in one household. Keep the first.

What to ask for before writing anything in its place. Ask whether the household shares more than DNA. The same kitchen, the same food budget, the same working hours, the same distance to anywhere safe to walk. Those are shared circumstances and they produce family patterns too. Nothing in an observed family pattern separates the two, so a note that names only one of them has chosen rather than found.

EXERCISE 3. Ten minutes and a whiteboard, and they meant to be kind. Start by saying so, because they did.

Draw a line across the board with four stops on it. Write at the first stop: a gene. At the second: a protein. At the third: whether that protein is made right now. At the fourth: what happens in a person.

Now go along it with them.

Stop one to stop two. A gene fixes the order of the parts in a protein. That step is solid and it is the step their sentence is about.

Stop two to stop three. The gene does not fix when the protein gets made. A signal can arrive at the cell, cross into it, and set the making of the protein going. Ask them what a signal lands on. They met it in the section on the cell. They will say a receptor. Good — that is this step, and it is one route of several.

Stop three to stop four. A protein has to hold its shape to work. Heat, acidity and the chemicals around it change that shape, and a protein that has lost its shape has lost its job.

Then stand back and ask them the question. Their sentence was about the first arrow. Which of the other two did it make a claim about? They will see that it made a claim about all of them.

Now the second half, which is about what the patient heard. She did not hear a statement about proteins. She heard "this is settled, and nothing you do matters". That is not what they said. It is what she has taken home, and she will act on what she took home.

Give them the replacement, and make them say it out loud before they leave the room.

"What you inherit is real and it is part of the picture. It sets some things about how your body handles food. It does not settle what happens next, and that is the part we work on together."

Then one last instruction, which is the one that sticks. Whenever they are about to say a sentence about a patient's body, they should finish it silently with "and therefore". Then listen to what comes out. If what comes out is "and therefore nothing can be done", the sentence needs rewriting before it is said.

E8 · Organs and systems that matter for metabolism

EXERCISE 1. The six are the gut, the liver, the pancreas, adipose tissue, muscle and the brain.

The gut cuts food into pieces small enough to cross its wall, and lets them through into the blood. What dissolves in water goes from there to the liver. What dissolves in fat takes another road and joins the blood further along.

The liver takes all the blood coming from the gut before anything else does. It works on what arrives and passes on what the body needs. It stores spare sugar as glycogen and turns glycogen back into sugar. It also makes bile, which breaks large blobs of fat into tiny ones.

The pancreas does two jobs. It sends a juice full of enzymes down a tube into the gut. And small islands inside it put insulin and glucagon straight into the blood.

Adipose tissue holds triglyceride until it is needed. It is one of the two tissues insulin acts on most, and it makes hormones of its own.

Muscle is the other tissue insulin acts on most, and it holds glycogen.

The brain runs on glucose and can use nothing else. It takes glucose from the blood without needing insulin.

Now the hormone. A hormone is a chemical that a gland puts into the fluid around its own cells. That fluid drains into the blood, and the blood carries it all over the body.

How it gets where it acts: it does not go anywhere in particular. It goes everywhere. It acts only where a cell carries a receptor shaped to hold it. When it binds, the receptor changes shape, and something happens inside that cell.

One check on your own answer. If you wrote that a hormone travels to its target organ, go back. That picture will make every later sentence about side effects come out wrong.

EXERCISE 2. Three sentences, and each one is wrong in a different way.

The first sentence is wrong as biology. Adipose tissue is not a bag. It makes hormones of its own and puts them into the blood, which makes it an organ that signals to the rest of the body. The word "simply" is doing the damage, and it is doing it in a leaflet a patient will read.

The second sentence is a claim with a number hidden in it. "Once they are full" asserts a capacity. Nothing in this section gives you one. No figure in the book behind this section would survive a check either, because every figure in it is stated with no study named. A quantity asserted without a measurement is the kind of sentence that sounds like physiology and is not.

It is also wrong about the loop. This section gives you a store that fills and a store that empties, several times a day, under two hormones that run opposite ways. The leaflet describes a one-way process. A store that only ever filled would not be a store.

The third sentence takes a mechanism and turns it into an instruction for the reader. That is a separate move and it needs its own evidence, which the leaflet does not offer.

What to put in its place. Say what adipose tissue does, in the same voice you would use for the liver. It holds triglyceride until it is needed. It releases it under a signal. It also makes hormones, which is why it counts as an organ rather than a place.

And take out the word "simply", wherever it appears. Nothing in this section is simple, and the word is only ever doing work on the reader rather than on the subject.

EXERCISE 3. Ten minutes and a whiteboard, and the sentence they used is the one everybody is taught. Say that first, because it is true and it stops them defending it.

Draw a line across the board and mark two ends. Write “sugar high” at one end and “sugar low” at the other. Nothing else yet.

Ask them what the pancreas does at each end. They will get insulin and glucagon in the right places. Write the two names under the two ends.

Now add one row under each: what it does to the stores. Insulin fills them. Glucagon empties them. Let them see that the picture is a loop and not a rule.

Now break their sentence, and do it by asking rather than telling. Ask which organ runs on glucose and can use nothing else. They will say the brain. Ask whether the brain needs insulin to take glucose in.

Let them answer, then give them the sentence from the book. The brain, the liver, the kidneys, red blood cells and the lining of the gut need no insulin to take glucose in. The liver still answers to insulin in other ways: insulin makes it store glycogen.

Now go back to the board and write the two tissues that do wait for insulin: muscle and fat.

Then ask the question that fixes the habit. If the brain needs no insulin, what was the key unlocking? They will see it. The key picture had one door in it. There are at least two kinds of cell, and only one of them needs the key to let sugar in.

Close with the rule you want them using tomorrow. Any sentence about a hormone names a tissue. Not “insulin lets sugar into cells”, but “insulin brings glucose carriers to the surface of muscle and fat cells”.

And tell them why it matters to the patient and not only to the exam. A patient who is told that insulin lets sugar into cells will conclude that without it the brain starves for want of a way in. That is not what happens, and a patient acting on it will do the wrong thing.

F1 · Reading a table

EXERCISE 1. Among the 18,750 women surveyed in state B during fieldwork conducted across 2019 to 2021, 39.6 per cent were recorded as overweight or obese. It is a property of a surveyed group, in one state, over a period. It is not a property of any woman, of the state’s whole population, or of a date.

EXERCISE 2. First, it describes the states, when the table reports surveyed samples.

Second, it treats the gap as a measured quantity. Nothing in the table tells you the precision of either figure. The difference may be solid or may be noise, and the table cannot tell you which.

A third and subtler point: “obesity” and “overweight or obese” are different categories, and the column heading says the second one.

F2 · Reading a figure, and the ways a figure misleads

EXERCISE 1. It would not. On a line chart a start printed on the axis and stated in the caption is enough, because the reader reads positions. On a bar chart the reader compares lengths, at a glance, before reading any caption.

Work out what the glance sees.

```
450 minus 400 = 50
800 minus 400 = 400
400 divided by 50 = 8
```

The lower primary bar is drawn 50 kilocalories long and the bar for children who are malnourished 400 long. That is eight times as long, for a value 1.78 times as large. A caption tells the reader the lengths are wrong. It does not make them right.

The first honest redraw: start the side axis at zero, so each bar is as long as its value.

The second: keep the axis from 400 and change the mark. Draw each value as a dot at the right height, with no bar under it. A dot carries its value by position. So the axis start no longer misleads, as long as it is printed on the axis and stated in the caption. This works because the top of each bar was at the right height all along.

What neither redraw tells you is whether the six values are right for these groups. That is a question about the Schedule, not about the chart.

EXERCISE 2. A slice shows a share of the total, not an amount. Two pies drawn the same size hide the fact that the total changed. Work out the amounts.

```
0.4 times 100 = 40
0.3 times 150 = 45
```

Forty crore rupees on grain in the first year, and 45 crore in the second. Spending on grain rose by 5 crore while its slice shrank. The presenter read a share as an amount.

This is a known fault of pie charts set side by side. The total changes from year to year while the pie stays one size. So the slices of two years can be compared as shares, never as amounts.

What to ask for: the amount in rupees for each part in each year, drawn as bars from zero and set side by side. Then the share and the amount can each be read for what it is.

EXERCISE 3. Three minutes, no jargon, and a line they can print.

Start with the answer. It is true for bar charts and not true for line charts.

Then the reason, as a picture. A bar tells you its number by how long it is. Your eye compares two bars by their length, in a glance. Cut the bottom off and the lengths go wrong. A short bar can look an eighth of a long one when it is more than half of it.

A line tells you its numbers by where the points sit, and you read them off the side. Nothing asks you to compare lengths. So a line chart that starts at 400 is honest, as long as the 400 is printed and the caption says so. But read the change off the numbers, not off how steep the line looks. The steepness is somebody's choice. If the line looks alarming, redraw it from zero to check the impression. That is a check, not a sign that the first drawing was dishonest.

Then the one thing to check on the chart in their story. Is it bars or a line? If it is bars, find the lowest number up the side. If that is not zero, the chart exaggerates every difference on it.

The quotable sentence: "A bar tells you its number by its length, so it has to start at zero. A line tells you by position, so read its numbers, not its steepness."

What not to say: that the chart's numbers are wrong. They may be exactly right. Only the drawing misleads.

1. Each bar is drawn up from 60, so its drawn length is its value minus 60.

```
80 minus 60 = 20
100 minus 60 = 40
```

Compare the drawn lengths.

```
40 divided by 20 = 2
```

The second bar is drawn twice as long. Now compare the values.

```
100 divided by 80 = 1.25
```

Its value is one and a quarter times as large. The drawing says twice, and the numbers say a quarter more.

2. Whatever the shape, its area is multiplied by the height's factor times the width's factor. Both were multiplied by the same number. So the area is multiplied by that number times itself.

```
3 times 3 = 9
1.5 times 1.5 = 2.25
0.5 times 0.5 = 0.25
```

Nine times, 2.25 times, and a quarter. The last one shrinks the picture. Its area shrinks faster than its height, as it grew faster in the other two.

In the notation of the section on powers, the first is this.

$$3^2 = 9$$

Working backwards, you need the number that gives 16 when it is multiplied by itself. That is the square root of 16.

```
4 times 4 = 16
```

The height was multiplied by 4. A picture four times as tall, kept in shape, covers 16 times the page.

3. List the numbered lines first. They start at 200 and go up in steps of 20: 200, 220, 240, 260, 280 and 300.

The third numbered line above the bottom is 260. That is A.

Halfway between 220 and 240 is half a step above 220.

```
20 divided by 2 = 10
220 plus 10 = 230
```

B is 230.

A quarter of the way from 280 to 300 is a quarter of a step above 280.

```
20 divided by 4 = 5
280 plus 5 = 285
```

C is 285. Now the gap.

```
285 minus 260 = 25
```

C is 25 above A. Notice that you never treated the bottom edge of the chart as zero. The bottom here is 200, and every value came from the printed numbers.

4. Each bar is drawn up from 400 kilocalories.

```
450 minus 400 = 50
600 minus 400 = 200
```

The meal bar is drawn 50 kilocalories long on the axis, and the ration bar 200 kilocalories long.

```
200 divided by 50 = 4
```

The ration bar looks four times as long as the meal bar.

```
600 divided by 450 = 1.3333…
```

Its value is about 1.33 times as large, which is a third more.

Redrawn from zero, the two bars are 450 and 600 kilocalories long, and the longer is 1.33 times the shorter. The slide showed a gap of four times. The Schedule sets a gap of a third.

One thing is the same on both charts. The tops of the bars sit 150 kilocalories apart.

```
600 minus 450 = 150
```

The difference read off the scale was never wrong. Only the lengths were.

5. The scale factor is the ratio of the two values.

```
800 divided by 500 = 1.6
```

The larger packet is 1.6 times as tall and 1.6 times as wide.

```
1.6 times 1.6 = 2.56
```

It covers 2.56 times the page, for 1.6 times the kilocalories.

For the areas to match the values, the area must grow by 1.6. So the scale factor must be the number that gives 1.6 when multiplied by itself. That is the square root of 1.6, from the section on powers. Check it.

```
1.265 times 1.265 = 1.600
```

A scale factor of about 1.265.

Now look at what that does to the height. The larger packet is only 1.265 times as tall as the smaller. A reader who compares heights sees about a quarter more, for a ration with three fifths more energy. No scale factor makes a picture honest to a reader who may compare either its height or its area. Draw bars.

6. The gap between the tops of two bars does not depend on where the axis starts. So use the gap.

```
60 minus 10 = 50
700 minus 450 = 250
```

Fifty millimetres of bar stand for 250 kilocalories.

```
250 divided by 50 = 5
```

One millimetre stands for 5 kilocalories. The unit is kilocalories divided by millimetres, which is kilocalories per millimetre.

Now the short bar. Its 10 millimetres stand for this much.

```
10 times 5 = 50
```

Fifty kilocalories of that bar are showing. Its value is 450, so the bar starts 50 below it.

```
450 minus 50 = 400
```

The side axis starts at 400 kilocalories. Check it with the long bar.

```
60 times 5 = 300
700 minus 300 = 400
```

The same 400. So a chart with no numbers on its side axis still gives its start away, if you know two of its values. Without them, its lengths cannot be read at all.

7. The step that broke is the second, even though its answer is right. Whatever the shape, its area is multiplied by the height's factor times the width's factor, and the working added them. The fourth line is where the break shows.

With 2, adding and multiplying give the same answer.

```
2 plus 2 = 4
2 times 2 = 4
```

With 3 they part.

```
3 times 3 = 9
```

The second picture covers 9 times the area of the original, not 6.

Why it looks reasonable: the first case came out right, and a method that has worked once gets trusted. Two plus two and two times two are the one pair of equal numbers where the error hides.

8. The step that broke is the last. The steepness on the page was read as a rate, without checking what each step across the page stands for.

Read the dates. The first two steps are five years each. The last three are one year each. All five are drawn the same width.

Work out the rate over each step: the change, divided by the years it took.

```
30 minus 20 = 10
10 divided by 5 = 2
40 minus 30 = 10
10 divided by 5 = 2
42 minus 40 = 2
2 divided by 1 = 2
44 minus 42 = 2
2 divided by 1 = 2
46 minus 44 = 2
2 divided by 1 = 2
```

Two a year, at every step. The quantity grew at the same rate throughout. The bend in the line is made entirely by the spacing.

Why it looks reasonable: the third line says the points are evenly spaced, and it reads like a neutral fact. It is the fact that should have stopped the reading, because the dates under those points are not evenly spaced.

9. The step that broke is the last. It compared the heights of bars that do not cover the same span of values.

Read the ranges. Each of the first four covers 20. The last is labelled "over 80", and the largest reading tells you where it really ends.

```
190 minus 80 = 110
110 divided by 20 = 5.5
```

The last bar covers a span of 110, five and a half times the span of each of the others. It is drawn the same width, so nothing on the page shows that.

Put it on the same footing as the others. Work out how many readings it holds for each 20 of its span, on average.

```
18 divided by 5.5 = 3.2727…
```

About 3 readings for each 20. Every one of the first four bars holds more than that: 6, 10, 14 and 12. Averaged over their span, the readings over 80 are the thinnest, not the most common.

That is an average, and the chart cannot say more. All 18 could sit between 80 and 100, and then that stretch would be the busiest of all. The broken step claimed something the drawing cannot show either way.

Why it looks reasonable: the five bars are the same width, so they look like five equal ranges. The label “over 80” reads like a range, and it never says where it stops.

What to check on any chart like this. Read the range printed under every bar. Check that each covers the same span. Where a label is open, such as “over 80”, ask where it ends. Then ask for that range split into steps of the same size as the rest.

10. The step that broke is the fifth. Where two lines cross on a figure with two side axes depends on the axes, not on the quantities.

Read each line at the crossing against its own axis. The crossing is 40 per cent of the way up.

```
0.4 times 100 = 40
0.4 times 20 = 8
```

Line A is at 40 and line B is at 8. They are not equal. They may not even be in the same unit.

Now change only the right axis, so that it runs from 0 to 10. Line B’s value of 8 now sits here.

```
8 divided by 10 = 0.8
```

Eighty per cent of the way up, while line A stays at 40 per cent. The lines no longer cross at year 4, and nothing about either quantity changed.

Why it looks reasonable: on a figure with one side axis, a crossing does mean the two are equal. The rule is sound there, and it has been carried to a figure it does not fit.

11. What to decide first: how the chart’s marks carry their numbers. They are points joined by a line, so they carry them by position. Nothing asks you to compare lengths. On a line chart, an axis starting at 400 is honest when the 400 is printed on the axis and stated in the caption. The colleague has taken the rule for bars and applied it to a line.

What to compute: the size of the change, with its unit, and how much of the chart’s height it fills.

```
470 minus 440 = 30
```

Thirty kilocalories. Six readings a year apart span five years.

```
30 divided by 5 = 6
```

Six kilocalories a year, on average. Now the share of the height.

```
500 minus 400 = 100
30 divided by 100 = 0.3
```

The change fills three tenths of the height. On an axis from zero to 500 it would fill six hundredths.

```
30 divided by 500 = 0.06
```

Now see when the colleague would be right. Draw the first and last values as bars from 400.

```
440 minus 400 = 40
470 minus 400 = 70
70 divided by 40 = 1.75
```

The last bar would look 1.75 times the first, for a rise of under 7 per cent.

```
30 divided by 440 = 0.0681…
```

That chart would be the dishonest one.

What this does not establish. It does not establish that the line gives a fair impression. How steep it looks was still somebody's choice, as the section on graphs showed. Redrawing it from zero, as that section advised, checks the impression. It does not make the first drawing dishonest. And it does not tell you whether 30 kilocalories matters. That is a question about meals, not about drawing.

12. What to compute: the rate in each district, which is the count divided by the population, as in the section on rates. The chart is an honest picture of counts. The press line uses it for a claim about burden, and a count cannot carry that.

Write the populations out. Twelve lakh is 12,00,000 and three lakh is 3,00,000.

```
1800 divided by 1200000 = 0.0015
0.0015 times 1000 = 1.5
```

In P, 1.5 cases for every 1,000 people.

```
900 divided by 300000 = 0.003
0.003 times 1000 = 3
```

In Q, 3 cases for every 1,000 people.

```
3 divided by 1.5 = 2
```

Measured as a rate, the burden is twice as high in Q. The press line has it the wrong way round. P has more cases because it has four times the people.

```
1200000 divided by 300000 = 4
```

What this does not establish. It does not establish that the counts are complete in both districts. A district that finds and records more of its cases shows a higher rate for that reason alone. And it does not say why Q's rate is higher. Before either district is named in public, ask how the cases were found and counted in each.

13. What to compute: whether the note's sentence is true, the change over the stretch the chart shows, and the change over the whole series.

The sentence first. Each of years 7, 8, 9 and 10 is below the year before it. The sentence is true.

The change over the chart's stretch, from year 6 to year 10.

```
7100 minus 7400 = -300
-300 divided by 4 = -75
```

A fall of 300 meals a day across four years, 75 a year on average.

```
300 divided by 7400 = 0.0405…
```

About 4 per cent of the year 6 figure.

The change over the whole series, from year 1 to year 10.

```
7100 minus 5000 = 2100
2100 divided by 5000 = 0.42
```

A rise of 2,100 meals a day, 42 per cent above year 1. Drawn whole, the chart shows a long climb that levels off and dips a little at the end. Drawn from year 6, it shows only the dip.

What this does not establish. The window is not a lie, since the note's sentence is true. The question is what the chart is being used to argue, and which window serves that argument.

It does not establish why meals fell. Meals served is a count. If enrolment fell by more than meals did, about 4 per cent, the meals for each enrolled child went up. So ask for enrolment in each year, and divide.

And it does not establish that the fall will continue. Four small falls after a long rise fit several stories, and these ten numbers cannot choose between them.

What to ask for: the whole series on one chart, and the reason the chart starts at year 6.

F3 · Checking a reference

EXERCISE 1. Three things are missing. There is no locator, so nothing tells a reader where to look in a document of several hundred articles. There is no evidence that anyone opened it. And there is no indication of which consolidated edition was consulted.

The edition matters here specifically. Article 279A was inserted in 2016, so any text predating that will not contain it.

To settle it: get a consolidated copy, check the latest amendment it cites, and search it for Article 279A. Then read clause (1), and check that it says what the draft claims. Record both the locator and which copy you used.

EXERCISE 2. There is no single correct answer, since it depends on the document you chose; the output is the record. Expect at least one to fail at act three or four.

Three patterns are common. A page number that does not contain the claim. A review cited for a finding that belongs to a single study. A source that supports a weaker statement than the one it is attached to.

F4 · Argument: premise, inference, conclusion

EXERCISE 1. The premise: countries where people eat more packaged food show more obesity. The conclusion: packaged food causes obesity.

The inference does not hold. A pattern across countries can come from something that moves both, such as income, or more people living in cities. And a pattern in country averages need not hold for people.

To repair it you would need two more premises. One ruling out a common cause. One connecting the country pattern to people. The conclusion may still be true; what has failed is this route to it.

EXERCISE 2. The spine of the explanation. A wrong fact can be replaced, and the argument survives, so attacking facts invites an endless supply of new ones. A broken inference means no fact of that kind could establish the conclusion. That ends the line rather than extending it.

Demonstrate it. Take a claim, grant every premise aloud, and show that the conclusion still does not follow. The granting is what makes it land.

F5 · How government works, in the shape you need to read a regulation

EXERCISE 1. The Constitution is changed by Parliament, by a special majority. For some parts the states must agree as well. A statute is changed by Parliament, or by a State Legislature. A rule or a regulation is changed by the Central Government, or by the body the statute gave the power to. A notification is issued by whoever the statute gave that power to, and published in the Gazette.

The ordering is useful because it predicts how quickly something can change, and who would have to be persuaded.

EXERCISE 2. Which government is it — the Union or a State? The Seventh Schedule splits the subject, so this has to be settled first.

Which instrument — was anything notified in the Gazette, and under which statutory power?

And who is bound? A binding rule names who commits a breach and what follows. An advisory names nobody and carries no consequence.

A report that answers none of these has not established that anything legally happened.

EXERCISE 3. The label requirement is a regulation, not a ministerial decision. It was written by the body an Act of Parliament created for that purpose. It used a power that Act granted.

Changing it means that body publishing a draft amendment and getting the Central Government's approval. It also means notifying it in the Gazette and laying it before Parliament. The Minister can push, and can be blamed, but cannot sign it away.

So the accurate story is not "Minister refuses". It is "here is how long this process takes, and here is where it has reached".

Glossary

Every term of art this book teaches, in the plain words it gets where it is first used, and the section that teaches it.

Term	Plain words	Section
absorption	a molecule passing through the wall of the gut and into the blood	E6
accuracy	how close a measured value comes to the true value	D7
Act	the name each statute passed by Parliament or a State Legislature is called by	F5
adipose tissue	the tissue whose cells hold triglyceride until it is needed	E6
ADP	what is left when ATP's last phosphate group is knocked off: the same molecule, one phosphate short	E6
alimentary canal	the gut tube	E8
allele	one of the versions a gene comes in	E7
alpha cell	the islet cell that makes glucagon	E8
amendment	a later change stitched into the law	F3
amino acid	one of a small fixed set of kinds of link a protein chain is built from	E5
amount-of-substance concentration	an amount of substance in a volume, such as millimoles per litre (mmol/L): it says how many	B5
anabolism	reactions that join small molecules into bigger ones, which costs energy	E6
argument	reasons, called premises, together with a further statement they are meant to establish	F4
at random	every member has exactly the same chance of being the one you pick	D1
atom	a tiny heavy centre with much lighter electrons around it	E1
ATP	adenosine triphosphate, the small molecule a cell uses to carry energy from one reaction to another	E5
Atwater general factors	one fixed energy value for each of protein, fat and carbohydrate, used whatever the food	E2
average rate of change	the change between the two ends of an interval divided by its length; it uses nothing in between	C7
base	(1) of a power, the number being multiplied, as the ten in 10^3 (A6); (2) in DNA, one of the four units, A, G, C and T, whose order along DNA is the information (E7)	A6, E7
base ten	the counting system in which each place is worth ten times the place to its right	A1
base unit	one of the seven units the SI builds on: the second, metre, kilogram, ampere, kelvin, mole and candela	B1

Term	Plain words	Section
beta cell	the islet cell that makes insulin	E8
bile	what the liver makes to break a large blob of fat into many tiny ones	E8
bin	a range of values grouped together into one bar	F2
blood sugar (blood glucose)	the glucose in the blood	E8
body mass index (BMI)	a person's mass in kilograms divided by the square of their height in metres, in kg/m^2	B6
bomb calorimeter	a sealed steel pot in a water tank, used to burn a weighed sample	E2
bond	an attraction between two atoms that holds them together	E1
boundary	the closed line you draw, with the system inside it and everything else outside	E3
calibration	measuring a machine against something already known, so its readings mean something	E2
calibration weight	a weight already checked and trusted to be correct	D7
calorie	a unit of energy outside the SI, whose size in joules was fixed by agreement, and two agreements are in use	B4
calorimeter	a machine that measures energy from the temperature change it causes	E2
capillaries	the smallest blood vessels	E8
catabolism	reactions that break a big molecule into smaller ones and let its energy go	E6
ceiling	(1) the level a saturating shape heads for and flattens against (C6); (2) of a test's scale, the top score, which no score passes (S57-R1-C12)	C6
cell	(1) of a table: where a row meets a column, the count of people who are both (D3); (2) a living cell: the smallest unit of a living thing (E5)	D3, E5
checking a reference	four acts, each of which can fail on its own: find the source, confirm it is the source named, find the exact passage pointed to, read it to see whether it supports the claim	F3
chord	the straight line laid across two points on a curve, whose slope is the average rate between them	C7
chromosome	one of the structures genes sit along, in a fixed order; a person gets two copies of each, one from each parent	E7
combustion	burning, a substance joining with oxygen	E2
complement	every outcome in the sample space that is not in the event; everything that is not it	D1
complementary	each one completes the other; A always faces T and C always faces G	E7
concentration	an amount of something held in a volume of something else	B5

Term	Plain words	Section
conclusion	the claim the premises are offered for	F4
conduction	heat moving through matter that is staying put, by contact	E4
conservation of energy	energy is never made and never destroyed; it changes form and it moves	E3
Constitution	the founding law that every statute must fit	F5
convection	heat moving because a liquid or a gas moves and takes it along	E4
convenience sample	whoever happens to be available	D6
conversion factor	a fraction whose top and bottom are the same quantity written in two different units, so it is equal to one	B2
correction	subtracting a known systematic error, such as a zero error, from the reading (VIM 2.53)	D7
covalent bond	a bond where two atoms share a pair of electrons	E1
crore	ten million, or a hundred lakh	A1
cytoplasm	the watery jelly a cell is a bag of	E5
decimal	place value carried on past the point: tenths, then hundredths, then thousandths	A3
delta	the standard way of writing "the change in", using the Greek letter written as a triangle	E4
denominator	(1) the number below the line, saying how many equal parts the whole was cut into (A2); (2) of a percentage, what it is a percentage of (F1)	A2, F1
derivative	the mathematicians' name for the rate at a moment, written dy/dx	C7
derived unit	a unit built out of the base units by multiplying and dividing	B1
deviation	a value minus the mean	D5
dimension	the kind of thing a quantity is: a length, a mass, a time, or a combination of these	B3
diploid	holding two matching copies of each chromosome, one inherited from each parent	E7
distribution	the set of values a quantity takes, across the people, occasions or measurements counted, together with how often each value turns up	D4
DNA	the long molecule a cell's instructions are written in	E7
domain	the inputs a rule will accept	C4
dominant	an allele that shows whenever it is present	E7
electron	one of the much lighter particles in the cloud round an atom's tiny heavy centre	E1
elimination	adding or subtracting the two equations, after multiplying where needed, so that one letter cancels	C3

Term	Plain words	Section
endocrine	putting a chemical straight into the blood rather than sending it down a tube	E8
endothermic	a change that takes heat in	E1
energy	the capacity to supply heat or do work	E1
energy balance	the energy inside the boundary at the end equals the energy inside at the start, plus everything that crossed in, minus everything that crossed out	E3
energy equivalent	how many joules one degree is worth in one particular calorimeter	E2
enzyme	a protein that makes one particular reaction go faster	E5
equation	a statement that two expressions are the same number	C1
error	what a measurement says minus what is actually true, or minus a trusted reference value (VIM 2.16: "measured quantity value minus a reference quantity value")	D7
estimate	a number made from part of the group and used to stand in for the whole	D6
event	any collection of outcomes from the sample space	D1
executive	the day-to-day running of government	F5
exocrine	sending a secretion down a tube to where it is needed	E8
exothermic	a change that gives heat out	E1
expanding a bracket	multiplying every term inside it by what stands outside	C1
exponent	the small raised number that says how many copies to multiply together	A6
exponential	equal steps in the input multiply the output by the same factor every time	C6
expression	a piece of algebra that stands for a number, like 2x + 5	C1
extracellular fluid	the fluid around a gland's own cells, into which it lets its hormone out	E8
flow	a rate that changes a stock, with a time on the bottom of its unit	C9
fraction	a division that has not yet been carried out; a sum you have not done yet	A2
free energy	loose energy, of which a cell cannot hold a useful amount	E6
frequency table	lists each value, or each band of values, against the count of times it occurred	D4
function	a rule that gives every input it accepts exactly one output; a machine with one slot in and one slot out	C4
gallbladder	a small bag next to the liver that holds bile, concentrates it and lets it out	E8
Gazette	the government's official newspaper	F5
gene	what fixes the order of the amino acids in one protein	E7

Term	Plain words	Section
general circulation	the blood that goes round the whole body	E8
genotype	what a person actually carries in their DNA, including the alleles that are not showing	E7
gland	an organ whose job is to make something and let it out	E8
glucagon	the hormone released when blood sugar is low, which empties the stores	E8
glucose	the single sugar a body reaches for first as fuel	E6
glycogen	a great many glucose molecules strung together, kept in the liver and the muscles	E6
grand total	what you get when you add up everything in the table	D3
graph	a drawing of a relationship between two quantities, made of points	C5
gross energy	all the energy a food gives up when it is burnt to nothing, also called ingested energy	E2
heat	energy moving, because one thing is hotter than another	E4
heat capacity	the joules one particular object needs to go up by one degree	E4
hepatic portal circulation	the route that takes every drop of blood leaving the gut through the liver before it goes anywhere else	E8
hepatic portal vein	the vein along which blood from the gut runs straight to the liver	E8
hormone	a chemical made in one part of the body that affects other parts some distance away	E8
hydrophilic	water-loving: mixes with water	E5
hydrophobic	water-hating: does not mix with water	E5
independent	finding out one of them happened tells you nothing new about the other	D2
inference	the move from the reasons to the conclusion	F4
infographic	a chart decorated with pictures	F2
instantaneous rate	how fast the quantity is changing at one moment: the steepness of the curve at that single point	C7
instrument	the word lawyers use for any one of the Constitution, a statute, a rule or regulation, or a notification	F5
insulin	the hormone released when blood sugar is high, which fills the stores	E8
integral	the total under a rate line, written with the sign $\int$	C8
intercept	the height of the line where the x quantity is zero	C5
International Table calorie	4.1868 joules, settled in 1956	B4
interquartile range	the third quartile minus the first	D5
ion	an atom left carrying a charge	E1

Term	Plain words	Section
ionic bond	a bond where one atom hands an electron to another and the opposite charges pull	E1
islets (pancreatic islets)	the small islands of another kind of cell scattered through the pancreas, which put chemicals straight into the blood	E8
joule	the SI unit of energy, written J	B1
kilocalorie	a thousand calories, the unit used for the energy in food, where it is often written just calories	B4
kilojoule	a thousand joules; kilo means a thousand	B4
kinetic energy	energy a thing has because it is moving	E1
lacteals	the lymph vessels of the villi, which take in what dissolves in fat	E8
lakh	a hundred thousand	A1
large store	fat: triglyceride, held in adipose tissue	E6
legislature	the body that makes law: Parliament for the country, a State Legislature for its own state	F5
ligand	the molecule that fits into a protein shaped to hold it	E5
linear	equal steps in the input add the same amount to the output every time	C6
lipolysis	the breakdown of stored triglycerides into free fatty acids and glycerol	E8
logarithm	the power you have to raise ten to, to get the number	A7
logarithmic axis	an axis on which equal distances stand for equal multiplications, not equal additions	A7
long tail	the side of a distribution that thins out slowly	D4
mark	anything drawn that stands for one number: a bar, a point, a slice, a picture	F2
mass concentration	a weight in a volume, such as milligrams per decilitre (mg/dL): it says how heavy	B5
mean	the sum of a set of values divided by how many there are, which everyday speech calls the average	D5
measurement	a number together with a unit	B1
measurement bias	an error that points one way and does not shrink when you collect more (VIM 2.18: an "estimate of a systematic measurement error")	D7
median	the middle value of the set once it is ordered from smallest to largest; with an even count, the mean of the two middle values	D5
metabolism	the name for every chemical reaction in a body added together	E6
metabolizable energy	what is left of a food's energy after the losses in faeces, gas and urine	E2
millimole	one thousandth of a mole	B5
mitochondria	the parts of a cell that make ATP	E5

Term	Plain words	Section
molar mass	how heavy one mole of a substance is, in grams per mole (g/mol); it belongs to the substance	B5
mole	a fixed count of things, molecules here	B5
molecule	a group of atoms held together by bonds	E1
monosaccharide	a single sugar molecule, which is what breaking a starch gives you	E6
mutually exclusive	they cannot both happen	D2
negative number	a number less than zero, as far below zero as the matching positive number is above it, written with a minus sign in front	A1
negative predictive value	the share of negative results that are right	D3
net flow	the inflow minus the outflow	C9
net metabolizable energy	metabolizable energy with the energy the body itself spends on digesting the food and handling it taken off	E2
notification	an order issued under a power a statute gives, and published in the Gazette	F5
nucleotide	the repeating unit DNA is built from: a base, a sugar and a phosphate group	E7
nucleus	the compartment that holds the cell's instructions, written in its DNA	E5
numerator	the number above the line, saying how many of the parts are being counted	A2
odds	how many times more likely the event is than its opposite: p divided by (1 minus p)	D1
order of magnitude	which power of ten a number is nearest to	A8
order-of-magnitude estimate	every input rounded to one digit and a power of ten, the answer claimed only to the nearest power of ten; it tests a figure rather than producing one	A8
origin	the point where the two axes of a graph cross	C5
outcome	one member of the sample space	D1
parameter	a number that describes the population, if every member of it could be measured	D6
percentage	a fraction whose denominator is one hundred; per cent means per hundred	A4
percentage change	the change divided by the value it started from, times one hundred	A4
percentage point	the plain difference between two percentages, as against per cent, which divides	A4
phenotype	what you can see about a person, the traits they show	E7
phospholipid	the one kind of molecule a membrane is built from, with one end that mixes with water and one that does not	E5

Term	Plain words	Section
pie chart	a circle standing for a whole, each slice's share of the circle being its share of the total	F2
place value	what a digit is worth because of where it sits	A1
plasma membrane	the skin round a cell, which decides what gets in and what stays out	E5
polysaccharide	a chain of single sugar molecules, such as starch	E6
population	the whole group a question is about	D5
positive predictive value	the share of positive results that are right, written P(condition present given test positive)	D3
potential energy	energy a thing has because of where it is, what it is made of, or what state it is in	E1
power	(1) repeated multiplication of one number by itself, as 10^3 is ten times ten times ten (A6); (2) over people, the capacity to get things done through other people, from any source, authority being only one (S57-R1-C16)	A6
precision	how closely repeated readings agree with each other	D7
premise	a reason offered	F4
prevalence	how many people in a group have the condition, written P(condition present), with no test involved	D3
probability	a number from 0 to 1 that states how often an outcome would turn up if the same chance process were repeated a great many times	D1
proportion	a part out of the whole it came from, never more than one	A5
protein	a folded chain of amino acids, whose shape is what it does	E5
quartile	the first quartile is the median of the lower half of the ordered values, the third the median of the upper half, the second the median of the whole set	D5
radiation	heat crossing as light or as the invisible light below red, with nothing needed in between	E4
random error	the part that varies unpredictably from one reading to the next (VIM 2.19)	D7
random sample	picked at random: every member of the population has exactly the same chance of being the one picked, and an actual chance process decides it, not the person picking	D6
range	(1) of a rule, the outputs it can give back (C4); (2) of a set of values, the largest minus the smallest (D4)	C4, D4
rate	a count divided by the time, or the number of people, it was counted over	A5
rate line	the line the rate over each interval makes across a figure with time along the bottom	C8
rate of change	a change divided by the time it took	C7

Term	Plain words	Section
ratio	two quantities of the same kind set side by side by dividing one by the other	A5
receptor	the protein a ligand fits into, which changes shape when it does	E5
recessive	an allele that shows only when both copies are that version	E7
reference	a pointer from a claim to the place that is supposed to support it	F3
regulation	what a body the statute set up may make, not Parliament; it binds because a statute said it could	F5
relative frequency	a count divided by the total number counted; the proportion, under a longer name	D4
resolution	the smallest step an instrument's display can move by (VIM 4.14: the "smallest change in a quantity being measured that causes a perceptible change in the corresponding indication")	D7
ribosome	the machine in a cell that builds proteins	E5
rise	the change up, between two points on a line	C5
rounding	choosing which place to stop at, and adjusting the last kept digit for what follows it	A3
rule	what the Central Government may make; like a regulation, it binds because a statute said it could	F5
run	the change across, between two points on a line	C5
running total	the amounts for successive intervals, each a rate times a time, added up as you go	C8
sample	the part of that group that was actually measured	D5
sample space	the complete list of everything that could happen	D1
sampling variation	a second random sample of the same size from the same population gives a statistic that does not match the first sample's exactly	D6
saturating	rises, then flattens against a level it does not pass	C6
scale	how much of the quantity one step along an axis stands for, chosen by whoever draws the figure	C5
scale factor	the number you multiply every length of a picture by	F2
scientific notation	a number between one and ten, times a power of ten	A6
sensitivity	the share of people with the condition that the test catches, written P(test positive given condition present)	D3
shape	where a distribution's values sit relative to each other	D4
SI	the international system of units, the one the world agreed to measure in	B1
signal transduction	a signal outside the cell converted into a signal inside it	E5
significant figures	the digits in a number that carry information about the quantity	A3
slope	how steep a line is: the change up divided by the change across	C5

Term	Plain words	Section
small store	glycogen, kept in the liver and the muscles	E6
solution (of a system)	the pair of values that satisfies both equations, not one	C3
solving for a letter	reaching an equation with that letter alone on one side and nowhere on the other	C2
special majority	a majority of all the members of a House, and two-thirds of those present and voting	F5
specific heat	the joules one kilogram of a material needs to go up by one degree	E4
specificity	the share of people without the condition that the test correctly clears, written P(test negative given condition absent)	D3
square root	the number which, multiplied by itself, gives you the one you started with	A6
square-root law	the spread of a sample's average equals the spread of the individual values in the population, divided by the square root of the sample size	D6
squaring	multiplying a number by itself	A6
standard deviation	the square root of the variance, in the same unit as the original values	D5
standard error of the mean	that spread of a random sample's average, over many samples of the same size	D6
starch	the stuff rice, wheat and potato are mostly made of	E6
statistic	the same kind of number, computed instead from a sample	D6
statute	a law enacted by Parliament or a State Legislature, each called an Act	F5
stock	a quantity that exists at a moment and can be measured at that moment, such as the water in a tank	C9
subject (of a formula)	the one letter got on its own; getting it there is making it the subject	C2
substitution	(1) putting a number in place of a letter, everywhere it appears (C1); (2) to solve two equations, rearranging one to give a letter on its own and putting that into the other (C3)	C1, C3
surroundings	everything outside the line you drew	E3
system	whatever you have drawn a line round, for the purpose of counting something	E3
system of two equations	a pair of equations that have to hold at the same time	C3
systematic error	pushes every reading the same way, by close to the same amount, every time	D7
table	values set out in a grid, each one identified by the row and the column it sits in	F1
tangent	the straight line that touches a curve at one point and runs along it, whose slope is the rate at that point	C7
temperature	how hot a thing is - a level, not an amount	E4

Term	Plain words	Section
thermochemical calorie	4.184 joules exactly	B4
triglyceride	the molecule fat is made of, in food and in a body alike	E6
trueness	how close the average of many repeated readings sits to the true value	D7
two-way table	a table that sorts the same people twice, one way down the columns and the other way across the rows	D3
unit	an agreed amount that everybody measures against	B1
unit of observation	what one row of a table stands for	F1
value axis	the axis a bar's length is read against: the side axis when bars stand up, the bottom axis when they lie flat	F2
variable	a letter written in place of a number	C1
variance	worked out from the squared deviations: add them, then divide by the count for a whole population, or by one less than the count for a sample	D5
villi	the tiny fingers lining the gut	E8
window	the stretch of time a chart over time shows, starting and ending where somebody chose	F2
work	a force moving something through a distance; lifting a weight is work	E1
x-axis	the flat line along the bottom of a graph	C5
y-axis	the upright line up the side of a graph	C5
zero error	an instrument reading something other than zero where it should read zero, as a thermometer in melting ice reading 0.3 °C	D7

The series: all 196 books

In build order: Book 0 first, then every Introductory rung, then every Intermediate rung, and so on; within a level, a subject comes after the subjects it depends on. "done" means that book's version 1 is finished. This is Book 0.

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OBESITY EXPERTISE · BOOK 0

Ground floor

Forty-three short sections that take you from place value to reading a statute: the arithmetic, units, graphs, probability, measurement, energy and biology that every later book in the series takes for granted. Every claim is checked against a source you can open. Every worked answer is recomputed. Every section ends with exercises and practice problems you can do.

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Dr. Harsh Maheshwari
MD Community Medicine Resident, AIIMS Raipur
contact@drharshmaheshwari.com
ORCID 0000-0001-8848-1918
drharshmaheshwari.com
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